

THE ANNALS of PROBABILITY

AN OFFICIAL JOURNAL OF
THE INSTITUTE OF MATHEMATICAL STATISTICS

Articles

On the Williams-Bjerknes tumour growth model I	MAURY BRAMSON AND DAVID ORD FEATH	173
The shape of the limit set in Richardson's growth model	RICHARD DURRETT AND THOMAS M. LIGGETT	186
Weak convergence of the empirical characteristic function	MICHAEL B. MARCUS	194
On the accompanying laws theorem in Banach spaces	ALOISIO ARAUJO, EVARIST GINÉ, V. MANDREKAR AND JOEL ZINN	202
Asymptotic properties of semigroups of measures on vector spaces	T. BYCZKOWSKI AND T. ŻAK	211
Tauberian theorems and the central limit theorem	N. H. BINGHAM	221
Brownian motions on the homeomorphisms of the plane	THEODORE E. HARRIS	232
Some classes of two-parameter martingales	MOSHE ZAKAI	255
Order convergence of martingales in terms of countably additive and purely finitely additive martingales	KENNETH ASTBURY	266
Ordering of distributions and rearrangement of functions	LUDGER RÜSCHENDORF	276
Arm-acquiring bandits	P. WHITTLE	284
Decision processes with total-cost criteria	STEPHEN DEMKO AND THEODORE P. HILL	293
Reflected Brownian motion on an orthant	J. MICHAEL HARRISON AND MARTIN I. REIMAN	302

Short Communications

On skew Brownian motion	J. M. HARRISON AND L. A. SHEPP	309
Construction of a martingale with given absolute value	M. T. BARLOW	314
Borel sets via games	D. BLACKWELL	321
The empirical discrepancy over lower layers and a related law of large numbers	F. T. WRIGHT	323
Comparison theorems for sample function growth	P. W. MILLAR	330
Approximation of product measures with an application to order statistics	R.-D. REISS	335
A Berry-Esseen theorem for linear combinations of order statistics	R. HELMERS	342

Correction Note

Correction to "The existence and uniqueness of stationary measures for Markov-renewal processes"	RONALD A. SCHAUFELE AND RONALD PYKE	348
--	-------------------------------------	-----

Vol. 9, No. 2—April 1981

THE INSTITUTE OF MATHEMATICAL STATISTICS

(Organized September 12, 1935)

The purpose of the Institute of Mathematical Statistics is to encourage the development, dissemination, and application of mathematical statistics.

OFFICERS

President:

Peter Bickel, Department of Statistics, University of California, Berkeley, California 94720

President-Elect:

Mark Kac, Rockefeller University, New York, New York 10021.

Past-President:

George E. P. Box, Department of Statistics, University of Wisconsin, 1210 W. Dayton St., Madison, Wisconsin 53706

Executive Secretary:

Martin Fox, Department of Statistics and Probability, Michigan State University, East Lansing, Michigan 48824

Treasurer:

Heebok Park, Department of Statistics, California State University, Hayward, California 94542
IMS Business Office, 3401 Investment Blvd., Suite 6, Hayward, California 94545

Program Secretary:

Richard Johnson, Department of Statistics, University of Wisconsin, 1210 West Dayton St., Madison, Wisconsin 53706.

Editor: *Annals of Statistics*

David V. Hinkley, Department of Applied Statistics, University of Minnesota, St. Paul Campus, St. Paul, Minnesota 55108

Editor: *Annals of Probability*

R. M. Dudley, Department of Mathematics, Room 2-245, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139.

Managing Editor:

Jagdish S. Rustagi, Department of Statistics, Ohio State University, Columbus, Ohio 43210.

Membership. Membership dues including a subscription to one *Annals* and *The Institute of Mathematical Statistics Bulletin* are \$35 per year for all members. Special rates of \$16.00 per year are available to students. Rates in each category are one-fourth higher for members who wish both *Annals* as well as the *Bulletin*. Inquiries regarding membership in the institute should be sent to the Treasurer at the business office.

