

THE ANNALS of MATHEMATICAL STATISTICS

FOUNDED AND EDITED BY H. C. CARVER, 1930-1938

EDITED BY S. S. WILKS, 1938-1949

THE OFFICIAL JOURNAL OF THE INSTITUTE
OF MATHEMATICAL STATISTICS

Contents

	PAGE
Unbiased estimation in convex families.....	P. J. BICKEL AND E. L. LEHMANN 1523
On an extended compound decision problem DENNIS C. GILLILAND AND JAMES F. HANNAN	1536
Estimation of parameters in a transient Markov chain arising in a reliability growth model.....	M. DUBMAN AND B. SHERMAN 1542
Admissible designs for polynomial spline regression WILLIAM J. STUDDEN AND D. J. VAN ARMAN	1557
Optimal and efficient designs of experiments.....	CORWIN L. ATWOOD 1570
The loose subordination of differential processes to Brownian motion BARTHEL W. HUFF	1603
Stochastic integrals and derivatives.....	DEAN ISAACSON 1610
Conditions for optimality and validity of simple least squares theory SUJIT KUMAR MITRA AND C. RADHAKRISHNA RAO	1617
Homogeneous Gauss-Markov random fields.....	EUGENE WONG 1625
Discrete dynamic programming with sensitive discount optimality criteria ARTHUR F. VEINOTT, JR.	1635
A note on estimating a unimodal density.....	EDWARD J. WEGMAN 1661
On a class of rank order tests for the parallelism of several regression lines PRANAB KUMAR SEN	1668
Minimax risk and unbiasedness for multiple decision problems of type I WILLEM SCHAAFSMA	1684
Characterizations of the linear exponential family in a parameter by recurrence rela- tions for functions of cumulants.....	D. C. DOSS 1721
A consistent estimator for the identification of finite mixtures.....	S. YAKOWITZ 1728
Some properties and an application of a statistic arising in testing correlation J. S. MEHTA AND JOHN GURLAND	1736
The maximum variance of restricted unimodal distributions....	HAROLD I. JACOBSON 1746
Some striking properties of binomial and beta moments.....	MORRIS SKIBINSKY 1753
On strong consistency of density estimates.....	J. VAN RYZIN 1765

Continued on back cover

Vol. 40, No. 5—October 1969

THE ANNALS OF MATHEMATICAL STATISTICS

Subscription Rates. Current issues are \$30 per volume (six issues of one calendar year) in the U.S. and Canada, \$22.50 per volume elsewhere. Single issues are \$5.75. Back numbers for all issues up to and including 1956 (Volume 27) are \$16.50 per volume, \$4.50 per issue; issues from 1957 through 1964 are \$22.00 per volume, \$5.75 per issue; from 1965 onward, \$33.00 per volume, \$5.75 per issue. Complete sets (Volumes 1 to 27) are \$275.00. If additional volumes are ordered at the same time complete sets are ordered, the price is \$16.50 per volume for Volume 28 to Volume 35 and \$22.00 per volume for Volume 36 onward.

Rates to members of the Institute of Mathematical Statistics are lower (see inside back cover).

Communications concerning **subscriptions, back numbers, payment of dues, membership, changes of address, etc.**, should be addressed to Professor George J. Resnikoff Treasurer, Department of Statistics, California State College, Hayward, California 94542.

Papers to be submitted for publication should be sent to the Editor, Z. W. Birnbaum, Department of Mathematics, University of Washington, Seattle, Washington 98105. **Abstracts and corrected galley proofs** should be sent to the Managing Editor, K. J. C. Smith, Department of Statistics, University of North Carolina, Chapel Hill, North Carolina 27514.

Preparation of manuscripts. Each manuscript should be typewritten, double spaced with wide margins at sides, top, and bottom, and the original should be submitted with two additional copies, on paper that will take corrections. Dittoed or mimeographed papers are acceptable only if completely legible. Footnotes should be reduced to a minimum, and where possible replaced by remarks in the text, or bibliography at the end of the paper; formulae in footnotes should be avoided. References should follow the style of 33 313-314.

