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ON THE WORK OF LECH DREWNOWSKI

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It is difficult to write a paper on a mathematical work of a person like Lech Drewnowski. First of all because of a large variety of subjects he worked on. Secondly, since several of his topics which seem to be completely pairwisely unrelated surprisingly turn out to be connected on a deep level: look for instance, on Drewnowski's work on Nikodým and Vitali-Hahn-Saks theorems — it is a root of his interest in barrelled spaces and one can easily observe its further interrelation with his work on series convergence in the context of the Orlicz-Pettis theorem. The third reason why our task is close to impossible is that Drewnowski's work is far from being completed: every year new and thorough papers appear so one cannot predict which of that existing papers should be reinterpreted in a new light of his recent discoveries. Last but not least it is difficult to write about our own Master and Teacher whom we owe so much. We do not present a systematic and detailed analysis of all Drewnowski's results. We have subjectively selected several topics and theorems due to him which are, in our private opinion, beautiful, important and also sufficiently near to our own interests.

Let us say that Drewnowski is a true "problem solver". When others feel that the solution is already complete and fully satisfactory, he works further and finds unexpected deeper results and he builds an extensive theory. While collaborating we experienced this phenomena several times. One could say that the "first solution" is only a motivation and starting point for Drewnowski's extensive research which leads to the "deep solution". In this paper we explain a little bit of motivation of the particular theorems but the emphasis will be put on further results inspired by Drewnowski's research. We will show how many people owe their inspiration to Drewnowski. Writing this survey we realized once again how lovely Lech's mathematics is.

Lech Drewnowski was born on 10 February 1944 in Kruszyna (now part of Belarus). He started an academic career at Adam Mickiewicz University in Poznań in 1966 when he had finished mathematical studies on the Faculty of Mathematics, Physics and Chemistry. Five years later, in 1971, Lech Drewnowski obtained Ph.D. defending the thesis *On some problems in the theory of spaces of integrable functions* which was prepared under a supervision of Władysław Orlicz. The thesis, published in 1972 in the Bulletin of the Polish Academy of Science as a three-part paper *Topological rings of sets, continuous set functions, integration*, had an essential influence on the measure theory and it belongs till now to the most frequently cited Drewnowski's works. Lech Drewnowski's career developed very quickly – in 1975 he earned postdoctoral degree (habilitation) and in 1985 he obtained a specifically Polish scientific highest grade: the title of the professor nowadays equivalent with the full professor position. Both his Ph.D. dissertation and the postdoctoral dissertation *Decompositions of set functions and existence of scalar measures topologically equivalent to vector measures* were honored by the Award of the Ministry of Science, Higher Education and Technology of Poland.

During his whole scientific activity Professor Drewnowski received numerous invitations to universities in Europe (Paris VI, Universidad de Sevilla, Universidad Complutense de Madrid, Universitá di Catania, Universität Trier) and USA (Michigan State University, University of South Carolina, University of Florida, University of Mississippi) where he had visiting positions. He obtained many scientific awards: Polish Mathematical Society Award for young mathematicians (1972), Stefan Banach Main Prize of the Polish Mathematical Society (1977), the Award of the Scientific Secretary of the Polish Academy of Sciences (1977, 1986), Award of the Third Department of the Polish Academy of Sciences (1978), the Award of the Minister of National Education (1988).

Up to now Lech Drewnowski has published 116 papers.

Let us explain that in all sections below the notation [DX] refers to the paper on the list of *Publications of Lech Drewnowski.*

1. Vector and scalar set functions

Results concerning measure theory form a big and important part of Lech Drewnowski work. Papers [D7], [D8], [D9] are his most frequently cited publications. As the Web of Science informs they were jointly cited 212 times. After forty years from printing these papers still inspire succeeding generations of mathematicians – from 2010 till 2013 authors of 35 articles put them in references. Drewnowski's contribution to a development of the measure theory was appreciated very quickly. Already the book [84], printed in 1975, refers to four Drewnowski's publications. The famous Diestel and Uhl monograph [25] published in 1977, i.e., less than ten years after beginning of Drewnowski's scientific activity, quotes thirteen his papers (and additionally six written with coauthors). Importance of his results was confirmed twenty five years later when the handbook [103] arose. Drewnowski's achievements are discussed in five chapters and again thirteen his publications are cited.

Drewnowski's papers refer to natural, classical problems which are a subject of interests of many investigators and which are in the main stream of measure theory. It is worth to emphasize that a lot of his results have a final character closing a problem.

1.1. FN-topologies and related topics – decompositions and convergence theorems for set functions

A topological ring of sets is a ring $\mathcal{R}$ of subsets included in some set together with a topology Γ providing continuity of the symmetric difference and intersection of sets. A topology Γ is said to be *Fréchet-Nikodým* if there exists a basis $\mathfrak{A}$ of neighborhoods of the empty set $\emptyset$ such that $B \in \mathfrak{A}$ whenever $\mathcal{R} \ni B \subset A \in \mathfrak{A}$. The most important example of an FN-tpology is related to a submeasure, i..e, to a set function $\eta : \mathcal{R} \to [0, \infty)$ satisfying the following three conditions:

1. $\eta(\emptyset) = 0$,
2. $\eta(A \cup B) \leqslant \eta(A) + \eta(B)$,
3. $A \subset B \;\Rightarrow\; \eta(A) \leqslant \eta(B)$.

The FN-topology $\Gamma(\eta)$ is then defined by a semimetric $d(A, B) = \eta((A \setminus B) \cup (B \setminus A))$ and $\overline{\{\emptyset\}}^{\Gamma(\eta)} = \{A \in \mathcal{R} : \eta(A) = 0\}$. Conversely, every FN-topology Γ is generated by some family of submeasures $(\eta_t)_{t \in T}$, i.e., sets $\bigcap_{t \in \Delta} \{A \in \mathcal{R} : \eta_t(A) \leqslant \varepsilon\}$ (where $\varepsilon > 0$ and $\Delta \subset T$ is finite) form a basis of Γ-neighborhoods of $\emptyset$. Beginnings of FN-topologies go back to Fréchet and Nikodým. They were used, probably for the first time, in proofs of the famous Vitali-Hahn-Saks and Nikodým theorems. Weber, the author of Chapter 16 in [103], says that “Today FN-topologies are considered an elegant and powerful tool in the measure theory.”

