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SPECIAL ISSUE OF THE AFRICAN DIASPORA J. MATH. DEVOTED TO:

Quasicrystals, Almost Periodic Patterns, Mean-periodic Functions and Irregular Sampling

by

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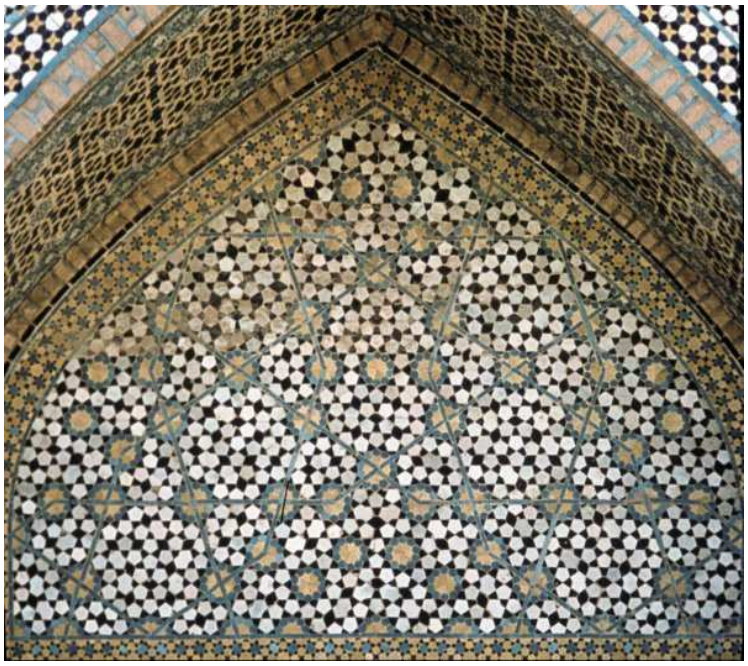
Abstract: Three properties of *quasicrystals* will be proved in this essay. Quasicrystals are *almost periodic patterns* (such patterns are carefully defined below). Every *mean-periodic function* whose spectrum is contained in a quasicrystal is almost periodic. Finally simple quasicrystals are *universal sampling sets*.



Portal from the Darb-i Imam Shrine at Isfahan, Iran (1453 AD).
[Peter J. Lu and Paul J. Steinhardt, "Decagonal and Quasicrystalline Tilings in Medieval Islamic Architecture," Science (2007).]

Welcome

The African Diaspora Journal of Mathematics is deeply honored to publish this special issue devoted to: 'QUASICRYSTALS, ALMOST PERIODIC PATTERNS, MEAN-PERIODIC FUNCTIONS AND IRREGULAR SAMPLING' by Yves Meyer.

Yves Meyer

It is with *emotion*, according to his own words, that the 2010 Gauss Prize winner, member of the African Diaspora and *alumnus* of the "Lycée Carnot" of Tunis, Tunisia, has submitted this monumental work to *The African Diaspora Journal of Mathematics* for publication.

The paper is at the heart of world scientific topicality: 2011 Nobel prize of chemistry was dedicated to quasicrystals.

It is for love of beauty and truth that Prof. Y. Meyer entered in mathematics. He was born in Tunis, Tunisia and he grew there up to 17. His childhood in Tunis had had a great influence upon his passion and love of Geometry. This passion never left him but it rather led him to significant achievements in various fields of mathematics. Motivated by the search of beauty in pure mathematics, many of his fundamental results appeared to be extremely useful in applications. For instance, his works on *wavelets* have had a longstanding impact on signal and image processing and his theory of *model sets* paved the road for the theory of quasicrystals. The improbable arrangement of atoms in certain alloys as described in the work of D. Shechtman, the 2011 Chemistry Nobel prize winner, follows the rules established by R. Penrose in 1974, for the construction of non periodic paving. The atoms in these alloys are what is now known as *Meyer sets* which were introduced by Y. Meyer in 1970. "*I was amazed when I realize that what I was doing for love of pure beauty, was useful*" he said. Whereas Penrose motivations for the quasicrystals trace back to Kepler, in the 17th century, it turns out that almost periodic tesselation were already used in the Islamic art of the 15th century . . . the art of Y. Meyer's childhood in Tunis.

Short biography[1]

Yves Meyer was born on 1939. He graduated from École Normale Supérieure, Paris, in 1960 and became a high school teacher until 1963. He then obtained a teaching assistantship at Université de Strasbourg from where he obtained his PhD in 1966. He was his own thesis supervisor, which, according to him, was not uncommon those days. He has been a professor at École Polytechnique, Université Paris-Dauphine and has also held a full research position at Centre National de la Recherche Scientifique (CNRS). His current position as Professor Emeritus at École Normale Supérieure de Cachan, France, comes after having served as a professor during 1999-2009 at the same institution. He is a member of the French Académie des Sciences and a foreign honorary member of the American Academy of Arts and Sciences. He has also been awarded a Doctorate Honoris causa by Universidad Autonoma de Madrid. In 2010, he was awarded the Carl Friedrich Gauss Prize for his "*fundamental contributions to number theory, operator theory and harmonic analysis, and his pivotal role in the development of wavelets and multiresolution analysis*".