Figures, charts, and diagrams should be professionally drawn on plain white paper or tracing cloth in black India ink twice the size they are to be printed.

Authors are asked to keep in mind the typographical difficulties of complicated mathematical formulae. The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between such pairs as zero and the letter O, the numeral 1 and the letter l, numeral 1 used as superscript and prime ('), alpha and a, kappa and k, mu and u, nu and v, eta and n, etc. Subscripts or superscripts should be clearly below or above the line. Bars above groups of letters (e.g., $\overline{\log x}$) and underlined letters (e.g., \underline{x}) are difficult to print and should be avoided. Symbols are automatically italicized by the printer and should not be underlined on manuscripts. Boldface letters may be indicated by underlining with a wavy line on the manuscript; boldface subscripts and superscripts are not available. Complicated exponentials should be represented with the symbol exp. In writing square roots the fractional exponent is preferable to the radical sign. Fractions are preferably written with the

solidus; thus $(a + b)/(c + d)$ rather than $\frac{a + b}{c + d}$, and ab/cd rather than $\frac{ad}{cd}$.

Authors will ordinarily receive only galley proofs. Fifty reprints without covers will be furnished free. Additional reprints and covers will be furnished at cost.

Mail to the *Annals of Mathematical Statistics* should be addressed to either the Editor, Managing Editor or the Treasurer, as described above. It should *not* be addressed to Waverly Press.

COMPOSED AND PRINTED AT THE
WAVERLY PRESS, INC., BALTIMORE, MARYLAND 21202 U.S.A.

Second-class postage paid at Baltimore, Maryland 21202 and at additional mailing offices.

Copyright © 1969 by the Institute of Mathematical Statistics

EDITORIAL STAFF

EDITOR

Z. W. BIRNBAUM

ASSOCIATE EDITORS

R. R. BAHADUR
PETER J. BICKEL
A. P. DEMPSTER
SYLVAIN EHRENFIELD
THOMAS S. FERGUSON
RONALD K. GETOOR

FRANKLIN A. GRAYBILL
W. J. HALL
JAMES F. HANNAN
DAVID L. HANSON
SAMUEL KARLIN
EUGENE LUKACS
ALBERT W. MARSHALL

INGRAM OLKIN
FRANK PROSCHAN
RONALD PYKE
JEROME SACKS
FRANK L. SPITZER
HENRY TEICHER

MANAGING EDITOR

K. J. C. SMITH

PAST EDITORS OF THE ANNALS

H. C. CARVER, 1930-1938	T. E. HARRIS, 1955-1958
S. S. WILKS, 1938-1949	WILLIAM KRUSKAL, 1958-1961
T. W. ANDERSON, 1950-1952	J. L. HODGES JR., 1961-1964
E. L. LEHMANN, 1953-1955	D. L. BURKHOLDER, 1964-1967

Published bimonthly by the Institute of Mathematical Statistics in February,
April, June, August, October and December.