Diestel and Uhl in [25] describe papers [D7], [D8], [D9], [D14], [D15] and [D19] as “masterful study of Fréchet-Nikodým topologies and their applications”. The first three papers are a part of Drewnowski’s Ph.D. thesis. They contain extensive investigations of FN-topologies and related to them exhaustive and order continuous extended real valued as well as group valued set functions (let us recall that *exhaustivity* of μ means $\mu(A_n) \to 0$ for disjoint A_n’s and order continuity is defined as follows: $A_n \downarrow \emptyset \;\Rightarrow\; \mu(A_n) \to 0$). Characterizations and relationships joining equi-continuity, order continuity and uniform exhaustivity of families of set functions are also presented there. Additionally one can find theorems concerning extensions of exhaustive, order continuous or σ-additive set functions from a ring onto the generated σ-ring (see [D8] Thm. 7.2, Thm. 7.3 and [D9] Thm. 9.2). Drewnowski continued an analysis of FN-topologies (for instance their completeness) and submeasures in later papers [D19] and [D26].

Decompositions of set functions is the next direction of Drewnowski’s interests which was developed in [D8] and [D14]. He paid a special attention to the Lebesgue and Hewitt-Yosida type decompositions for various classes of submeasures and exhaustive group valued additive set functions μ defined on rings. Paper [D14] presents a very general and unified approach to decompositions which essence is included in Theorem 3.11 saying that every such μ can be uniquely decomposed onto a sum of two additive exhaustive set function where the first component is additive and the second is singular with respect to some family of sets (called an additivity). Choosing appropriate additivities one can obtain the Lebesgue and Hewitt-Yosida decompositions for μ. It is also worth to recall Theorem 4.1 from [D14] concerning very general decompositions of exhaustive submeasures.

Classical Vitali-Hahn-Saks and Nikodým theorems (their more general form is presented in Theorem 1.1) about properties of a set function being the pointwise limit of σ-additive (real valued) set functions belong to the most important results in the measure theory. They were generalized in various directions (the assumption of σ-additivity was weakened and the range $\mathbb{R}$ was replaced by infinite dimensional Banach spaces or complete topological groups). Several early Drewnowski's papers contain results belonging to the above area (see [D7] Thm. 3.2 or [D14] Thm. 2.14') but his most spectacular theorem in this direction is the following ([D10] Theorem).

Theorem 1.1. *Let $\mathcal{R}$ be a σ-ring of sets, G a Hausdorff topological commutative group and let (μ_n) be a sequence of additive G-valued set functions such that there exists the limit $\mu_0(A) = \lim_{n\to\infty} \mu_n(A)$ for each $A \in \mathcal{R}$. Then the following statements hold and are equivalent.*

(BJ) *If each μ_n is exhaustive then the family $\{\mu_n : n \in \mathbb{N}\}$ is uniformly exhaustive.*

(VHS) *Let Γ be an FN-topology on $\mathcal{R}$. If each μ_n, $n \geqslant 1$, is exhaustive and Γ-continuous, then the family $\{\mu_n : n = 0, 1, 2, \dots\}$ is equi-Γ-continuous.*

(N) *If each μ_n, $n \geqslant 1$, is σ-additive, then the family $\{\mu_n : n = 0, 1, 2, \dots\}$ is uniformly σ-additive.*

The Author of [D10] discovered very important relationship joining exhaustivity and σ-additivity.

Theorem 1.2. *Let $\mathcal{R}$ be a σ-ring and let G denote a a metrizable commutative topological group. An additive set function $\mu : \mathcal{R} \to G$ is exhaustive if and only if each disjoint sequence $(A_n) \subset \mathcal{R}$ contains a subsequence (A_{n_k}) such that μ is σ-additive on the σ-ring generated by (A_{n_k}).*

It is worth to add that the assumption of metrizability is essential. Later we will see that this circle of ideas has a deep connection with barreledness properties – see Section 4 (especially 4.1) below – and the problems described above were considered by Drewnowski from a different point of view.

1.2. The control measure problem

Suppose that $\mathcal{R}$ be a ring of sets and let G denote a Hausdorff commutative topological group. We will say that a set function $m : \mathcal{R} \to G$ is *controlled* by an additive set function $\mu : \mathcal{R} \to [0, \infty)$ (or that μ is a control measure for m) if $\mu(A_n) \to 0 \;\Rightarrow\; m(A_n) \to 0$. The question if there exists a control measure for a given m is known as the control measure problem. A particular case when m is an exhaustive submeasure is a very famous Maharam's problem formulated in [95]. The problem waited for a solution around sixty years. Efforts of Kalton, Roberts, Farah and Talagrand led to the solution in the negative. The authors of [79] showed that an exhaustive submeasure η is controlled by an additive set function if and only if η is uniformly exhaustive, i.e., for given $\varepsilon > 0$ there exists

$n \in \mathbb{N}$ such that $\min_{1 \leqslant i \leqslant n} \eta(A_i) \leqslant \varepsilon$ whenever $A_1, \ldots, A_n$ are disjoint. Later on Talagrand constructed in [123] a non-zero exhaustive submeasure on the algebra of clopen subsets of the Cantor set that is not uniformly exhaustive.

It occurs that many σ-additive vector valued set functions on σ-algebras are controlled by σ-additive (real valued positive) set functions. Saying about σ-additive set functions defined on a σ-algebra Σ of sets we will use the name "measure". In 1955 Bartle, Dunford and Schwartz showed in [6] that measures with values in normed spaces have control measures. One of the most spectacular results concerning control measures is due to Rybakov (see [110]). He proved that for every measure $m : \Sigma \to X$, where X is a normed space, the total variation of a real valued measure $x^* \circ m$ controls m for some continuous linear functional x^* on X. Moreover a family of such x^*'s forms a dense G_δ-set in the dual X^* (see [129]). Drewnowski obtained in [D9] (see also [D15] 3.2. Theorem) an interesting generalization of Rybakov's theorem emphasizing a role of the so-called countable chain condition. We will need an additional notation – for a set function m on Σ we distinguish the family of m-null sets $\mathcal{N}(m) = \{A \in \Sigma : m(B \cap A) = 0 \text{ for every } B \in \Sigma\}$.