Subscription Rates. Current volumes (six issues per calendar year) of the *Annals of Probability* are \$44.00. Current volumes (six issues per calendar year) of the *Annals of Statistics* are \$50.00. Members of the Institute of Mathematical Statistics pay different rates (see above). Single issues are \$8.00. Back numbers of both *Annals* and the *Annals of Mathematical Statistics* (Volumes 1 through 43) may be purchased from the Treasurer.

The Annals of Probability, Volume 9, Number 2, April 1981. Published bimonthly in February, April, June, August, October, and December by The Institute of Mathematical Statistics, IMS Business Office, 3401 Investment Blvd., Suite 6, Hayward, California 94545.

Mail to the *Annals of Probability* should be addressed to either the Editor, Managing Editor or the Treasurer, as described above. It should *not* be addressed to Waverly Press.

Printed at the
WAVERLY PRESS, INC., BALTIMORE, MARYLAND 21202 U.S.A.

Second-class postage at Hayward, California and at additional mailing offices

Copyright © 1981 by the Institute of Mathematical Statistics

EDITORIAL STAFF

EDITOR

R. M. DUDLEY

ASSOCIATE EDITORS

ALEXANDRA BELLOW

SIMEON M. BERMAN

RABI BHATTACHARYA

MIKLÓS CSÖRGŐ

C. A. DOLÉANS-DADE

VÁCLAV FABIAN

C. C. HEYDE

NARESH C. JAIN

MARK KAC

HARRY KESTEN

THOMAS G. KURTZ

M. R. LEADBETTER

THOMAS M. LIGGETT

WALTER PHILIPP

STANLEY SAWYER

DAVID O. SIEGMUND

JOHN B. WALSH

STEPHEN JAMES WOLFE

EDITORIAL ASSISTANT

CYNTHIA FRIEDMAN

MANAGING EDITOR

JAGDISH S. RUSTAGI

EDITORIAL ASSISTANT

MARY SHERRILL

DOROTHY GARBIN TONJES

PAST EDITORS

ANNALS OF MATHEMATICAL STATISTICS

H. C. CARVER, 1930-1938

S. S. WILKS, 1938-1949

T. W. ANDERSON, 1950-1952

E. L. LEHMANN, 1953-1955

T. E. HARRIS, 1955-1958

WILLIAM KRUSKAL, 1958-1961

J. L. HODGES, JR., 1961-1964

D. L. BURKHOLDER, 1964-1967

Z. W. BIRNBAUM, 1967-1970

INGRAM OLKIN, 1970-1972

ANNALS OF PROBABILITY

RONALD PYKE, 1972-1975

PATRICK BILLINGSLEY, 1976-1978

ANNALS OF STATISTICS

INGRAM OLKIN, 1972-1973

I. R. SAVAGE, 1974-1976

RUPERT G. MILLER, JR., 1977-1979

EDITORIAL POLICY

The main purpose of the *Annals of Probability* and the *Annals of Statistics* is to publish contributions to the theory of probability and statistics and to their applications. The emphasis is on importance and interest, not formal novelty and correctness. Especially appropriate are authoritative expository papers and surveys of areas in vigorous development. All papers are refereed.

IMS INSTITUTIONAL MEMBERS

AEROSPACE CORPORATION
El Segundo, California

ARIZONA STATE UNIVERSITY
Tempe, Arizona

BELL TELEPHONE LABORATORIES, TECHNICAL LIBRARY
Murray Hill, N.J.

BOWLING GREEN STATE UNIVERSITY, DEPT. OF MATHEMATICS
Bowling Green, Ohio

CALIFORNIA STATE UNIVERSITY, FULLERTON, DEPARTMENT OF MATHEMATICS
Fullerton, California

CALIFORNIA STATE UNIVERSITY, HAYWARD, DEPARTMENT OF STATISTICS
Hayward, California

CASE WESTERN RESERVE UNIVERSITY, DEPARTMENT OF MATHEMATICS
Cleveland, Ohio

CORNELL UNIVERSITY, DEPARTMENT OF MATHEMATICS
Ithaca, New York

FLORIDA STATE UNIVERSITY, DEPARTMENT OF STATISTICS
Tallahassee, Florida

GENERAL MOTORS CORPORATION, RESEARCH LABORATORIES
Warren, Michigan

GEORGE WASHINGTON UNIVERSITY, DEPARTMENT OF STATISTICS
Washington, D.C.

