

THE ANNALS of PROBABILITY

AN OFFICIAL JOURNAL OF
THE INSTITUTE OF MATHEMATICAL STATISTICS

Special Invited Paper

Gaussian and their subordinated self-similar random generalized fields	R. L. DOBRUSHIN	1
--	-----------------	---

Articles

Approximation theorems for independent and weakly dependent random vectors	ISTVÁN BERKES AND WALTER PHILIPP	29
An improvement of Strassen's invariance principle	P. MAJOR	55
Envelopes of vector random processes and their crossing rates	DANIELE VENEZIANO	62
Some stability results for vector valued random variables	J. KUELS AND JOEL ZINN	75
Laws of large numbers for $D[0,1]$	PETER Z. DAFFER AND ROBERT L. TAYLOR	85
Sign changes of the difference of convex functions and their application to large deviation rates	JAMES LYNCH	96
A strong law for variables indexed by a partially ordered set with applications to isotone regression	F. T. WRIGHT	109
Lower bounds for nonlinear prediction error in moving average processes	MAREK KANTER	128

Short Communications

Pointwise ergodicity of the basic contact process	DAVID GRIFFEATH	139
A relation between Brownian bridge and Brownian excursion	WIM VERVAAT	143
Laws of large numbers for tight random elements in normed linear spaces	R. L. TAYLOR AND DUAN WEI	150
A strong law of large numbers of subsequences of random elements in separable Banach spaces	A. BOZORGNIA AND M. BHASKARA RAO	156
Continuous versions of regular conditional distributions	SANDY ZABELL	159
A functional relationship between the different r -means for indicator functions	D. LANDERS AND L. ROGGE	166
Some remarks on a mixing condition	M. ROSENBLATT	170
Maximum in the Lévy-Baxter theorem for Gaussian random fields	TAKAYUKI KAWADA	173
Conditional expectation and ordering	EP DE JONGE	179
An inequality in p -functions	DAVID MOUNTFORD	184
An inverse balayage problem for Brownian motion	A. F. KARR AND A. O. PITTINGER	186

Vol. 7, No. 1—February 1979

THE INSTITUTE OF MATHEMATICAL STATISTICS

(Organized September 12, 1935)

The purpose of the Institute of Mathematical Statistics is to encourage the development, dissemination, and application of mathematical statistics.

OFFICERS

President:

Samuel Karlin, Department of Mathematics, Stanford University, Stanford, California 94305

President-Elect:

George E. P. Box, Department of Statistics, University of Wisconsin, 1210 W. Dayton St., Madison, Wisconsin 53706

Past-President:

Elizabeth Scott, Department of Statistics, University of California, Berkeley, California 94720

Executive Secretary:

Martin Fox, Department of Statistics and Probability, Michigan State University, East Lansing, Michigan 48824

Treasurer:

Heebok Park, Department of Statistics, California State University, Hayward, California 94542

IMS Business Office, 3401, Investment Blvd., Suite 6, Hayward, California 94545

Program Secretary:

William L. Harkness, Pennsylvania State University, University Park, Pennsylvania 16801

Editor: *Annals of Statistics*

Rupert G. Miller, Jr., Department of Statistics, Stanford University, Stanford, California 94305

Editor: *Annals of Probability*

Richard M. Dudley, Room 2-245, Massachusetts Institute of Technology, Cambridge, MA 02139

Managing Editor:

Donald R. Truax, Department of Mathematics, University of Oregon, Eugene, Oregon 94703.

Membership. Membership dues including a subscription to one *Annals* and *The Institute of Mathematical Statistics Bulletin* are \$27.00 per year for residents of the United States or Canada and \$20.00 per year for residents of other countries. Special rates of \$15.00 per year are available to students. Rates in each category are one-third higher for members who wish both *Annals* as well as the *Bulletin*. Inquiries regarding membership in the institute should be sent to the Treasurer at the business office.

Subscription Rates. Current volumes (six issues per calendar year) of the *Annals of Probability* and the *Annals of Statistics* are each \$37.00. Members of the Institute of Mathematical Statistics pay different rates (see above). Single issues are \$8.00. Back numbers of both *Annals* and the *Annals of Mathematical Statistics* (Volumes 1 through 43) may be purchased from the Treasurer.

