# THE ANNALS of MATHEMATICAL STATISTICS

FOUNDED AND EDITED BY H. C. CARVER, 1930-1938 EDITED BY S. S. WILKS, 1938-1949

### THE OFFICIAL JOURNAL OF THE INSTITUTE OF MATHEMATICAL STATISTICS

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126	Laramie, Wyoming (Annual Meeting)	August 25-28, 1970	July 6, 1970	AMS
127	Hanover, West Ger- many (European Regional)	August 19–26, 1970	June 30, 1970	
	University Park, Penn. (Eastern Regional)	April 21–23, 1971		ASA, ENAR
	Columbia, Missouri (Central Regional)	Spring, 1971		
	Ames, Iowa (Central Regional)	Spring, 1972		ASA, ENAR

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