INDIANA UNIVERSITY, MATHEMATICS DEPT.
Bloomington, Indiana

INSTITUTE FOR DEFENSE ANALYSES
Arlington, Virginia

INTERNATIONAL BUSINESS MACHINES CORPORATION
Armonk, New York

IOWA STATE UNIVERSITY, STATISTICAL LABORATORY
Ames, Iowa

JOHNS HOPKINS UNIVERSITY, DEPARTMENT OF BIostatistics, DEPARTMENT OF MATHEMATICAL SCIENCES
Baltimore, Maryland

KANSAS STATE UNIVERSITY, STATISTICS DEPARTMENT
Manhattan, Kansas

MARQUETTE UNIVERSITY, MATHEMATICS AND STATISTICS DEPARTMENT
Milwaukee, Wisconsin

MASSACHUSETTS INSTITUTE OF TECHNOLOGY MATHEMATICS DEPARTMENT
Cambridge, Massachusetts

MIAMI UNIVERSITY, DEPARTMENT OF MATHEMATICS
Oxford, Ohio

MICHIGAN STATE UNIVERSITY, DEPARTMENT OF STATISTICS AND PROBABILITY
East Lansing, Michigan

NATIONAL SECURITY AGENCY
Fort George G. Meade, Maryland

NEW MEXICO STATE UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES
Las Cruces, New Mexico

NORTHERN ILLINOIS UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES
De Kalb, Illinois

NORTHWESTERN UNIVERSITY, DEPARTMENT OF MATHEMATICS
Evanston, Illinois

OHIO STATE UNIVERSITY, DEPARTMENT OF STATISTICS
Columbus, Ohio

OREGON STATE UNIVERSITY, DEPARTMENT OF STATISTICS
Corvallis, Oregon

PENNSYLVANIA STATE UNIVERSITY, DEPARTMENT OF STATISTICS
University Park, Pennsylvania

PRINCETON UNIVERSITY, DEPARTMENT OF STATISTICS
Princeton, New Jersey

PURDUE UNIVERSITY LIBRARIES
Lafayette, Indiana

QUEEN'S UNIVERSITY, DEPT. OF MATHEMATICS AND STATISTICS
Kingston, Ontario, Canada

RICE UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES
Houston, Texas

THE ROCKEFELLER UNIVERSITY
New York, New York

SANDIA CORPORATION, SANDIA BASE
Albuquerque, New Mexico

SIMON FRASER UNIVERSITY, MATHEMATICS DEPARTMENT
Burnaby, Canada

SOUTHERN ILLINOIS UNIVERSITY, MATHEMATICAL STUDIES
Edwardsville, Illinois

SOUTHERN METHODIST UNIVERSITY, DEPARTMENT OF STATISTICS
Dallas, Texas

STANFORD UNIVERSITY, GIRSHICK MEMORIAL LIBRARY
Stanford, California

STATE UNIVERSITY OF NEW YORK, BUFFALO, DEPARTMENT OF STATISTICS
Amherst, New York

TEMPLE UNIVERSITY, MATHEMATICS DEPARTMENT
Philadelphia, Pa

TEXAS TECH UNIVERSITY, DEPARTMENT OF MATHEMATICS
Lubbock, Texas 79409

THE TOBACCO INSTITUTE
 Washington, D.C.

UNION OIL COMPANY OF CALIFORNIA, UNION RESEARCH CENTER
 Brea, California

UNIVERSITY OF ALBERTA, DEPARTMENT OF MATHEMATICS
 Edmonton, Alberta, Canada

UNIVERSITY OF ARIZONA, DEPARTMENT OF MATHEMATICS AND COMMITTEE ON STATISTICS
 Tucson, Arizona

UNIVERSITY OF BRITISH COLUMBIA, DEPARTMENT OF MATHEMATICS
 Vancouver, B.C., Canada

UNIVERSITY OF CALGARY, MATHEMATICS DEPARTMENT
 Calgary 44, Alberta, Canada

UNIVERSITY OF CALIFORNIA, BERKELEY, STATISTICAL LABORATORY
 Berkeley, California

UNIVERSITY OF CALIFORNIA, DAVIS, DIVISION OF STATISTICS
 Davis, California

UNIVERSITY OF CINCINNATI, DEPARTMENT OF MATHEMATICAL SCIENCES
 Cincinnati, Ohio

UNIVERSITY OF GUELPH, MATHEMATICS AND STATISTICS DEPARTMENT
 Guelph, Ontario, Canada

