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A HOLOMORPHIC REPRESENTATION OF THE SEMIDIRECT SUM OF SYMPLECTIC AND HEISENBERG LIE ALGEBRAS

STEFAN BERCEANU

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Abstract. A representation of the Jacobi algebra by first order differential operators with polynomial coefficients on a Kähler manifold which as set is the product of the complex multidimensional plane times the Siegel ball is presented.

1. Introduction

In this paper we construct a holomorphic polynomial first order differential representation of the Lie algebra which is the semidirect sum $\mathfrak{h}_n \rtimes \mathfrak{sp}(2n, \mathbb{R})$, on the manifold $\mathbb{C}^n \times \mathcal{D}_n$, different from the extended metaplectic representation [6]. The case $n = 1$ corresponding to the Lie algebra $\mathfrak{h}_1 \rtimes \mathfrak{su}(1,1)$ was considered in [3]. The natural framework of such an approach is furnished by the so called coherent state (CS)-groups, and the semi-direct product of the Heisenberg-Weyl group with the symplectic group is an important example of a mixed group of this type [11]. We use Perelomov's coherent state aproach [12]. Previous results concern the hermitian symmetric spaces [2] and semisimple Lie groups which admit CS-orbits [4]. The case of the symplectic group was previously investigated in [1], [6], [5], [10], [12]. Due to lack of space we do not give here the proofs, but in general the technique is the same as in [3], where also more references are given. More details and the connection of the present results with the squeezed states [13] will be discussed elsewhere.

2. The Differential Action of the Jacobi Algebra

The Heisenberg-Weyl (HW) group is the nilpotent group with the $2n+1$-dimensional real Lie algebra $\mathfrak{h}_n =< \mathrm{i}s1 + \sum_{i=1}^{n}(x_i a_i^+ - \bar{x}_i a_i) >_{s\in\mathbb{R}, x_i\in\mathbb{C}}$, where a_i^+ (a_i) are the boson creation (respectively, annihilation) operators.

Table 1: *The generators of the symplectic group: operators, matrices, and bifermion operators*

$\boldsymbol{K}^+_{ij}$	$K^+_{ij} = \frac{\mathrm{i}}{2}\left(\begin{array}{cc} 0 & e_{ij}+e_{ji} \\ 0 & 0 \end{array}\right)$	$\frac{1}{2}a^+_i a^+_j$
$\boldsymbol{K}^-_{ij}$	$K^-_{ij} = \frac{\mathrm{i}}{2}\left(\begin{array}{cc} 0 & 0 \\ e_{ij}+e_{ji} & 0 \end{array}\right)$	$\frac{1}{2}a_i a_j$
$\boldsymbol{K}^0_{ij}$	$K^0_{ij} = \frac{1}{2}\left(\begin{array}{cc} e_{ij} & 0 \\ 0 & -e_{ji} \end{array}\right)$	$\frac{1}{4}(a^+_i a_j + a_j a^+_i)$

We consider the realization of the Lie algebra of the group $\mathrm{Sp}(2n,\mathbb{R})$ [1], [6]

$$\mathfrak{sp}(2n,\mathbb{R}) = \langle \sum_{i,j=1}^{n}(2a_{ij}K^0_{ij} + b_{ij}K^+_{ij} - \bar{b}_{ij}K^-_{ij})\rangle, \quad a^* = -a, \quad b^t = b. \tag{1}$$

With the notation: $\boldsymbol{X} := \mathrm{d}\pi(X)$, we have the correspondence: $X \in \mathfrak{sp}(2n,\mathbb{R}) \rightarrow \boldsymbol{X}$, where the real symplectic Lie algebra $\mathfrak{sp}(2n,\mathbb{R})$ is realized as $\mathfrak{sp}(2n,\mathbb{C}) \cap \mathfrak{u}(n,n)$

$$X = \left(\begin{array}{cc} a & b \\ \bar{b} & \bar{a} \end{array}\right) \leftrightarrow \quad \boldsymbol{X} = \sum_{i,j=1}^{n}(2a_{ij}\boldsymbol{K}^0_{ij} + z_{ij}\boldsymbol{K}^+_{ij} - \bar{z}_{ij}\boldsymbol{K}^-_{ij}),\ b = \mathrm{i}z. \tag{2}$$