Theorem 1.3. *Let X be a Banach space and let $\mathcal{M}$ denotes a family of measures on Σ with values in X having the property that every family of disjoint sets in $\Sigma \setminus \bigcap_{m \in \mathcal{M}} \mathcal{N}(m)$ is at most countable. There exist sequences $(c_n) \in \ell^1$ and $(m_n) \in \mathcal{M}$ such that $m_0(\cdot) = \sum_{n=1}^{\infty} c_n m_n(\cdot)$ is a control measure for each $m \in \mathcal{M}$.*

Applying the above theorem to the family $\mathcal{M} = \{x^* \circ m : \|x^*\| \leqslant 1\}$ we will find measures $x_n^* \circ m$ which produce a functional $x^* = \sum_{n=1}^{\infty} c_n x_n^*$ described in the Rybakov's theorem. The paper [D15] contains two extensions of the Bartle-Dunford-Schwartz result to measures taking their values in locally convex topological vector spaces. Namely, Drewnowski obtained the following result.

Theorem 1.4. *A measure $m : \Sigma \to X$ taking values in a Hausdorff locally convex topological vector space X has a control measure if and only if every family of disjoint sets in $\Sigma \setminus \mathcal{N}(m)$ is at most countable.*

Moreover, any X-valued measure on a σ-algebra has a control measure if and only if every family $(x_i)_{i \in I} \subset X \setminus \{0\}$ such that every its countable subfamily $(x_i)_{i \in J \subset I}$ is summable, is countable, that is $\operatorname{card} I \leqslant \aleph_0$ (in particular, it is the case when X admits a coarser metrizable linear topology).

1.3. The Pettis integral

Drewnowski joined to investigations of the Pettis integrability after publication of Talagrand's monograph [122] which systematized a knowledge about the Pettis integral dispersed in the literature. The author of [D47] discovered several conditions equivalent to the Pettis integrability.

Suppose that μ is a finite measure on a σ-algebra of subsets of S and X is a Banach space. Let us recall that a function $f : S \to X$ is scalarly μ-integrable if $x^* \circ f \in L^1(\mu)$ for each $x^* \in X^*$ and such a function has the *Dunford integral* (D) $\int_A f\, d\mu \in X^{**}$ if the following holds $\big((\text{D}) \int_A f\, d\mu\big)(x^*) = \int_A (x^* \circ f)\, d\mu$ for all

$x^* \in X^*$. We say that f as above is *Pettis integrable* if its Dunford integral belongs to X. Every scalarly integrable function f defines an operator $T_f : X^* \to L^1(\mu)$ given by the formula $T_f(x^*) = x^* \circ f$. It is known that if f is Pettis integrable then T_f is always weakly compact. We say that f is determined by a subspace $Y \subset X$ if for every $x^* \in Y^\perp$ the equality $x^* \circ f = 0$ holds μ-a.e. (for details concerning all above notions we refer [103] Chapter 12). Drewnowski noticed that the Pettis integrability is related to a special type of Banach spaces Y called Corson spaces. We say that a Banach space Y is a *Corson space* whenever each family of closed convex subsets of Y has a nonempty intersection if every its countable subfamily has nonempty intersection; weakly compactly generated spaces – in particular reflexive spaces – are Corson. Drewnowski proved in [D47] the following result.

Theorem 1.5. *For a scalarly integrable $f : S \to X$ such that the operator T_f is weakly compact the following are equivalent.*

(a) *f is Pettis integrable.*
(b) *f is determined by a weakly compactly generated subspace $Y \subset X$.*
(c) *f is determined by a Corson subspace $Y \subset X$.*

Another, more general, characterization of the Pettis integrability is related to two notions introduced by Drewnowski. The first is called the *Corson envelope* $\widetilde{X}$ of X, i.e., $\widetilde{X} = \bigcup \overline{Y}^{**}$ where the sum runs over all Corson subspaces Y in X and $\overline{Y}^{**}$ means the $\sigma(X^{**}, X^*)$-closure of Y, and the second is said to be a weak***-core* of a scalarly integrable $f : S \to X$, i.e., for $A \in \Sigma$ we define $\operatorname{cor}_f^{**}(A) = \bigcap\{\overline{\operatorname{conv}}^{**} f(A \setminus N) : \mu(N) = 0\}$. It occurs that if $f : S \to X$ is scalarly integrable and the operator T_f is weakly compact, then f is Pettis integrable whenever $\operatorname{cor}_f^{**}(A) \cap \widetilde{X} \neq \emptyset$ or $\{(\mathrm{D}) \int_A f \, d\mu : A \in \Sigma\} \subset \widetilde{X}$.

Drewnowski investigated also the whole space of Pettis integrable functions (see Section 4.2).

1.4. Ranges of measures and Baire category results

Properties of sets $m(\Sigma)$ where Σ is a σ-algebra of sets and a measure (= σ-additive set function) m takes values in a topological vector space X (or in a topological group G) were investigated intensively for many years. Some obtained results remains valid in more general situations, e.g., for algebras of sets or exhaustive and additive functions m. The Lapunov convexity theorem opens a long list of papers devoted to ranges of vector measures. It says (see [88]) that a finite dimensional vector measure m has compact range and the range is convex for an atomless m. This fundamental theorem has many different proofs (see [65], [52]) but the most elegant, using the Krein-Milman theorem, is due to Lindenstrauss (see [90]).

Wnuk modifying Gerencsér's idea showed in [131] that if every atomless measure m defined on an arbitrary σ-algebra with values in a metrizable complete topological vector space X has either convex or closed range, then X is finite dimensional. There exist a lot of examples of atomless measures with values in infinite dimensional Banach spaces whose ranges are neither closed nor convex.

On the other hand $m(\Sigma)$ is always compact whenever m is purely atomic (and even if m is group valued) — see [100], [66]. Similarly, $m(\Sigma)$ is conditionally compact when m is an indefinite Bochner integral with respect to real valued positive measure μ and moreover $\overline{m(\Sigma)}$ is convex for atomless μ (see [128]).