[1] From `www.icm2010.in/prize-winners-2010`. See therein *"The laudations"* by I. Daubechies and *"The work profile"* by R. Ramachandran and also *Yves Meyer, lauréat 2010 du prix Gauss* by S. Jaffard – Images des Mathématiques, CNRS, 2010, for more details.

Interview with Professor Yves Meyer

by **M. Mboup**

I was greatly honored when you sent me your seminal paper for submission to The African Diaspora Journal of Mathematics. The members of the Editorial Board of ADJM and I, wish to extend their thanks to you for that and also for accepting this Interview.
Now, let me wish you a very warm and friendly welcome to this journal and back in the Diaspora.

"I propose with pleasure and emotion" these were your words when submitting your paper to the African Diaspora Journal of Mathematics. Why this is so special to you?

The older I get the more I feel I belong to Tunis, Tunisia. I arrived to North Africa when I was five years old and I left when I was seventeen. I spent there the most important years of my life. In a sense I belong to the African Diaspora.

Publishing this article in The African Diaspora Journal of Mathematics means reaching my journey's end. I am returning to the holy place where my personality was built. My return to Tunis is also a return to the extraordinary beauty of Islamic art. Quasicrystals are already present in the medieval Islamic art.

Did your childhood environment in Tunis play a role in your love for mathematics?

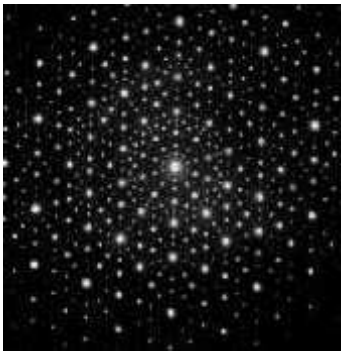
In Tunis there were two major highschools. *Collège Sadiki* was the first highschool created in Tunisia. This was achieved by Kheireddine Pacha in 1875. Habib Bourguiba[2] studied there. My *Lycée Carnot* was founded later on in 1889. It is the place where I studied. The teachers were outstanding. I was mostly attracted by humanities. I eventually switched to mathematics. What certainly played a seminal role in my way of thinking was the everyday life in Tunis. The Tunis of my childhood was a melting pot where people from all over the Mediterranean sea had found a peaceful exile. Italian, Maltese, Sepharades, Berberes, Arabs and French were living together. As a child I was obsessed by the desire of crossing the frontiers between these distinct ethnic groups. But I was limited by my ignorance of the languages which were spoken in the streets of Tunis. As an adult I like to cross the borders between distinct mathematical cultures. As I said before I came back to France when I was seventeen. I am not rooted in France. My sister who is one year older could not stand a country carrying on with a cruel and insane war in Algeria and moved to Marocco after graduating. The natural choice would have been Tunis but it was the time when Habib Bourguiba and Général de Gaulle initiated an absurd and bloody fight about Bizerte, a main naval basis in Tunisia. Cooperation agreements were cancelled. My sister spent her whole life teaching there. She speaks arabic.

Mathematics, Truth and Beauty: "Beauty is truth, truth beauty,"[3]*. Would you say that as for mathematics ?*

When I was a teenager I wished to be an artist, a painter. I am still overwhelmed by the beauty of some paintings. But I had to get a job. I was more gifted for mathematics than for anything else. I chose mathematics. When I began my research work I wished that my mathematics would reflect my artistic sensibility. My Ph.D. did not meet such standards which left me unhappy. My subsequent work on Pisot and Salem numbers gave me an

[2] Habib Bourguiba (1903–2000) was the President of Tunisia from 1956 to 1987.
[3] J. Keats (1795-1821), "Ode on a Grecian Urn"

immense pleasure and my "model sets" were art pieces. I pictured these model sets on sheets of papers stuck together with tape. Later on R. Coifman convinced me to attack Calderón's conjectures. Coifman explained me the relevance of this problem in operator theory. Calderón's intellectual construction was spendid. Calderón's program relied on the proof of the boundedness of the Cauchy kernel on Lipschitz curves, which I was able to prove in collaboration with Coifman and Alan McIntosh. The search for beauty did not play a leading role in the third part of my work on wavelets. I then wished that my research be useful for science and technology.



Diffraction diagram of a quasicrystal with 5-fold rotational symmetry - Wikipedia

Your paper got caugth up in the world scientific topicality[4]*, with the Nobel Prize of chemistry. The word "quasicrystal" refers to three names: Yves Meyer (in 1969), Roger Penrose (in 1974) and Dan Shechtman (in 1982).*

D. Shechtman who is a chemist was unaware of Penrose's work. Roger Penrose was unaware of my earlier contribution which was published by North Holland, in English [20]. Penrose should not be blamed since there were no pictures of model sets in my book. I discovered the role of Pisot and Salem numbers as inflation factors of model sets. But I did not address the issue of designing model sets with a pentagonal symmetry. This was achieved by Penrose. Rediscovery often happens in science. The anonymous referee of the seminal paper [7][5] by M. Duneau and A. Katz observed that their construction was already present in my book [20] (Denis Gratias, oral communication). Then the Ariadne's clew was found and I was given more credit than I deserve.