The *Annals of Probability*, Volume 7, Number 1, February 1979. Published bimonthly in February, April, June, August, October, and December by The Institute of Mathematical Statistics, IMS Business Office, 3401 Investment Blvd., Suite 6, Hayward, California 94545.

Mail to the *Annals of Probability* should be addressed to either the Editor, Managing Editor or the Treasurer, as described above. It should *not* be addressed to Waverly Press.

Printed at the
WAVERLY PRESS, INC., BALTIMORE, MARYLAND 21202 U.S.A.

Second-class postage at Hayward, California and at additional mailing offices

Copyright © 1979 by the Institute of Mathematical Statistics

EDITORIAL STAFF

EDITOR

R. M. DUDLEY

ASSOCIATE EDITORS

RICHARD E. BARLOW
SIMEON M. BERMAN
RABI BHATTACHARYA
C. C. HEYDE
NARESH C. JAIN

MARK KAC
HARRY KESTEN
THOMAS G. KURTZ
M. R. LEADBETTER
THOMAS LIGGETT

WALTER PHILIPP
DAVID O. SIEGMUND
HOWARD TAYLOR
JOHN B. WALSH

MANAGING EDITOR

D. R. TRUAX

NOTICE

Manuscripts submitted for the *Annals of Probability* should be sent to the Editor at the following address:

R. M. DUDLEY
Department of Mathematics
Room 2-245
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

PAST EDITORS
ANNALS OF MATHEMATICAL STATISTICS

H. C. CARVER, 1930-1938
S. S. WILKS, 1938-1949
T. W. ANDERSON, 1950-1952
E. L. LEHMANN, 1953-1955
T. E. HARRIS, 1955-1958

WILLIAM KRUSKAL, 1958-1961
J. L. HODGES, JR., 1961-1964
D. L. BURKHOLDER, 1964-1967
Z. W. Birnbaum, 1967-1970
INGRAM OLKIN, 1970-1972

ANNALS OF PROBABILITY
RONALD PYKE, 1973-1975
PATRICK BILLINGSLEY, 1976-1978

ANNALS OF STATISTICS
INGRAM OLEIN, 1972-1973
I. R. SAVAGE, 1974-1976

EDITORIAL POLICY

The main purpose of the *Annals of Probability* and the *Annals of Statistics* is to publish contributions to the theory of probability and statistics and to their applications. The emphasis is on importance and interest, not formal novelty and correctness. Especially appropriate are authoritative expository papers and surveys of areas in vigorous development. All papers are refereed.