Bartle, Dunford and Schwartz showed in [6] conditional weak compactness of $m(\Sigma)$ for a Banach space valued m and later on Tweddle extended this result to measures with values in a complete locally convex topological vector space (see [127]). The assumption of local convexity is very important because Turpin constructed in [126] a complete metrizable topological vector space X_T and a measure transferring the Lebesgue measurable sets onto an unbounded set in X_T. Turpin's example is rather curious because, as Drewnowski proved in [D8], $m(\Sigma)$ is bounded if m carries Σ into a locally pseudo-convex space X (i.e., the topology of X is generated by a family of p-homogeneous seminorms with, in general, various p's). To be precise we should recall that Drewnowski's result is more general.

Theorem 1.6. *If $\mathcal{R}$ is a ring of sets and $m : \mathcal{R} \to G$ is additive and exhaustive, then $m(\mathcal{R})$ is additively bounded, i.e., for every neighborhood U of zero there exists n such that $m(\mathcal{R}) \subset U^n$ (where $U^1 = U$ and $U^{n+1} = U^n + U$).*

It is easy to check that a subset A of a topological vector space X is additively bounded if and only if $\sup_{a\in A} \|a\| < \infty$ for every continuous F-seminorm $\|\cdot\|$ on X. Now it is clear that families of additively bounded and bounded subsets coincide in a locally pseudo-convex space (in particular, in a locally bounded space). The space $L^0(\mu)$ of equivalence classes of real functions measurable with respect to a finite measure μ considered with the topology of convergence in measure is a classical example of a non locally bounded space. Hence after publication of Drewnowski's result it was natural to ask if $L^0(\mu)$-valued measures have bounded ranges. After long efforts, finally, Talagrand and, independently, Kalton, Peck and Roberts (see [121], [77]) found a solution in the affirmative. The paper [D43] contains an essential generalization of the Talagrand-Kalton-Peck-Roberts result. There was showed that if $L^0(Z)$ means the space of all Bochner measurable functions from a finite measure space (Ω, Σ, μ) to a Banach space Z equipped with the topology of μ-convergence, then every measure $m : \Sigma \to L^0(Z)$ has a bounded range.

The use of the Baire category theorem in the measure theory has a long story (it goes back to S. Saks). A part of this story is a question how "big" (in the Baire category sense) can be sets of measures possessing a fixed property, e.g., is it typical that an interior of the range of a measure m is nonempty or the range is convex or closed? Similarly we can ask how "big" is a family of sets A such that $m(A) \in Y$ where Y is a linear subspace or $m(\Sigma \cap A)$ is not relatively compact. Such type of problems was considered in [3] and [4]. Results obtained by R. Anantharaman and K.M. Garg inspired papers [D63] and [D89] which content can be, roughly speaking described as follows (see [D89], p. 103):

"Let X be a topological abelian group or vector space in which 'small' subsets has been distinguished, satisfying some mild conditions. For instance, 'small' may mean 'finite', or 'finite dimensional', or 'precompact', or 'bounded'. Furthermore,

let $m : \Sigma \to X$ be a measure with a non-small range. Then, given a small (or σ-small) set $H \subset X$, we prove that on most of the sets from Σ the values assumed by m are outside of H. Likewise, the range of m over most of the sets from Σ is non-small (and even non-σ-small). Here 'most of the sets from Σ' means that the class of sets involved is residual ..."

As we have already explained a measure (or a submeasure) $\lambda : \Sigma \to [0, \infty)$ defines a complete semimetric d_λ on Σ by $d_\lambda(A, B) = \lambda(A \bigtriangleup B)$ where $\bigtriangleup$ is the symmetric difference. It was shown in [D63] that the following theorem holds.

Theorem 1.7. *Let $\mathcal{C}$ be a subset of Σ such that*

(a) *every λ-atom is in $\mathcal{C}$;*
(b) *if $E, F \in \mathcal{C}$ are disjoint, then $E \cup F \in \mathcal{C}$;*
(c) *if $E, F \in \mathcal{C}$ and $E \subset F$, then $F \setminus E \in \mathcal{C}$.*

Then:

(i) *If $\mathcal{C}$ has a nonempty interior in (Σ, d_λ), then $\mathcal{C} = \Sigma$.*
(ii) *If $\mathcal{C}$ is an F_σ-set in (Σ, d_λ), then either $\mathcal{C} = \Sigma$ or $\mathcal{C}$ is of the first category.*

The above theorem allows us to simplify several proofs presented in [3] and [4] and it has many further applications. For instance, let f be a positive Σ-measurable function. If f is not λ- essentially bounded (not λ-integrable), then $\mathcal{C} = \{A \in \Sigma : f \text{ is } \lambda - \text{essentially bounded on } A\}$ ($\mathcal{C} = \{A \in \Sigma : \int_A f \, d\lambda < \infty\}$) is of first category in (Σ, d_λ). Consider a measure $m : \Sigma \to X$ (X is a Banach space) and assume m is λ-absolutely continuous. If the total variation $\operatorname{var} m$ is not σ-finite, then $\mathcal{C} = \{A \in \Sigma : \operatorname{var} m \text{ is } \sigma - \text{finite on } A\}$ is nowhere dense in (Σ, d_λ). Similarly, if $m(\Sigma)$ is not relatively (norm) compact, then $\mathcal{C} = \{A \in \Sigma : m(\Sigma \cap A) \text{ is relatively compact}\}$ is nowhere dense in (Σ, d_λ).

It turned out that considerations presented in [D63] can be generalized and it was done in [D89]. Assume that $\mathcal{H}(X)$ is a class of subsets of a (Hausdorff) topological vector space X satisfying the following four conditions:

1. $\mathcal{H}(X)$ contains finite subsets of X,
2. $K \subset H \in \mathcal{H}(X) \;\Rightarrow\; K \in \mathcal{H}(X)$,
3. $K, H \in \mathcal{H}(X) \;\Rightarrow K \pm H \in \mathcal{H}(X)$,
4. $H \in \mathcal{H}(X) \;\Rightarrow\; \overline{H} \in \mathcal{H}(X)$.

Basic examples of classes $\mathcal{H}(X)$ are the following: finite subsets of X, sets of finite dimension in X, precompact subsets of X, bounded subsets of X. Let us formulate two interesting results from [D89].