IMS INSTITUTIONAL MEMBERS

- AEROSPACE CORPORATION
El Segundo, California
- ARIZONA STATE UNIVERSITY
Tempe, Arizona
- ARTHUR D. LITTLE, INC.
Cambridge, Massachusetts
- BALLISTIC RESEARCH LABS.
Aberdeen Proving Ground, Md.
- BELL TELEPHONE LABORATORIES, INC.,
TECHNICAL LIBRARY
New York, N. Y.
- BOEING AIRPLANE COMPANY
Seattle, Washington
- CASE WESTERN RESERVE UNIVERSITY, DE-
PARTMENT OF MATHEMATICS
Cleveland, Ohio
- CATHOLIC UNIVERSITY OF AMERICA, STAT.
LAB., MATH. DEPT.
Washington, D. C.
- CHEVRON RESEARCH CO.
Richmond, California
- CIBA PHARMACEUTICAL CO.
Summit, New Jersey
- COLORADO STATE UNIVERSITY, DEPARTMENT
OF MATHEMATICS AND STATISTICS
Fort Collins, Colorado
- CORNELL UNIVERSITY, MATHEMATICS DE-
PARTMENT
Ithaca, New York
- FLORIDA STATE UNIVERSITY, DEPARTMENT
OF STATISTICS
Tallahassee, Florida
- FORD MOTOR COMPANY
Dearborn, Michigan
- GENERAL MOTORS CORPORATION, RESEARCH
LABORATORIES
Warren, Michigan
- GEORGE WASHINGTON UNIVERSITY, DE-
PARTMENT OF STATISTICS
Washington, D. C.
- W. R. GRACE AND COMPANY, RESEARCH
DIVISION
Clarksville, Maryland
- INDIANA UNIVERSITY, THE LIBRARY
Bloomington, Indiana
- INTERNATIONAL BUSINESS MACHINES COR-
PORATION
Armonk, New York
- IOWA STATE UNIVERSITY, STATISTICAL
LABORATORY
Ames, Iowa
- JOHNS HOPKINS UNIVERSITY, DEPARTMENT
OF BIostatISTICS, DEPARTMENT OF
STATISTICS
Baltimore, Maryland
- KANSAS STATE UNIVERSITY, DEPARTMENT
OF STATISTICS
Manhattan, Kansas
- KNOLLS ATOMIC POWER LABORATORY
Schenectady, New York
- LOCKHEED AIRCRAFT CORPORATION, ENGI-
NEERING LIBRARY
Burbank, California
- MICHIGAN STATE UNIVERSITY, DEPARTMENT
OF STATISTICS
East Lansing, Michigan
- MINNESOTA MINING AND MANUFACTURING
COMPANY, APPLIED MATHEMATICS AND
STATISTICS
St. Paul, Minnesota
- NATIONAL CASH REGISTER COMPANY, RE-
SEARCH DEPARTMENT
Dayton, Ohio
- NATIONAL SECURITY AGENCY
Fort George G. Meade, Maryland
- NEW MEXICO STATE UNIVERSITY, DEPART-
MENT OF MATHEMATICAL SCIENCES
Las Cruces, New Mexico
- NORTHWESTERN UNIVERSITY, DEPARTMENT
OF MATHEMATICS
Evanston, Illinois
- OHIO STATE UNIVERSITY, STATISTICS LAB-
ORATORY, DEPARTMENT OF MATHE-
MATICS
Columbus, Ohio
- OKLAHOMA STATE UNIVERSITY, DEPART-
MENT OF MATHEMATICS
Stillwater, Oklahoma
- OREGON STATE UNIVERSITY, DEPARTMENT
OF STATISTICS
Corvallis, Oregon
- PENNSYLVANIA STATE UNIVERSITY, DE-
PARTMENT OF MATHEMATICS LIBRARY
University Park, Penna.
- PRINCETON UNIVERSITY, DEPT. OF MATH.,
SECT. OF MATH. STAT.
Princeton, New Jersey
- PROCTOR & GAMBLE COMPANY, TECHNICAL
LIBRARY
Cincinnati, Ohio
- PURDUE UNIVERSITY LIBRARIES
Lafayette, Indiana
- THE RAND CORPORATION
Santa Monica, California
- ROCKEFELLER UNIVERSITY
New York, N. Y.
- ROCKLAND STATE HOSPITAL
Orangeburg, New York
- SANDIA CORPORATION, Sandia Base
Albuquerque, New Mexico
- SOUTHERN METHODIST UNIVERSITY
Department of Statistics, Dallas, Texas
- STANFORD UNIVERSITY, GIRSHICK MEMO-
RIAL LIBRARY
Stanford, California
- SYSTEM DEVELOPMENT CORPORATION
Santa Monica, California
- TEXAS A & M UNIVERSITY, INSTITUTE OF
STATISTICS
College Station, Texas
- TRW SYSTEMS, INC., Redondo Beach
Redondo Beach, California
- UNION OIL COMPANY OF CALIFORNIA, UNION
RESEARCH CENTER
Brea, California
- KENT STATE UNIVERSITY, DEPT. OF MATH.
Kent, Ohio
- SIMON FRASER UNIVERSITY, DEPT. OF MATH.
Burnaby, B.C., Canada
- STATE UNIVERSITY OF NEW YORK AT BUF-
FALO, DEPT. OF STATISTICS
Amherst, N. Y.