UNIVERSITY OF ILLINOIS AT CHICAGO CIRCLE, DEPARTMENT OF MATHEMATICS
 Chicago, Illinois

UNIVERSITY OF ILLINOIS, MATHEMATICS DEPT.
 Urbana, Illinois

UNIVERSITY OF IOWA, DIVISION OF MATHEMATICAL SCIENCES
 Iowa City, Iowa

UNIVERSITY OF MANITOBA, DEPARTMENT OF STATISTICS
 Winnipeg, Manitoba, Canada

UNIVERSITY OF MARYLAND, DEPARTMENT OF MATHEMATICS
 College Park, Maryland

UNIVERSITY OF MICHIGAN, DEPARTMENT OF STATISTICS
 Ann Arbor, Michigan

UNIVERSITY OF MINNESOTA, SCHOOL OF STATISTICS
 Minneapolis, Minnesota

UNIVERSITY OF MISSOURI, DEPARTMENT OF STATISTICS
 Columbia, Missouri

UNIVERSITY OF MISSOURI AT ROLLA, DEPARTMENT OF MATHEMATICS
 Rolla, Missouri

UNIVERSITY OF MONTREAL, DEPARTMENT OF MATHEMATICS
 Montreal, Quebec, Canada

UNIVERSITY OF NEBRASKA, MATHEMATICS AND STATISTICS DEPARTMENT
 Lincoln, Nebraska

UNIVERSITY OF NEW MEXICO, DEPARTMENT OF MATHEMATICS AND STATISTICS
 Albuquerque, New Mexico

UNIVERSITY OF NORTH CAROLINA, DEPARTMENT OF STATISTICS
 Chapel Hill, North Carolina

UNIVERSITY OF OREGON, MATHEMATICS DEPARTMENT
 Eugene, Oregon

UNIVERSITY OF OTTAWA, DEPARTMENT OF MATHEMATICS
 Ottawa, Ontario, Canada

UNIVERSITY OF SOUTH CAROLINA, DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
 Columbia, South Carolina

UNIVERSITY OF TEXAS, DEPARTMENT OF MATHEMATICS
 Austin, Texas

UNIVERSITY OF TEXAS, MATHEMATICS DEPT.
 San Antonio, Texas

UNIVERSITY OF UTAH, DEPT. OF MATHEMATICS
 Salt Lake City, Utah

UNIVERSITY OF VICTORIA, DEPT. OF MATHEMATICS
 Victoria, British Columbia, Canada

UNIVERSITY OF VIRGINIA, DEPT. OF MATHEMATICS
 Charlottesville, Virginia

UNIVERSITY OF WASHINGTON, DEPARTMENT OF MATHEMATICS
 Seattle, Washington

UNIVERSITY OF WATERLOO, STATISTICS DEPARTMENT
 Waterloo, Ont., Canada

UNIVERSITY OF WISCONSIN, MADISON, DEPARTMENT OF STATISTICS
 Madison, Wisconsin

UNIVERSITY OF WISCONSIN, MILWAUKEE, DEPARTMENT OF MATHEMATICS
 Milwaukee, Wisconsin

VIRGINIA COMMONWEALTH UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES
 Richmond, Virginia

WAYNE STATE UNIVERSITY, DEPARTMENT OF MATHEMATICS
 Detroit, Michigan

WEST CHESTER STATE COLLEGE
 West Chester, Pennsylvania

WESTINGHOUSE ELECTRIC CORPORATION, RESEARCH LABORATORIES
 Pittsburgh, Pennsylvania

THE ANNALS OF PROBABILITY

INSTRUCTIONS FOR AUTHORS

Submission of Papers. Papers to be submitted for publication should be sent to the Editor of the *Annals of Probability*. (For current address, see the latest issue of the *Annals*.) The original (or xerox copy) should be submitted with two additional copies on paper that will take ink corrections. The manuscript will *not* normally be returned to the author; when expressly requested by the author, one copy of the manuscript will be returned.