Theorem 1.8. *Suppose that λ is an order continuous submeasure on Σ.*

1. *Let $m_n : \Sigma \to X$ be measures λ-absolutely continuous and such that $m_n(\Sigma) \notin \mathcal{H}(X)$ for $n = 1, 2, \ldots$. Then, for every sequence (L_n) of sets being a countable union of elements from $\mathcal{H}(X)$ (resp. a countable union of closed sets from $\mathcal{H}(X)$), the class $\{A \in \Sigma : m_n(A) \notin L_n \text{ for each } n\}$ is residual (resp. dense and G_δ) in (Σ, d_λ).*

2. *Let a measure $m : \Sigma \to X$ be λ-absolutely continuous and such that $m(\Sigma) \notin \mathcal{H}(X)$. Then the class $\{A \in \Sigma : m(\Sigma \cap A) \in \mathcal{H}(X)\}$ has an empty interior in (Σ, d_λ).*

One of consequences of the above result is the following

Corollary 1.9. *Let $m_n : \Sigma \to X$, $(n = 1, 2, \dots)$ be measures such that each m_n has infinite range. Then, for any two sequences (x_n) and (y_n) in X, there exists $A \in \Sigma$ such that $m_n(A) \neq x_n$ and $m_n(S \setminus A) \neq y_n$ for every n.*

The Baire category method was also the main tool in [D81] where we can find extensions and deep analysis of ideas from [4] concerning sets of vector measures having everywhere infinite total variations or noncompact ranges.

2. Orthogonally additive functionals

One of the first fields of interest of Lech Drewnowski was a description of orthogonally additive functionals — the topic he studied already with his Ph. D. tutor the well-known Polish mathematician Władysław Orlicz. The result turned out to be striking and still inspire other mathematicians: the papers [D4], [D5], [D6] of Drewnowski and Orlicz obtained more than 30 citations till now. The newest one is the paper [17], see also [8] or [87].

Let us consider a finite measure space (Ω, Σ, μ). We say that two elements x, y in the space of measurable functions $L^0(\mu)$ are orthogonal $x \perp y$ if $x \cdot y = 0$ in $L^0(\mu)$. A function $f : \mathbb{R} \times \Omega \to \mathbb{R}$ is called Carathéodory function if $f(r, \cdot)$ is μ-measurable for every fixed $r \in \mathbb{R}$ and $f(\cdot, t)$ is continuous for μ-a.e. $t \in \Omega$. Moreover, a map $\xi : A \to \mathbb{R}$ acting on a subset of $L^0(\mu)$ is called *orthogonally additive* if and only if $\xi(x + y) = \xi(x) + \xi(y)$ for every $x \perp y$. A map $\xi : A \to \mathbb{R}$ as above is called *continuous* if for any sequence $(x_n) \subset A$ such that $x_n \xrightarrow{\mu} x$ and there is $y \in L^0(\mu)$ with $|x_n|, |x| \leqslant y$, then

$$\xi(x_n) \to \xi(x).$$

A subset A is called *solid* if $x \in A$ whenever $|x| \leqslant |y|$ for some $y \in A$.

The main result is the following representation theorem [D4], [D5], [D6]:

Theorem 2.1. *Let E be a solid subset closed with respect to addition of orthogonal elements of the space of measurable functions $L^0(\mu)$ for some finite measure space (Ω, Σ, μ).*

A map $\xi : E \to \mathbb{R}$ is orthogonally additive and continuous if and only if

$$\xi(x) = \int_\Omega f(x(t), t) \mu(dt),$$

for some Carathéodory function f, $f(0, t) = 0$ a.e.

There are several generalizations of this result directly or indirectly related to the above theorem. For instance, for operators $T : C(K) \to X$, X a suitable Banach space, see [50], [7]. For operators $T : L_p(\mu, X) \to Y$; X, Y Banach spaces, see [98]. For operators on Sobolev spaces $T : W_n^p[a,b] \to L_1[a,b]$ see [96], [18]. In [8] the representation of orthogonally additive polynomials on a Banach lattice is given generalizing several earlier results.

The results above depend on the notion of orthogonality. It is worth mentioning that there is a related notion in arbitrary Banach spaces. Namely in a real Banach space X two vectors x, y are called orthogonal $x \perp y$ if and only if

$$\|x + \lambda y\| \geqslant \|x\| \qquad \forall \lambda \in \mathbb{R}.$$

Then Sundaresan in [120] proved the following theorem:

Theorem 2.2. *Let X be a Banach space and let Y be a locally convex space and $T : X \to Y$ be continuous. The map T is orthogonally additive if and only if*

(a) *T is linear whenever X is not isometric to a Hilbert space;*
(b) *$\exists\, \xi \in Y, \exists\, G : X \to Y$ linear and continuous such that*

$$T(x) = \|x\|^2 \xi + G(x),$$

whenever X is a Hilbert space.

3. Topological vector spaces

In the late seventies there was a growing interest in non-locally convex spaces. It appeared several deep results of Kalton, Peck, Roberts on that topic. In that atmosphere Drewnowski developed also in Poznań a systematic program of studying p-Banach and more general non-locally convex F-spaces (i.e., metrizable complete) to which he attracted a group of his pupils. The programme was loosely connected to earlier research in Poznań on modular spaces and Orlicz spaces. It was the time Kalton visited for the first time Poznań to meet Władysław Orlicz and work with Lech Drewnowski. A little bit later at some conference Kalton was wearing a T-shirt with a statement "$L_p, 0 < p < 1$" showing his enthusiasm for the non-locally convex case — the research programme which was summarized in the monograph [78]. The topic was then very "hot". The emphasis was put on finding to which extent the locally convex theory using a lot of duality can be transferred to the non-locally convex case where the Hahn-Banach theorem fails in general and so duality tools are not available.

This was the time of a very fast development of Drewnowski's seminar and time of a very intensive work of its participants: we meet in the seminar room of the Poznań branch of the Institute of Mathematics of the Polish Academy of Science at 9 am and work till 2 pm every Thursday. First, we presented new results (also own results) and later there were systematic presentations of some topics from existing books (topics like distributions, martingales, tensor products, H_p spaces and many more, elements of local theory of Banach spaces, probability methods) — in the meantime we discussed all the topics including politics which was also hot (Solidarity movement period). We learn a lot but we have also a lot of fun.