As for the future of mathematics, H. Poincaré warned already at his epoch against the danger of specialization. And concerning the applied mathematics, he advised: C'est du côté de la Nature qu'il faut diriger le gros de notre armée ["it is on the side of Nature that we must direct the bulk of our army"]. What Yves Meyer would say today, as for the future of mathematics ?

In 1643 Pierre Fermat wrote to Marin Mersenne[6]:

> *Vous me demandez si le nombre*
>
> 100 895 598 169
>
> *est premier ou non, et une méthode pour découvrir, dans l'espace d'un jour, s'il est premier ou composé. A cette question, je réponds que ce nombre est composé et se fait du produit de ces deux : 898 423 et 112 303, qui sont premiers.*

This letter is just incredible. In 1977 Ron Rivest (MIT), Adi Shamir (Weizmann Institute) and Leonard Adleman (University of Southern California) proposed the RSA scheme in cryptography. The RSA scheme is based on the computational load of factorization. But

[4] The paper was already under review at the time when the Nobel Prize was awarded to D. Shechtman.

[5] The references given in this interview are those from the paper.

[6] ["You ask whether the number 100 895 598 169 is prime or not, and for a method to discover, within one day, whether it is prime or composite. To this question, I answer that the number is composite and is the product of these two: 898 423 and 112 303 which are primes."]

three centuries before RSA was designed Pierre Fermat and Marin Marenne were addressing the same computational issue of factorizing very large numbers into a product of primes. This example teaches us that some main discoveries in applied sciences may result from idle problems which look completely artificial.

Number theory, topology, algebraic geometry, group theory, ... all these traditional pure mathematical chapters are more and more the basis of many applications such as in cryptology, internet, database etc. Is it the sign of a convergence of pure and applied mathematics?

Number theory, group theory, and modular forms are converging in the Langlands program which belongs to pure mathematics. This clearly shows that convergence in pure mathematics does not rely on applied sciences. But the solution of an applied problem often depends on many pieces of pure mathematics from distinct fields.

Simulation and functional modelling of high-dimensional complex systems, such as e.g. the human brain, are major challenges that absorb an increasing part of worldwide both pure and applied mathematical research activities. Is such a strong focus good for mathematics?

I had the chance of meeting David Hubel (Nobel Prize in medicine, 1981) and of attending Donald Geman's graduate course. David Hubel unveiled the functioning of the primary visual cortex of mammals. Donald Geman is trying to "break the neural code". It is clear that understanding the functioning of the human brain is the most fascinating problem of the century and that a lot of mathematics will be needed for achieving this goal. It is also clear that the whole development of mathematics cannot be directed or shaped by this endeavour. The Langlands program is one of the greatest endeavour in mathematics and has nothing to do with cognitive sciences.

In 2010, you were awarded the Gauss Prize[7] *"for fundamental contributions to number theory, operator theory and harmonic analysis, and his pivotal role in the development of wavelets and multiresolution analysis". How do you explain such a wide spectrum, from pure mathematics to concrete applications as the image compression standard jpeg2000?*

My wide spectrum can be explained by my taste for challenging problems and my reluctance to develop theories. I am attracted by specific issues. I like to enter a new field in mathematics where I have to start from scratch, as a begginer. I am a nomad, and I like to give up my own culture for adopting other ones. In Tunis I liked crossing cultural and ethnic borders.

'A Meyer set is a sampling set': This bridges the gap in between your earlier work on number theory, your work on wavelet decomposition and your current interest in compress sensing in signal processing. Would you say that it is the same seed or as H. Poincaré that la Mathématique est l'art de donner le même nom à des choses différentes ? ["The Mathematic is the art of giving the same name to different things"]

It is not true that a Meyer set is a sampling set as stated in this interview. But that claim alludes to a theorem which will be found in this volume. Unexpected connections between two completely different problems provide us with the most exciting mathematical emotions. It is not true that "universal sampling sets" as defined by Alexander Olevskii and

[7] The Carl Friedrich Gauss Prize for Applications of Mathematics is to honor scientists for "outstanding mathematical contributions that have found significant applications outside of mathematics". It is awarded once every four years at the International Congress of Mathematicians.

Alexander Ulanovskii are model sets and it is not clear that the converse is true. My discovery does not back Poincaré's assertion.

As discovered by Peter Lu of Harvard University, the quasicrystals were already present in the Islamic art, back in the 15th century, the art of your childhood in Tunis. . .

Reading Peter Lu's paper was like a fairy tale. I felt that my accidental life was justified.

Acknowledgement

I would like to express my deepest gratitude to both Professor Robert V. Moody and Professor Jeffrey C. Lagarias for their valuable and kind help during the reviewing process of this special issue. I also would like to thank so much the anonymous referes for their contribution.

The publication of this issue would not have been possible without the precious help of Professor Toka Diagana, Executive Editor of ADJM. Thank you for your patience. My thanks go also to Dr. Gisèle Mophou and Dr. Aissa Wade both of whom are co-Managing editors for this journal and to Dr. Mila Nikolova.