UNITED STATES STEEL CORPORATION
 LIBRARY
 Monroeville, Pennsylvania
 UNIVERSITY OF ALBERTA, DEPT. OF MATHE-
 MATICS
 Edmonton, Alberta, Canada
 UNIVERSITY OF ARIZONA
 Tucson, Arizona
 UNIVERSITY OF BRITISH COLUMBIA, DE-
 PARTMENT OF MATHEMATICS
 Vancouver, B. C.
 UNIVERSITY OF CALIFORNIA, DEPARTMENT
 OF MATHEMATICS
 Irvine, California
 UNIVERSITY OF CALIFORNIA, DEPARTMENT
 OF MATHEMATICS
 Santa Barbara, California
 UNIVERSITY OF CALIFORNIA, DEPARTMENT
 OF MATHEMATICS
 Riverside, California
 UNIVERSITY OF CALIFORNIA, DEPT. OF
 MATHEMATICS
 Davis, Calif.
 UNIVERSITY OF CALIFORNIA, STATISTICAL
 LABORATORY
 Berkeley, California
 UNIVERSITY OF GEORGIA, DEPARTMENT OF
 STATISTICS
 Athens, Georgia
 UNIVERSITY OF ILLINOIS AT CHICAGO
 CIRCLE, DEPARTMENT OF MATHEMATICS
 Chicago, Illinois
 UNIVERSITY OF ILLINOIS, SERIALS DEPART-
 MENT
 Urbana, Illinois
 UNIVERSITY OF IOWA
 Iowa City, Iowa
 UNIVERSITY OF MANITOBA, DEPARTMENT
 OF STATISTICS
 Winnipeg 19, Manitoba, Canada
 UNIVERSITY OF MARYLAND, DEPARTMENT OF
 MATHEMATICS
 College Park, Maryland
 UNIVERSITY OF MASSACHUSETTS, DEPT. OF
 MATH & STAT
 Amherst, Mass.

UNIVERSITY OF MICHIGAN, DEPARTMENT
 OF MATHEMATICS
 Ann Arbor, Michigan
 UNIVERSITY OF MINNESOTA, DEPARTMENT
 OF STATISTICS
 Minneapolis, Minnesota
 UNIVERSITY OF MISSOURI, DEPARTMENT OF
 STATISTICS
 Columbia, Missouri
 UNIVERSITY OF MISSOURI AT ROLLA, DE-
 PARTMENT OF MATHEMATICS
 Rolla, Missouri
 UNIVERSITY OF MONTREAL, DEPARTMENT
 OF MATHEMATICS
 Montreal, Quebec, Canada
 UNIVERSITY OF NEW MEXICO, ALBUQUERQUE,
 New Mexico
 Albuquerque, New Mexico
 UNIVERSITY OF NORTH CAROLINA, DEPART-
 MENT OF STATISTICS
 Chapel Hill, North Carolina
 UNIVERSITY OF PENNSYLVANIA, LIBRARY
 Philadelphia, Pa.
 UNIVERSITY OF ROCHESTER
 Rochester, New York
 UNIVERSITY OF WASHINGTON, LABORATORY
 OF STATISTICAL RESEARCH
 Seattle, Washington
 UNIVERSITY OF WISCONSIN, MADISON, DE-
 PARTMENT OF STATISTICS
 Madison, Wisconsin
 UNIVERSITY OF WISCONSIN, MILWAUKEE,
 DEPARTMENT OF MATHEMATICS
 Milwaukee, Wisconsin
 WAYNE STATE UNIVERSITY, DEPARTMENT
 OF MATHEMATICS
 Detroit, Michigan
 WESTAT RESEARCH, INC.
 Bethesda, Maryland
 WESTINGHOUSE ELECTRIC CORPORATION,
 RESEARCH LABORATORIES
 Pittsburgh, Penna.