3.1. Minimal spaces

One of the important contributions of Drewnowski to the theory of non-locally convex spaces were his papers on minimal spaces. A topological vector space is *minimal* if and only if there is no strictly weaker vector Hausdorff topology on the space. In the locally convex setting these are exactly arbitrary products of one dimensional spaces. Drewnowski in his paper [D25] introduced the notion of *q-minimal* spaces as those for which every Hausdorff quotient is minimal (i.e., there is no non-trivial weaker vector topology at all). It was discovered among others by Drewnowski that minimal or q-minimal spaces play in the theory of non-locally convex F-spaces a role of finite dimensional spaces in the theory of Banach spaces.

As explained in [78] the following results hold:

- (Kalton-Shapiro [80, Th. 3.2]) A non-locally convex F-space is minimal if and only if it has no basic sequence.
- (Kalton [78, Th. 4.10]) Non-locally convex p-Banach space is q-minimal if and only if it is *atomic*, i.e., does not contain a proper closed infinite dimensional subspace.

The question on the existence of a q-minimal non-locally convex F-space (or q-minimal non-locally convex topological vector space) is still open. This problem as well as the problem of existence of an atomic F-space was posed in [107, p. 114]. The first example of a minimal non-locally convex F-space is given in a paper [76] where the famous method of Gowers and Maurey [58] is used (the same method was used for obtaining Banach spaces without an unconditional basic sequence).

In [D25] Drewnowski proved that:

Theorem 3.1. *Let Z be an F-space. Assume that X, Y are closed subspaces of Z such that any F-space V isomorphic simultaneously to subspaces of X and Y is automatically q-minimal. Then $X+Y$ is closed.*

The result is a non-locally convex version of the results of Gurarii [62], Rosenthal [108] (for Banach spaces) and Diestel and Lohman [24] (for general locally convex spaces). In fact a kind of "reverse" is also proved [D42]:

Theorem 3.2. *Let Y be a separable F-space and X be a non-minimal subspace of Y with infinite dimensional Y/X. Then there exists $Z \subseteq Y$, $X \cap Z = \{0\}$, such that $X + Z \neq \overline{X+Z} = Y$.*

This is a generalization to the non-locally convex case of the results of Murray [99] and Mackey [94] (for more references see the introduction [D42]).

The paper [D25] of Drewnowski found some applications. First, it is used in a paper [9] in the proof of the following result:

Theorem 3.3. *Let X be a Fréchet non-Montel space (i.e., it has closed bounded non-compact sets). If all subspaces of X have the density property then $X \subseteq Z \times \omega$, where Z is a Banach space.*

Let us recall that X has the *density property*, if every bounded set in the strong dual X' is metrizable — this condition is related to ultraproducts of locally convex spaces and was used by Bierstedt and Bonet in their study of distinguishedness of Köthe sequence spaces.

Second application is even more surprising — in fact, some stability results for basic sequences from the paper [D25] are used in order to prove in [26] that there exists a quotient X of C^∞ such that there is a non-splitting "good" (in the sense explained in [26]) exact sequence of Fréchet spaces of the form:

$$0 \longrightarrow X \longrightarrow Y \longrightarrow C^\infty(\mathbb{R}^d) \longrightarrow 0.$$

Operator $T : X \to Y$ between Banach spaces is called *strictly singular* if there is no infinite dimensional subspace Z of X such that T restricted to Z is an isomorphism. This class of operators (in fact, an operator ideal) introduced by Kato is quite important in the theory of operators on Banach spaces. A classical characterization of strictly singular operators reads as follows: a map $T : X \to Y$ is strictly singular if and only if for every infinite dimensional closed subspace Z of X there is another closed subspace $V \subset Z$ such that T restricted to V is compact [57, Th. III.2.1]. In the paper [D30] the following generalization to F-spaces is shown:

Theorem 3.4. *Let X, Y be F-spaces, $T : X \to Y$ a continuous linear operator. A map T is not an isomorphism on any non-q-minimal subspace of X if and only if for every closed subspace Z of X there is a further closed subspace $V \subset Z$ such that T restricted to V is compact.*

3.2. Mackey topology

In the paper [D36] Drewnowski together with his pupil M. Nawrocki studied the so-called Mackey topology for F-spaces. Let us recall that the *Mackey topology* of a topological vector space X is the strongest locally convex topology giving the same dual. In case of F-spaces it is just the strongest locally convex topology weaker than the original one. In the study of non-locally convex F-spaces it is important to find the Mackey topology (or its completion called the *Mackey hull*) which allows to use the Fréchet space theory. There is a large literature on that subject for concrete function F-spaces. For instance J. Shapiro in [116] found the Mackey hull for the Hardy space H^p for $0 < p < 1$. Kalton [74] found it for separable sequence Orlicz spaces ℓ_φ. In [D36] the Mackey topology was found in the non-separable case, i.e., for Orlicz functions φ without the so-called Δ_2-condition. The result was used later on by Nawrocki [101] in his study of the Orlicz-Pettis property for Orlicz and other non-locally convex F-spaces.

3.3. Bases

It is an important fact that in a Fréchet space every Schauder basis in the weak topology is automatically a Schauder basis in the original Fréchet space topology (for more general results in locally convex spaces see [69, 14.3.3, 14.3.4]). It was first observed by Stiles [117] that this result fails in some non-locally convex F-spaces like ℓ_p, $0 < p < 1$ (more examples was found by Shapiro [115]). Drewnowski proved in [D29] the following striking result:

Theorem 3.5. *Every non-locally convex F-space X which has a basis in the weak topology contains also a sequence which is a basis in the weak topology but not in the F-space topology.*

We say that a sequence $(x_n)_{n\in\mathbb{N}}$ is *topologically independent* if for any sequence of real scalars $(\lambda_n)_{n\in\mathbb{N}}$ the equality $\sum_{n=0}^{\infty} \lambda_n x_n = 0$ implies that $\lambda_n = 0$ for every $n \in \mathbb{N}$. A topologically independent sequence is called a *quasi-basis* for a real topological vector space X whenever it is linearly dense in X. In [D35] Drewnowski and his coauthors showed how to construct quasi-basis from an arbitrary countable, linearly dense linearly independent sequence. In particular, they proved:

Theorem 3.6. *Every separable topological vector space has a quasi-basis.*

3.4. Locally balanced topological groups

In [D79] Drewnowski obtained a very easy proof of several results on the so-called *Nevanlinna class*, i.e., a family of holomorphic functions on the unit disc satisfying

$$\|f\|_0 := \lim_{r\to 1} \int_{-\pi}^{\pi} \log^+ |f(re^{it})| dt < \infty$$

with the 0-neighborhood basis of its group topology given by balls with respect to $\|\cdot\|_0$. The *Smirnov class* is a subclass of the Nevanlinna class consisting of functions for which integrals above are uniformly integrable for all $r \in (0,1)$. Drewnowski just look at the Nevanlinna class as a general topological group with balanced basis of neighborhoods of zero. This abstract approach allows him to prove easily the result of Shapiro and Shields that the Smirnov class N^+ is the largest topological vector subspace of the Nevanlinna class N. The result of Yanagihara that N^+ is not locally bounded is provided with an easy proof. He also proved that no "reasonable" complete metrizable vector topology exists on N. This approach allows to get similar results in the several variable case as well, comp. [102] and for other classes see [64].