IMS INSTITUTIONAL MEMBERS

- AEROSPACE CORPORATION**
El Segundo, California
- ARIZONA STATE UNIVERSITY**
Tempe, Arizona
- BELL TELEPHONE LABORATORIES, TECHNICAL LIBRARY**
Murray Hill, N.J.
- BOWLING GREEN STATE UNIVERSITY, DEPT. OF MATHEMATICS**
Bowling Green, Ohio
- CALIFORNIA STATE UNIVERSITY, FULLERTON, DEPARTMENT OF MATHEMATICS**
Fullerton, California
- CALIFORNIA STATE UNIVERSITY, HAYWARD, DEPARTMENT OF STATISTICS**
Hayward, California
- CASE WESTERN RESERVE UNIVERSITY, DEPARTMENT OF Mathematics**
Cleveland, Ohio
- CORNELL UNIVERSITY, DEPARTMENT OF MATHEMATICS**
Ithaca, New York
- FLORIDA STATE UNIVERSITY, DEPARTMENT OF STATISTICS**
Tallahassee, Florida
- FORD MOTOR COMPANY, ENGINEERING AND RESEARCH LIBRARY**
Dearborn, Michigan
- GENERAL MOTORS CORPORATION, RESEARCH LABORATORIES**
Warren, Michigan
- GEORGE WASHINGTON UNIVERSITY, DEPARTMENT OF STATISTICS**
Washington, D.C.
- INDIANA UNIVERSITY, MATHEMATICS DEPT.**
Bloomington, Indiana
- INSTITUTE FOR DEFENSE ANALYSES**
Arlington, Virginia
- INTERNATIONAL BUSINESS MACHINES CORPORATION**
Armonk, New York
- IOWA STATE UNIVERSITY, STATISTICAL LABORATORY**
Ames, Iowa
- JOHNS HOPKINS UNIVERSITY, DEPARTMENT OF BIostatistics, DEPARTMENT OF MATHEMATICAL SCIENCES**
Baltimore, Maryland
- KANSAS STATE UNIVERSITY, STATISTICS DEPARTMENT**
Manhattan, Kansas
- LOUISIANA STATE UNIVERSITY, DEPARTMENT OF EXPERIMENTAL STATISTICS**
Baton Rouge, Louisiana
- MARQUETTE UNIVERSITY, MATHEMATICS AND STATISTICS DEPARTMENT**
Milwaukee, Wisconsin
- MASSACHUSETTS INSTITUTE OF TECHNOLOGY MATHEMATICS DEPARTMENT**
Cambridge, Massachusetts
- MIAMI UNIVERSITY, DEPARTMENT OF MATHEMATICS**
Oxford, Ohio
- MICHIGAN STATE UNIVERSITY, DEPARTMENT OF STATISTICS AND PROBABILITY**
East Lansing, Michigan
- NATIONAL SECURITY AGENCY**
Fort George G. Meade, Maryland
- NEW MEXICO STATE UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES**
Las Cruces, New Mexico
- NORTHERN ILLINOIS UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES**
De Kalb, Illinois
- NORTHWESTERN UNIVERSITY, DEPARTMENT OF MATHEMATICS**
Evanston, Illinois
- OHIO STATE UNIVERSITY, DEPARTMENT OF STATISTICS**
Columbus, Ohio
- PENNSYLVANIA STATE UNIVERSITY, DEPARTMENT OF STATISTICS**
University Park, Pennsylvania
- PRINCETON UNIVERSITY, DEPARTMENT OF STATISTICS**
Princeton, New Jersey
- PURDUE UNIVERSITY LIBRARIES**
Lafayette, Indiana
- QUEEN'S UNIVERSITY, DEPT. OF MATHEMATICS, KINGSTON**
Ontario, Canada
- RICE UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES**
Houston, Texas
- THE ROCKEFELLER UNIVERSITY**
New York, New York
- SANDIA CORPORATION, SANDIA BASE**
Albuquerque, New Mexico
- SIMON FRASER UNIVERSITY, MATHEMATICS DEPARTMENT**
Burnaby, Canada
- SOUTHERN ILLINOIS UNIVERSITY, MATHEMATICAL STUDIES**
Edwardsville, Illinois
- SOUTHERN METHODIST UNIVERSITY, DEPARTMENT OF STATISTICS**
Dallas, Texas
- STANFORD UNIVERSITY, GIRSHICK MEMORIAL LIBRARY**
Stanford, California
- STATE UNIVERSITY OF NEW YORK, BUFFALO, DEPARTMENT OF STATISTICS**
Amherst, New York