Preparation of Manuscripts. Manuscripts should be typewritten, entirely double-spaced, including references, with wide margins at sides, top and bottom. Dittoed or mimeographed papers are acceptable only if completely legible; xerox copies are preferable. When technical reports are submitted, all extraneous sheets and covers should be removed.

Submission of Reference Papers. Copies (preferably two) of unpublished or not easily available papers cited in the manuscript should be submitted with the manuscript.

Title and Abbreviated Title. The title should be descriptive and as concise as is feasible, i.e., it should indicate the topic of the paper as clearly as possible, but every word in it should be pertinent. An abbreviated title to be used as a running head is also required, and should be given below the main title. This should normally not exceed 35 characters. For example, a title might be "A Limit Theorem for Conditioned Recurrent Random Walk Attracted to a Stable Law," with the running head "Limit Theorem for Recurrent Random Walk" or possibly "Recurrent Random Walk Attracted to a Stable Law," depending on the emphasis to be conveyed.

Summary. Each manuscript is required to contain a summary which will be printed immediately after the title, clearly separated from the rest of the paper. Its main purpose is to inform the reader quickly of the nature and results of the paper; it may also be used as an aid in retrieving information. The length of a summary will clearly depend on the length and difficulty of the paper, but in general it should not exceed 150 words. It should be typed on a separate page, under the heading "Summary," followed by the title of the paper. Formulas should be used as sparingly as possible. The summary should not make reference to results or formulas in the body of the paper—it should be self-contained.

Footnotes. Footnotes should be reduced to a minimum and, where possible, should be replaced by remarks in the text or in the references; formulas in footnotes should be avoided. Footnotes in the text should be identified by superscript numbers and typed together, double-spaced on a separate page.

Key Words. Included as the first footnote on page 1 should be the headings:

American Mathematical Society 1970 subject classifications. Primary—; Secondary—. Key words and phrases.

The classification numbers representing the primary and secondary subjects of the article may be found with instructions for its use, as an Appendix to *Mathematical Reviews* Index to Volume 39, June 1970. (See, also, *The Notices of the American Mathematical Society*, June 1970; *Bulletin of the Institute of Mathematical Statistics*, September 1974; or a current index issue of *Mathematical Reviews*.) The key words and phrases should describe the subject matter of the article; generally they should be taken from the body of the paper.

Identification of Symbols. Manuscripts for publication should be clearly prepared to insure that all symbols are properly identified. Distinguish between "oh" and "zero"; "ell" and "one"; "kappa" and "kay," etc. Indicate also when special type is required (Greek, German, script, boldface, etc.); other letters will be set in italics. Acronyms should be introduced sparingly.

Figures and Tables. Figures, charts, and diagrams should be prepared in a form suitable for photographic reproduction and should be professionally drawn twice the size they are to be printed. (These need not be submitted until the paper has been accepted for publication.) Tables should be typed on separate pages with accompanying footnotes immediately below the table.

Formulas. Fractions in the text are preferably written with the solidus or negative exponent;

thus, $(a + b)/(c + d)$ is preferred to $\frac{a + b}{c + d}$, and $(2\pi)^{-1}$ or $1/(2\pi)$ to $\frac{1}{2\pi}$. Also, $a^{b(c)}$ and $a_{b(c)}$ are preferred to a^{bc} and a_{bc} , respectively.

Complicated exponentials should be represented with the symbol exp. A fractional exponent is preferable to a radical sign.

References. References should be typed double-spaced and should follow the style:

[5] Doob, J. L. (1944). The elementary Gaussian processes. *Ann. Math. Statist.* 15 229–282.

In textual material, the format "... Doob (1944) ..." is normally preferred to "... Doob [5]..." Multiple references can be distinguished as "... Doob (1944a)..." Abbreviations for journals should be taken from a current index issue of *Mathematical Reviews*.

Proofs. Author will ordinarily receive galley proofs. Corrected galley proofs should be sent to the Managing Editor of the *Annals of Probability*. (For current address, see the latest issue of the *Annals*.)