3.5. Descriptive set theory and continuity

The paper [D104] was inspired by the earlier work [82], where the authors asked the following question.

Problem 3.7. *Let X be a separable Fréchet space covered by a family $(K_\alpha : \alpha \in \mathbb{N}^{\mathbb{N}})$ of its compact subsets with $K_\alpha \subseteq K_\beta$ whenever $\alpha \leqslant \beta$ pointwisely. Is it then possible to find a linear functional f on X that is continuous on each of the sets K_α, and yet is discontinuous on X?*

The paper [D104] is very characteristic for the work of Lech Drewnowski: the author looked at the problem in a very systematic way and then using simple but very effective tools he solved it completely, by far in a more general setting than one could expect. In this case he showed that:

Theorem 3.8. *Let X be an arbitrary F-space (so not necessarily locally convex and not necessarily separable) covered by a family $(E_\alpha : \alpha \in \mathbb{N}^\mathbb{N})$ (a so-called resolution) of its arbitrary (not necessarily compact) subsets with $E_\alpha \subseteq E_\beta$ whenever $\alpha \leqslant \beta$ pointwisely. Then for arbitrary linear topological vector space Y every linear map $T : X \to Y$ that is continuous on each of the sets E_α is automatically continuous on X.*

The proof is a beautiful mathematical gem: the author first prove that T has a closed kernel and then he applies it to the map $G : X \times Y \to Y$, $G(x, y) = y - T(x)$, and the resolution $(E_\alpha \times Y)$ of $(X \times Y)$. The kernel of G is equal to the graph of T. Closed graph theorem does the work since without loss of generality we may assume that Y is an F-space as well.

In fact in [82] the problem was already solved in the negative if all K_α are absolutely convex. This led Kąkol to suspect that might be if K is a compact set in a Fréchet space X and a linear functional f on X is continuous on K then it must be continuous on the absolute convex closed hull $\overline{\text{aco}}K$ of K. It is easy to show that in general the answer is no, but Drewnowski showed that the answer is yes whenever $\overline{\text{aco}}K$ is contained in the linear span of K.

3.6. Measures with values in topological Riesz spaces

Suppose that Σ denotes a σ-algebra of subsets of a set S. If E is a vector lattice, then a family of additive set functions $m : \Sigma \to E$ is an ordered linear space with respect to the natural partial order $m_1 \leqslant m_2 \iff m_1(A) \leqslant m_2(A)$ for all $A \in \Sigma$. By an *absolute majorant* for m we mean an additive set function $\gamma : \Sigma \to E_+$ such that $\pm m \leqslant \gamma$ (E_+ is the positive cone in E). The smallest absolute majorant for m, if exists, is called the *modulus* of m and denoted $|m|$. It is easily seen that if E is Dedekind complete and $m : \Sigma \to E$ has order bounded range, then the modulus $|m|$ exists and is given by each of the equalities:

$$
\begin{aligned}
|m|(A) = \sup\Big\{\sum_{D\in\pi} |m(D)| : \pi \in \Pi(A)\Big\} &= \sup\{m(B) - m(A \setminus B) : B \subset A\} \\
= \sup\{m(B) - m(C) : B, C \subset A\} &= \sup\{|m(B) - m(C)| : B, C \subset A\}
\end{aligned}
$$

where $\Pi(A)$ stands for all Σ-partitions of A. Moreover, whenever the above formulas make sense for each A, they define a set function which is precisely the modulus of m. When this is the case, we will say that the modulus $|m|$ *exists properly*, or that m has a *proper modulus*. It is still an open question whether a measure with the order bounded range may have a non-proper modulus.

In [60], [86], [112] and [114] the following natural problems were raised and investigated: conditions providing existence of the modulus, a form of the modulus, what properties of m are inherited by $|m|$. These problems were attacked in [D94] and [D102]. Clearly order boundedness of a range is a necessary condition for existence of the modulus but this condition is not sufficient. In [D102] one can find an example of a measure with values in c (i.e., in the Banach lattice of convergent sequences) that has no absolute majorant. The paper [D94] shows that if m is the indefinite Bochner integral of a function f with respect to a signed measure $\nu : \Sigma \to \overline{\mathbb{R}}$, then the modulus $|m|$ exists properly and $|m|(A) = (\mathrm{B}) \int_A |f|\, d|\nu|$ where $|f|(s) = |f(s)|$ and $|\nu|$ is the total variation of ν. As it was proved in [D94] similar result holds for the indefinite Pettis integral. We cannot expect full analogy because for every Banach lattice E not order isomorphic to a (closed) sublattice in some $C(K)$ there exists a Pettis integrable function $f : \mathbb{N} \to E$ whose indefinite Pettis integral is not order bounded (see [D94] Remark 4). A worse situation may happen – there exists a Pettis integrable function whose modulus is not weakly measurable (a suitable example, due Fremlin, is also presented in [D94]). However the following result is true.

Theorem 3.9. *Let m be the indefinite Pettis integral of a strongly measurable function f with respect to a positive measure ν. Then the following statements are equivalent:*

(a) *$|m|$ exists properly and is σ-additive.*
(b) *$|m|$ exists and is σ-additive.*
(c) *$|f|$ is Pettis integrable.*
(d) *$|f|$ is Pettis integrable and its indefinite Pettis integral is the modulus of m.*
(e) *The series $\sum_n |(\mathrm{P}) \int_{A_n} f\, d\nu|$ converges for every disjoint sequence (A_n) in Σ.*

If these conditions hold, then $|m|(A) = \sup_B (\mathrm{B}) \int_{A\cap B} |f|\, d\nu$ where B runs over the family of sets on which f is Bochner integrable.