The Jacobi algebra is the the semi-direct sum $\mathfrak{g}^J := \mathfrak{h}_n \rtimes \mathfrak{sp}(2n,\mathbb{R})$, where $\mathfrak{h}_n$ is an ideal in $\mathfrak{g}^J$, i.e. $[\mathfrak{h}_n, \mathfrak{g}^J] = \mathfrak{h}_n$, determined by the commutation relations

$$[a_i, a^+_j] = \delta_{ij}, \qquad [a_i, a_j] = [a^+_i, a^+_j] = 0 \tag{3a}$$

$$[K^-_{ij}, K^-_{kl}] = [K^+_{ij}, K^+_{kl}] = 0, \qquad 2[K^0_{ji}, K^0_{kl}] = K^0_{jl}\delta_{ki} - K^0_{ki}\delta_{lj} \tag{3b}$$

$$2[K^-_{ij}, K^+_{kl}] = K^0_{kj}\delta_{li} + K^0_{lj}\delta_{ki} + K^0_{ki}\delta_{lj} + K^0_{li}\delta_{kj} \tag{3c}$$

$$2[K^-_{ij}, K^0_{kl}] = K^-_{il}\delta_{kj} + K^-_{jl}\delta_{ki}, \qquad 2[K^+_{ij}, K^0_{kl}] = -K^+_{ik}\delta_{jl} - K^+_{jk}\delta_{li} \tag{3d}$$

$$2[a_i, K^+_{kj}] = \delta_{ik}a^+_j + \delta_{ij}a^+_k, \qquad 2[K^-_{kj}, a^+_i] = \delta_{ik}a_j + \delta_{ij}a_k \tag{3e}$$

$$2[K^0_{ij}, a^+_k] = \delta_{jk}a^+_i, \quad 2[a_k, K^0_{ij}] = \delta_{ik}a_j, \quad [a^+_k, K^+_{ij}] = [a_k, K^-_{ij}] = 0. \tag{3f}$$

Perelomov's coherent state vectors associated to the group G^J with Lie algebra the Jacobi algebra, based on the complex N-dimensional manifold , $N = \frac{n(n+3)}{2}$

$$M := \mathrm{HW}/\mathbb{R} \times \mathrm{Sp}(2n,\mathbb{R})/\mathrm{U}(n);\ M = \mathcal{D} := \mathbb{C}^n \times \mathcal{D}_n \tag{4}$$

are defined as

$$e_{z,W} = \exp(\boldsymbol{X})e_0,\ \boldsymbol{X} := \sum_i z_i a_i^+ + \sum_{ij} w_{ij}\boldsymbol{K}_{ij}^+,\ z \in \mathbb{C}^n, \qquad W \in \mathcal{D}_n. \quad (5)$$

The non-compact hermitian symmetric space $X_n = \mathrm{Sp}(2n,\mathbb{R})/\mathrm{U}(n)$ admits a realization as a bounded homogeneous domain, precisely the Siegel ball [7], [8]

$$\mathcal{D}_n := \left\{W \in M(2n,\mathbb{C}); W = W^t, 1 - W\bar{W} > 0\right\}. \quad (6)$$

The extremal weight vector e_0 verify the equations

$$a_i e_o \;=\; 0, \qquad i = 1,\cdots,n \quad (7a)$$

$$\boldsymbol{K}_{ij}^+ e_0 \;\neq\; 0; \qquad \boldsymbol{K}_{ij}^- e_0 \;=\; 0; \qquad \boldsymbol{K}_{ij}^0 e_0 \;=\; \frac{k}{4}\delta_{ij}e_0. \quad (7b)$$