- TEMPLE UNIVERSITY, MATHEMATICS DEPARTMENT
Philadelphia, Pennsylvania
- TEXAS A & M UNIVERSITY, DEPT. OF MATHEMATICS
College Station, Texas
- TEXAS TECH UNIVERSITY, DEPARTMENT OF MATHEMATICS
Lubbock, Texas
- THE TOBACCO INSTITUTE
Washington, D.C.
- UNION OIL COMPANY OF CALIFORNIA, UNION RESEARCH CENTER
Brea, California
- UNITED STATES ARMY RESEARCH AND DEVELOPMENT CENTER
Aberdeen Proving Ground, Maryland
- UNIVERSITY OF ALBERTA, DEPARTMENT OF MATHEMATICS
Edmonton, Alberta, Canada
- UNIVERSITY OF ARIZONA, DEPARTMENT OF MATHEMATICS
Tucson, Arizona
- UNIVERSITY OF BRITISH COLUMBIA, DEPARTMENT OF MATHEMATICS
Vancouver, B.C., Canada
- UNIVERSITY OF CALGARY, MATHEMATICS DEPARTMENT
Calgary, Alberta, Canada T2N 1N4
- UNIVERSITY OF CALIFORNIA, BERKELEY, STATISTICAL LABORATORY
Berkeley, California
- UNIVERSITY OF CINCINNATI, DEPARTMENT OF MATHEMATICAL SCIENCES
Cincinnati, Ohio
- UNIVERSITY OF GUELPH, MATHEMATICS AND STATISTICS DEPARTMENT
Guelph, Ontario, Canada
- UNIVERSITY OF ILLINOIS AT CHICAGO CIRCLE, DEPARTMENT OF MATHEMATICS
Chicago, Illinois
- UNIVERSITY OF ILLINOIS, MATHEMATICS DEPT.
Urbana, Illinois
- UNIVERSITY OF IOWA, DIVISION OF MATHEMATICAL SCIENCES
Iowa City, Iowa
- UNIVERSITY OF MANITOBA, DEPARTMENT OF STATISTICS
Winnipeg, Manitoba, Canada
- UNIVERSITY OF MARYLAND, DEPARTMENT OF MATHEMATICS
College Park, Maryland
- UNIVERSITY OF MASSACHUSETTS, MATHEMATICS AND STATISTICS DEPARTMENT
Amherst, Massachusetts
- UNIVERSITY OF MICHIGAN, DEPARTMENT OF STATISTICS
Ann Arbor, Michigan
- UNIVERSITY OF MINNESOTA, SCHOOL OF STATISTICS
Minneapolis, Minnesota
- UNIVERSITY OF MISSOURI, DEPARTMENT OF STATISTICS
Columbia, Missouri
- UNIVERSITY OF MISSOURI AT ROLLA, DEPARTMENT OF MATHEMATICS
Rolla, Missouri
- UNIVERSITY OF MONTREAL, DEPARTMENT OF MATHEMATICS
Montreal, Quebec, Canada
- UNIVERSITY OF NEBRASKA, MATHEMATICS AND STATISTICS DEPARTMENT
Lincoln, Nebraska
- UNIVERSITY OF NEW MEXICO, DEPARTMENT OF MATHEMATICS AND STATISTICS
Albuquerque, New Mexico
- UNIVERSITY OF NORTH CAROLINA, DEPARTMENT OF STATISTICS
Chapel Hill, North Carolina
- UNIVERSITY OF OREGON, MATHEMATICS DEPARTMENT
Eugene, Oregon
- UNIVERSITY OF OTTAWA, DEPARTMENT OF MATHEMATICS
Ottawa, Ontario, Canada
- UNIVERSITY OF SOUTH CAROLINA, DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
Columbia, South Carolina
- UNIVERSITY OF TEXAS, DEPARTMENT OF MATHEMATICS
Austin, Texas
- UNIVERSITY OF TEXAS, MATHEMATICS DEPT.
San Antonio, Texas
- UNIVERSITY OF UTAH, DEPT. OF MATHEMATICS
Salt Lake City, Utah
- UNIVERSITY OF VICTORIA, DEPT. OF MATHEMATICS
Victoria, British Columbia, Canada
- UNIVERSITY OF WASHINGTON, DEPARTMENT OF MATHEMATICS
Seattle, Washington
- UNIVERSITY OF WATERLOO, STATISTICS DEPARTMENT
Waterloo, Ont., Canada
- UNIVERSITY OF WISCONSIN, MADISON, DEPARTMENT OF STATISTICS
Madison, Wisconsin
- UNIVERSITY OF WISCONSIN, MILWAUKEE, DEPARTMENT OF MATHEMATICS
Milwaukee, Wisconsin
- VIRGINIA COMMONWEALTH UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES
Richmond, Virginia
- WAYNE STATE UNIVERSITY, DEPARTMENT OF MATHEMATICS
Detroit, Michigan
- WEST CHESTER STATE COLLEGE
West Chester, Pennsylvania
- WESTINGHOUSE ELECTRIC CORPORATION, RESEARCH LABORATORIES
Pittsburgh, Pennsylvania

THE ANNALS OF PROBABILITY

INSTRUCTIONS FOR AUTHORS

Submission of Papers. Papers to be submitted for publication should be sent to the Editor of the *Annals of Probability*. (For current address, see the latest issue of the *Annals*.) The original (or xerox copy) should be submitted with two additional copies on paper that will take ink corrections. The manuscript will *not* normally be returned to the author; when expressly requested by the author, one copy of the manuscript will be returned.

