

# THE ANNALS *of* PROBABILITY

AN OFFICIAL JOURNAL OF  
THE INSTITUTE OF MATHEMATICAL STATISTICS

## Articles

Excursions of stationary Gaussian processes above high moving barriers . . . . .	SIMEON M. BERMAN	365
High level occupation times for continuous Gaussian processes . . . . .	NORMAN A. MARLOW	388
A convergence theorem for extreme values from Gaussian sequences . . . . .	ROY E. WELSCH	398
The maximal oscillation problem for regenerative phenomena . . . . .	DAVID GRIFFEATH	405
On a class of secretary problems . . . . .	ANTHONY G. MUCCI	417
Invariance principles for the law of the iterated logarithm for martingales and processes with stationary increments . . . . .	C. C. HEYDE AND D. J. SCOTT	428
Boundary crossing probabilities associated with Motoo's law of the iterated logarithm . . . . .	MICHAEL J. WICHURA	437
Inequalities for branching processes . . . . .	BRUCE W. TURNBULL	457

## Short Communications

Inequalities for multitype branching processes . . . . .	BRUCE W. TURNBULL	475
A Poisson-type limit theorem for mixing sequences of dependent 'rare' events . . . . .	R. M. MEYER	480
A note on Fine's axioms for qualitative probability . . . . .	FRED S. ROBERTS	484
An almost sure invariance principle for multivariate Kolmogorov-Smirnov statistics . . . . .	PRANAB KUMAR SEN	488
On an $L_p$ version of the Berry-Essén theorem for independent and $m$ -dependent variables . . . . .	R. V. ERICKSON	497
A note on the rate of convergence and its applications . . . . .	RASUL A. KHAN	504

*Continued on back cover*

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## Articles

- On some analogues to linear combinations of order statistics in the linear model.....P. J. BICKEL
- Log-linear models for frequency data: sufficient statistics and likelihood equations.....SHELBY J. HABERMAN
- Open-ended tests for Koopman-Darmois families.....GARY LORDEN
- Some limit theorems with applications in sampling theory.....LARS HOLST
- Optimal stopping and sequential tests which minimize the maximum expected sample size.....TZE LEUNG LAI
- Events which are almost independent  
R. L. DYKSTRA, JOHN E. HEWETT AND W. A. THOMPSON, JR.
- Application of the theory of products of problems to certain patterned covariance matrices.....STEVEN F. ARNOLD
- On some test criteria for covariance matrix.....HISAO NAGAO
- The non-singularity of generalized sample covariance matrices.....MORRIS L. EATON AND MICHAEL D. PERLMAN
- Asymptotic non-null distributions of the likelihood ratio criteria for covariance matrix under local alternatives.....NARIAKI SUGIURA
- Hiding and covering in a compact metric space  
ROBERT J. McELIECE AND EDWARD C. POSNER

## Short Communications

- An extension of the Hájek-Rényi inequality for one maximum of partial sums.....DOMINIK SZYNAL
- On a theorem of Bahadur on the rate of convergence of point estimators.....JAMES C. FU
- On a multiple decision rule.....KHURSHEED ALAM
- On some properties of Hammersley's estimator of an integer mean.....RASUL A. KAHN
- Distinctness of the eigenvalues of a quadratic form in a multivariate sample.....MASASHI OKAMOTO
- An optimality property of Scheffé bounds.....ROBERT BOHRER
- Nonparametric selection procedures for symmetric location parameter populations.....MALAY GHOSH
- Estimation of the covariance function of a homogeneous process on the sphere.....ROCH ROY
- Constructions for some classes of neighbor designs  
F. K. HWANG
- On the notion of value for games with infinitely many stages.....SHMUEL ZAMIR

## Note

- Correction to "A moment problem for order statistics".....JOSEPH B. KADANE

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