Proposition 1. *The differential action of the generators of the Jacobi algebra is*

$$\boldsymbol{a} \;=\; \frac{\partial}{\partial z}, \qquad \boldsymbol{a}^+ \;=\; z + W\frac{\partial}{\partial z} \quad (8a)$$

$$\mathbb{K}^- \;=\; \frac{\partial}{\partial W}, \qquad \mathbb{K}^0 \;=\; \frac{k}{4}1 + \frac{1}{2}\frac{\partial}{\partial z}\otimes z + \frac{\partial}{\partial W}W \quad (8b)$$

$$\mathbb{K}^+ \;=\; \frac{k}{2}W + \frac{1}{2}z\otimes z + \frac{1}{2}(W\frac{\partial}{\partial z}\otimes z + z\otimes\frac{\partial}{\partial z}W) + W\frac{\partial}{\partial W}W. \quad (8c)$$

Proof: The calculation is an application of the formula $\mathrm{Ad}(\exp X) = \exp(\mathrm{ad}_X)$. We have used the convention: $\left[\left(\frac{\partial}{\partial W}W\right)f(W)\right]_{kl} := \frac{\partial f(W)}{\partial w_{ki}}w_{il},\ W = (w_{ij})$.

3. The Group Action

The displacement operator, i.e., $D(\alpha) := \exp(\alpha a^+ - \bar{\alpha}a)$, has the addition property

$$D(\alpha_2)D(\alpha_1) = \mathrm{e}^{\mathrm{i}\theta_h(\alpha_2,\alpha_1)}D(\alpha_2+\alpha_1), \qquad \theta_h(\alpha_2,\alpha_1) := \mathrm{Im}(\alpha_2\bar{\alpha}_1).$$

Concerning the real symplectic group, we extract from [1], [6]

Remark 2. *To every $g \in \mathrm{Sp}(2n,\mathbb{R})$, $g \to g_c \in \mathrm{Sp}(2n,\mathbb{C})\cap\mathrm{U}(n,n)$, or denoted just by g, $g = \begin{pmatrix} a & b \\ \bar{b} & \bar{a}\end{pmatrix}$, where $aa^* - bb^* = 1,\ ab^t = ba^t,\ a^*a - b^t\bar{b} = 1,\ a^t\bar{b} = b^*a$.*

We consider a particular case of the positive discrete series representation [9] of $\mathrm{Sp}(2n,\mathbb{R})$ and let us denote $\underline{S}(Z) = S(W)$. The vacuum is chosen such that equation (7b) is satisfied. Here

$$
\begin{aligned}
\underline{S}(Z) &= \exp(\sum z_{ij}\boldsymbol{K}^{+}_{ij} - \bar{z}_{ij}\boldsymbol{K}^{-}_{ij}), \qquad Z=(z_{ij}) && \text{(9a)}\\
S(W) &= \exp(W\boldsymbol{K}^{+})\exp(\eta\boldsymbol{K}^{0})\exp(-\bar{W}K^{-}) && \text{(9b)}\\
W &= Z\tanh\frac{\sqrt{Z^{*}Z}}{\sqrt{Z^{*}Z}} && \text{(9c)}\\
Z &= \frac{\operatorname{arctanh}\sqrt{WW^{*}}}{\sqrt{WW^{*}}}W = \frac{1}{2}\frac{1}{\sqrt{WW^{*}}}\log\frac{1+\sqrt{WW^{*}}}{1-\sqrt{WW^{*}}} && \text{(9d)}\\
\eta &= \log(1-WW^{*}) = -2\log\cosh\sqrt{ZZ^{*}}. && \text{(9e)}
\end{aligned}
$$

Perelomov's un-normalized CS-vectors for $\mathrm{Sp}(2n,\mathbb{R})$ are

$$
e_Z := \exp(\sum z_{ij}\boldsymbol{K}^{+}_{ij})e_0 = \pi\begin{pmatrix} 1 & \mathrm{i}Z\\ 0 & 1\end{pmatrix}e_0, \quad Z=(z_{ij}), \quad Z=Z^{t}. \tag{10}
$$