Preparation of Manuscripts. Manuscripts should be typewritten, entirely double-spaced, including references, with wide margins at sides, top and bottom. Dittoed or mimeographed papers are acceptable only if completely legible; xerox copies are preferable. When technical reports are submitted, all extraneous sheets and covers should be removed.

Submission of Reference Papers. Copies (preferably two) of unpublished or not easily available papers cited in the manuscript should be submitted with the manuscript.

Title and Abbreviated Title. The title should be descriptive and as concise as is feasible, i.e., it should indicate the topic of the paper as clearly as possible, but every word in it should be pertinent. An abbreviated title to be used as a running head is also required, and should be given below the main title. This should normally not exceed 35 characters. For example, a title might be "A Limit Theorem for Conditioned Recurrent Random Walk Attracted to a Stable Law," with the running head "Limit Theorem for Recurrent Random Walk" or possibly "Recurrent Random Walk Attracted to a Stable Law," depending on the emphasis to be conveyed.

Summary. Each manuscript is required to contain a summary which will be printed immediately after the title, clearly separated from the rest of the paper. Its main purpose is to inform the reader quickly of the nature and results of the paper; it may also be used as an aid in retrieving information. The length of a summary will clearly depend on the length and difficulty of the paper, but in general it should not exceed 150 words. It should be typed on a separate page, under the heading "Summary," followed by the title of the paper. Formulas should be used as sparingly as possible. The summary should not make reference to results or formulas in the body of the paper—it should be self-contained.

Footnotes. Footnotes should be reduced to a minimum and, where possible, should be replaced by remarks in the text or in the references; formulas in footnotes should be avoided. Footnotes in the text should be identified by superscript numbers and typed together, double-spaced on a separate page.

Key Words. Included as the first footnote on page 1 should be the headings:

American Mathematical Society 1970 subject classifications. Primary—; Secondary—.
Key words and phrases.

The classification numbers representing the primary and secondary subjects of the article may be found with instructions for its use, as an Appendix to *Mathematical Reviews* Index to Volume 39, June 1970. (See, also, *The Notices of the American Mathematical Society*, June 1970; *Bulletin of the Institute of Mathematical Statistics*, September 1974; or a current index issue of *Mathematical Reviews*.) The key words and phrases should describe the subject matter of the article; generally they should be taken from the body of the paper.

Identification of Symbols. Manuscripts for publication should be clearly prepared to insure that all symbols are properly identified. Distinguish between "oh" and "zero"; "ell" and "one"; "kappa" and "kay," etc. Indicate also when special type is required (Greek, German, script, boldface, etc.); other letters will be set in italics. Acronyms should be introduced sparingly.

Figures and Tables. Figures, charts, and diagrams should be prepared in a form suitable for photographic reproduction and should be professionally drawn twice the size they are to be printed. (These need not be submitted until the paper has been accepted for publication.) Tables should be typed on separate pages with accompanying footnotes immediately below the table.

Formulas. Fractions in the text are preferably written with the solidus or negative exponent;

thus, $(a + b)/(c + d)$ is preferred to $\frac{a + b}{c + d}$, and $(2\pi)^{-1}$ or $1/(2\pi)$ to $\frac{1}{2\pi}$. Also, $a^{b(c)}$ and $a_{b(c)}$ are pre-

ferred to a^{b_c} and a_{b_c} , respectively. Complicated exponentials should be represented with the symbol exp. A fractional exponent is preferable to a radical sign.

References. References should be typed double-spaced and should follow the style:

[5] Doob, J. L. (1944). The elementary Gaussian processes. *Ann. Math. Statist.* 15 229–282.

In textual material, the format "... Doob (1944) ..." is normally preferred to "... Doob [5]..." Multiple references can be distinguished as "... Doob (1944a)..." Abbreviations for journals should be taken from a current index issue of *Mathematical Reviews*.

Proofs. Author will ordinarily receive galley proofs. Corrected galley proofs should be sent to the Managing Editor of the *Annals of Probability*. (For current address, see the latest issue of the *Annals*.)