

# THE ANNALS *of* MATHEMATICAL STATISTICS

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## *Contents*

	PAGE
Asymptotic linearity of a rank statistic in regression parameter . . . JANA JUREČKOVÁ	1889
On the matrix renewal function for Markov renewal processes . . . . . J. KEILSON	1901
On the probability of large deviations and exact slopes . . . . . GERALD L. SIEVERS	1908
Inequalities of Chebyshev type involving conditional expectations C. L. MALLOWS AND DONALD RICHTER	1922
Contributions to the $k$ -sample problem: a symmetric statistic . . . . . M. SCHULZER	1933
Asymptotic behavior of Wilcoxon type confidence regions in multiple linear regression HIRA LAL KOUL	1950
The Markov inequality for sums of independent random variables S. M. SAMUELS	1980
The law of the iterated logarithm for mixing stochastic processes WALTER PHILIPP	1985
Asymptotic normality of simple linear rank statistics under alternatives II VÁCLAV DUPAČ AND JAROSLAV HÁJEK	1992
On Bartlett's test and Lehmann's test for homogeneity of variances NARIAKI SUGIURA AND HISAO NAGAO	2018
On the distributions of the ratios of the roots of a covariance matrix and Wilks' cri- terion for tests of three hypotheses K. C. S. PILLAI, S. AL-ANI AND G. M. JOURIS	2033
Asymptotic normality of linear combinations of functions of order statistics GALEN R. SHORACK	2041
Asymptotic expansions of the distributions of the likelihood ratio criteria for covariance matrix . . . . . NARIAKI SUGIURA	2051
Consistent estimates of the parameters of a linear system WILLIAM N. ANDERSON, JR., GEORGE B. KLEINDORFER, PAUL R. KLEINDORFER AND MICHAEL B. WOODROOFE	2064
Jackknifing U-statistics . . . . . JAMES N. ARVESEN	2076
Rank order tests for multivariate paired comparisons HAROLD D. SHANE AND MADAN L. PURI	2101
Weak qualitative probability on finite sets . . . . . PETER C. FISHBURN	2118

*Continued on back cover*

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Meet- ing No.	Place	Date	Abstract Deadline*	Joint With
124	Dallas, Texas (Central Regional)	April 8-10, 1970	February 17, 1970	ASA
125	Chapel Hill, N.C. (Eastern Regional)	May 5-7, 1970	March 16, 1970	ASA, ENAR
126	Laramie, Wyoming (Annual Meeting)	August 25-28, 1970	July 6, 1970	AMS
	University Park, Penn. (Eastern Regional)	April 21-23, 1971		ASA, ENAR
	Columbia, Missouri (Central Regional)	Spring, 1971		.....
	Ames, Iowa (Central Regional)	Spring, 1972		ASA, ENAR

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