Remark 3. *For $g\in\mathrm{Sp}(2n,\mathbb{R})$, the following relations between the normalized and un-normalized Perelomov's CS-vectors hold*

$$
\underline{S}(Z)e_0 = \det(1-WW^{*})^{k/4}e_W \tag{11}
$$

$$
e_g := \pi(g)e_0 = (\det\bar{a})^{-k/2}e_Z = \left(\frac{\det a}{\det\bar{a}}\right)^{\frac{k}{4}}\underline{S}(Z)e_0, Z=\frac{1}{\mathrm{i}}b\bar{a}^{-1} \tag{12}
$$

$$
S(g)e_{W/\mathrm{i}} = \det(Wb^{*}+a^{*})^{-k/2}e_{Y/\mathrm{i}} \tag{13}
$$

where $W\in\mathcal{D}_n$, and $Z\in\mathbb{C}^n$ in (12) *are related by equations* (9c), (9d)*, and the linear-fractional action of the group* $\mathrm{Sp}(2n,\mathbb{R})$ *on the unit ball $\mathcal{D}_n$ in* (13) *is*

$$
Y := g\cdot W = (a\,W+b)(\bar{b}\,W+\bar{a})^{-1} = (Wb^{*}+a^{*})^{-1}(b^{t}+Wa^{t}). \tag{14}
$$

Let us introduce the notation $\tilde{A} := \begin{pmatrix} A\\ \bar{A}\end{pmatrix}$ and

$$
\mathcal{D}(Z) = \mathrm{e}^{X} = \begin{pmatrix} \cosh\sqrt{Z\bar{Z}} & \dfrac{\sinh\sqrt{Z\bar{Z}}}{\sqrt{Z\bar{Z}}}Z\\ \dfrac{\sinh\sqrt{\bar{Z}Z}}{\sqrt{\bar{Z}Z}}\bar{Z}Z & \cosh\sqrt{\bar{Z}Z}\end{pmatrix}, \qquad X := \begin{pmatrix} 0 & Z\\ \bar{Z} & 0\end{pmatrix}. \tag{15}
$$

Remark 4. *The following* (*Holstein-Primakoff-Bogoliubov*) *equation is true:* $\underline{S}^{-1}(Z)\tilde{a}\underline{S}(Z) = \mathcal{D}(Z)\tilde{a}$.

Remark 5. *If* D *is the displacement operator and* $\underline{S}(Z)$ *is defined by* (9a)*, then*

$$D(\alpha)\underline{S}(Z) = \underline{S}(Z)D(\beta), \qquad \tilde{\beta} = \mathcal{D}(-Z)\tilde{\alpha}, \qquad \tilde{\alpha} = \mathcal{D}(Z)\tilde{\beta}. \tag{16}$$

Let us introduce the notation

$$S(g) = \underline{S}(Z, A) := \exp(\sum 2a_{ij}\boldsymbol{K}^0_{ij} + z_{ij}\boldsymbol{K}^+_{ij} - \bar{z}_{ij}\boldsymbol{K}^-_{ij}). \tag{17}$$

Remark 6. *If* S *denotes the representation of* $\mathrm{Sp}(2n, \mathbb{R})$*, in the matrix realization of Table 1, we have* $S^{-1}(g)\,\tilde{a}\,S(g) = g \cdot \tilde{a}$*, and*

$$S(g)D(\alpha)S^{-1}(g) = D(\alpha_g), \qquad \alpha_g = a\,\alpha + b\,\bar{\alpha}. \tag{18}$$

Lemma 7. *The normalized and un-normalized Perelomov's coherent state vectors*

$$\Psi_{\alpha,W} := D(\alpha)S(W)e_0, \qquad e_{z,W'} := \exp(za^+ + W'\boldsymbol{K}^+)e_0$$

are related by the relation

$$\Psi_{\alpha,W} = \det(1 - W\bar{W})^{k/4}\exp(-\frac{\bar{\alpha}}{2}z)e_{z,W}, \qquad z = \alpha - W\bar{\alpha}. \tag{19}$$

Comment 8. *Starting from* (19)*, we obtain the expression of the reproducing kernel* $K = K(\bar{x}, \bar{V}; y, W)$

$$\begin{aligned}(e_{x,V}, e_{y,W}) &= \det(U)^{k/2}\exp\frac{1}{2}[2 < x, Uy > + \langle V\bar{y}, Uy\rangle + \langle x, UW\bar{x}\rangle] \\ U &= (1 - W\bar{V})^{-1}.\end{aligned} \tag{20}$$

From the following proposition we can see the holomorphic action of the Jacobi group $G^J := \mathrm{HW} \rtimes \mathrm{Sp}(2n, \mathbb{R})$ on the manifold (4)

Proposition 9. *Let us consider the action* $S(g)D(\alpha)e_{z,W}$*, where* $g \in \mathrm{Sp}(2n, \mathbb{R})$*, and the coherent state vector is defined in* (5)*. Then we have*

$$S(g)D(\alpha)e_{z,W} = \lambda e_{z_1,W_1}, \qquad \lambda = \lambda(g, \alpha; z, W) \tag{21}$$

$$z_1 = (Wb^* + a^*)^{-1}(z + \alpha - W\bar{\alpha}) \tag{22}$$

$$W_1 = g \cdot W = (aW + b)(\bar{b}W + \bar{a})^{-1} = (Wb^* + a^*)^{-1}(b^t + Wa^t) \tag{23}$$

$$\lambda = \det(Wb^* + a^*)^{-k/2}\exp(\frac{\bar{x}}{2}z - \frac{\bar{y}}{2}z_1)\exp\mathrm{i}\theta_h(\alpha, x) \tag{24}$$

$$x = (1 - W\bar{W})^{-1}(z + W\bar{z}), \qquad y = a(\alpha + x) + b(\bar{\alpha} + \bar{x}). \tag{25}$$

Corollary 10. *The action of the Jacobi group G^J on the manifold* (4) *is given by* (21), (22)*. The composition law in G^J is*

$$(g_1,\alpha_1,t_1)\circ(g_2,\alpha_2,t_2) = (g_1\circ g_2, g_2^{-1}\cdot\alpha_1+\alpha_2, t_1+t_2+\mathrm{Im}(g_2^{-1}\cdot\alpha_1\bar{\alpha}_2)). \quad (26)$$

The proof of Proposition 9 is based on the previous assertions of this section.

4. The Scalar Product

Following the general prescription for CS-groups [4], we calculate the Kähler potential f as the logarithm of the reproducing kernel K, and the Kähler two-form

$$\begin{aligned} f =& -\frac{k}{2}\log\det(1-W\bar{W}) + \bar{z}_i(1-W\bar{W})^{-1}_{ij}z_j \\ &+ \frac{1}{2}[z_i[\bar{W}(1-W\bar{W})^{-1}]_{ij}z_j + \bar{z}_i[(1-W\bar{W})^{-1}W]_{ij}\bar{z}_j] \end{aligned} \quad (27)$$

$$\begin{aligned} -\mathrm{i}\omega =& \frac{k}{2}\mathrm{tr}[(1-W\bar{W})^{-1}\mathrm{d}W\wedge(1-\bar{W}W)^{-1}\mathrm{d}\bar{W}] \\ &+ \mathrm{tr}[\mathrm{d}z^t\wedge(1-\bar{W}W)^{-1}\mathrm{d}\bar{z}] \\ &- \mathrm{tr}[\mathrm{d}\bar{z}^t(1-W\bar{W})^{-1}\wedge\mathrm{d}W\bar{x}] + cc \\ &+ \mathrm{tr}[\bar{x}^t\mathrm{d}W(1-\bar{W}W)^{-1}\wedge\mathrm{d}\bar{W}x]. \end{aligned} \quad (28)$$

Applying the technique of Chapter IV in [8] and the property extracted from the first reference [5] p. 398, we find out for the density of the volume form

$$Q = \det(1-W\bar{W})^{-(n+2)}. \quad (29)$$

Now we determine the scalar product. If $f_\psi(z) := (e_{\bar{z}},\psi)$, then

$$(\phi,\psi) = \Lambda\int_{z\in\mathbb{C}^n;1-W\bar{W}>0;W=W^t}\bar{f}_\phi(z,W)f_\psi(z,W)QK^{-1}\mathrm{d}z\mathrm{d}W \quad (30)$$

$$\mathrm{d}z = \prod_{i=1}^{n}\mathrm{d}\Re z_i\mathrm{d}\mathrm{Im}z_i, \qquad \mathrm{d}W = \prod_{1\le i\le j\le n}\mathrm{d}\Re w_{ij}\mathrm{d}\mathrm{Im}w_{ij}. \quad (31)$$

We take in (30) $\phi,\psi=1$, we change the variable $z=(1-W\bar{W})^{1/2}x$, we apply equations (A1), (A2) in Bargmann [1] and Theorem 2.3.1 p. 46 in [8]

$$\int_{1-W\bar{W}>0,W=W^t}\det(1-W\bar{W})^{\lambda}\mathrm{d}W = J_n(\lambda)$$

and we find for Λ in (30) (below $p := (k-3)/2 - n > -1$)

$$\Lambda = \pi^{-n} J_n^{-1}(p), J_n(p) = 2^n \pi^{\frac{n(n+1)}{2}} \prod_{i=1}^{n} \frac{\Gamma(2p+2i)}{\Gamma(2p+n+i+1)}. \tag{32}$$

Proposition 11. *Let us consider the Jacobi group G^J with the composition rule* (26)*, acting on the coherent state manifold* (4) *via* (22)–(25)*. The manifold M has the Kähler potential* (27) *and the G^J-invariant Kähler two-form ω given by* (28)*. The Hilbert space of holomorphic functions $\mathcal{F}_K$ associated to the holomorphic kernel $K : M \times \bar{M} \to \mathbb{C}$ given by* (20) *is endowed with the scalar product* (30)*, where the normalization constant Λ is given by* (32) *and the density of volume given by* (29).

Proposition 12. *Let $h = (g, \alpha) \in G^J$, and let us consider the representation $\pi(h) = S(g)D(\alpha)$, $g \in \mathrm{Sp}(2n, \mathbb{R})$, $\alpha \in \mathbb{C}^n$, and making use of the notation $x = (z, W) \in \mathcal{D}$. Then the continuous unitary representation $(\pi_K, \mathcal{H}_K)$ attached to the positive definite holomorphic kernel K defined by* (20) *is $(\pi_K(h).f)(x) = J(h^{-1}, x)^{-1} f(h^{-1}.x)$, where the cocycle $J(h^{-1}, x)^{-1} = \lambda(h^{-1}, x)$ with λ defined by equations* (21)-(25) *and the function f belongs to the Hilbert space of holomorphic functions $\mathcal{H}_K \equiv \mathcal{F}_K$ endowed with the scalar product* (30).

Comment 13. *The value of Λ given by* (32) *corresponds to the one given in* (7.16) *in* [3]*, by taking above $n = 1$, $k \to 4k$. Note that p defying the normalization constant Λ in* (32) *for the Jacobi group is related with $q = \frac{k}{2} - n - 1$ in*

$$(\phi, \psi)_{\mathcal{F}_\mathcal{H}} = \Lambda_1 \int_{1 - W\bar{W} > 0; W = W^t} \bar{f}_\phi(W) f_\psi(W) \det(1 - W\bar{W})^q \mathrm{d}W \tag{33}$$

defining the normalization constant $\Lambda_1 = J_n^{-1}(q)$ for the group $\mathrm{Sp}(2n, \mathbb{R})$ by the relation $p = q - \frac{1}{2}$. It is well known [5], [8] *that the* **admissible set** *for k for the space of functions $\mathcal{F}_\mathcal{H}$ endowed with the scalar product* (33) *is the set $\Sigma = \{0, 1, \cdots, n-1\} \cup ((n-1), \infty)$. The integral* (33) *deals with a non-negative scalar product if $k \geq n - 1$, in which the domain of convergence $k \geq 2n$ is included, and the separate points $k = 0, 1, ..., n-1$.*

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Stefan Berceanu
Institute for Physics and Nuclear Engineering
Department of Theoretical Physics
PO BOX MG-6, Bucharest-Magurele
ROMANIA
E-mail address:
`Berceanu@theor1.theory.nipne.ro`