It is a non trivial work to find a measure whose modulus is not a measure (i.e., it is not σ-additive) First example was constructed by G. Groenewegen (see [60]) and the next one was shown in [D94]. An essential part of [D102] is devoted to an analysis of the so-called property (S) (which was introduced by G. Groenewegen, compare [60] and [61]). The property allows to activate a machinery producing a lot of examples of measures whose modulus exists and is not σ-additive. Let us recall that a Banach lattice E has the *property (S)* if for every subseries convergent series $\sum_n x_n$ with order bounded set $\{\sum_{n\in M} x_n : M \subset \mathbb{N} \text{ finite}\}$, the series $\sum_n |x_n|$ is convergent. $C(K)$-spaces and Banach lattices with order continuous norms are typical examples of spaces with the property (S) while nonseparable Orlicz spaces $L^\varphi[0,1]$ have not the property. The machinery mentioned above is described in [D102]:

Theorem 3.10. *Let E be a σ-Dedekind complete Banach lattice without property (S). If the σ-algebra Σ admits an atomless probability measure, then there exists an atomless measure $m : \Sigma \to E$ with a non-σ-additive proper modulus.*

Papers [112] and [114] contain conditions providing σ-additivity of the modulus (but these sufficient conditions can be used only for particular classes of Banach lattices). Some necessary and sufficient conditions for σ-additivity of $|m|$ were obtained in [D102]. One of them is the following.

Theorem 3.11. *Let E a topological Riesz space ($=$ a locally solid Riesz space in the terminology of [2]) have a basis of neighborhoods of zero consisting of solid order closed sets. If a measure $m : \Sigma \to E$ has a proper modulus $|m|$, then $|m|$ is a measure if and only if m is absolutely exhaustive (i.e. for every disjoint sequence (A_j) the series $\sum_j |m(A_j)|$ is Cauchy).*

3.7. Structure of topological Riesz spaces

Inspired by some results of Lavrič (see [89]) Drewnowski investigated in [D100] F-lattices ($=$ metrizable complete topological Riesz spaces) E that have the *property (λ)*: every nondecreasing function $f : [0,1] \to E$ has at most countably many points of discontinuity. He simplified and shortened Lavrič construction of an increasing function $f : [0,1] \to \ell^\infty$ without any points of continuity and, applying this construction, extended two Lavrič's results: a characterization of order continuity as well as a relationship between separability and the property (λ). Namely, he showed that in the class of σ-Dedekind complete F-lattices $(E, \|\cdot\|)$ the (λ) property is just equivalent to the order continuity of $\|\cdot\|$ and every separable F-lattice has the property (λ). Moreover, Drewnowski found a negative answer to Lavrič's question: if every Banach lattice E which contains no order copy of ℓ^∞ must have the property (λ). It turns out that the space of those functions $f : [0,1] \to \mathbb{R}$ that are continuous from the left, continuous from the right at zero, and the right-hand limit $f(t+)$ exists for each $t \in [0,1)$ forms a Banach lattice with respect to the sup norm which does not contain any order copy of ℓ^∞ and has no property (λ). It was also obtained in [D100] that the property (λ) is inherited from E onto spaces of vector valued functions $L^p(\mu, E)$ (for arbitrary μ and $p \in [1,\infty)$) and $C(K,E)$ for some special compact sets K (e.g., Eberlein compacts).

One of the most important classes of topological Riesz spaces (E,τ) are spaces with the *order* (respectively, *σ-order*) *continuous topologies* τ, i.e., nets (respectively sequences) of positive elements decreasing to zero are τ-convergent to zero. These two classes are different — the quotient norm topology in ℓ^∞/c_0 is σ-order continuous but not order continuous. There exists around two hundred various characterizations of Banach lattices whose norm topology is order continuous (see [97] and [132]). One of the main characterization is due to G.J. Lozanovskii and it says that a σ-Dedekind complete Banach lattice E has order continuous norm topology if and only if E contains no order copy of ℓ^∞ and if and only if E contains no isomorphic ($=$ linearly homeomorphic) copy of ℓ^∞. Similar situation holds with copies of c_0 – a Banach lattice E does not contain any order copy of c_0 if and only if E does not contain any isomorphic copy of c_0 if and only if E is a *KB-space* (i.e., bounded increasing nets of positive elements are convergent).

Paper [D93] contains important extensions of the above equivalences to a broad class of topological Riesz spaces X of λ-measurable functions satisfying some completeness type assumptions. A space X considered in [D93] is equipped with a Hausdorff locally solid topology, and it forms an order ideal in the Riesz space $L^0(\lambda)$ of λ-equivalence classes of measurable scalar functions where λ is a semi-finite measure. A theory of topological vector spaces of Bochner measurable functions $L(\eta, E)$ was developed in [D101]. The theory is sufficiently general to include topological Riesz spaces of measurable functions as a particular case because there are considered functions which are almost everywhere limits of simple functions with values in an F-space E and equality almost everywhere is taken with respect to a special type of a submeasure η. The main parts of [D101] are devoted to investigations of these properties of spaces $L(\eta, E)$ which are related to the order continuity of their topology, to a counterpart of KB-spaces, and to the topological completeness.

It is also worth to quote a characterization of σ-order continuity formulated in [D102]:

Theorem 3.12. *A σ-Dedekind complete topological Riesz space E has σ-order continuous topology if and only if order bounded monotone Cauchy sequences are convergent and there is no positive linear isomorphic embedding of the Banach lattice c of convergent scalar sequences into E.*

It is interesting that the phrase "E contains no positive copy of c" cannot be replaced with "E contains no order isomorphic copy of c".

The paper [D112] contains studies of F-lattices that are curious from many points of view. Namely it was constructed an F-lattice λ_0 consisting of sequences and its separable order ideal λ_{00} such that (see [D112] 15. Conclusion):

(a) λ_0 and λ_{00} are quasi-L_0-like, i.e., for every neighborhood U of zero in λ_0 (resp. in λ_{00}) there exists a finite-codimensional subspace L in λ_0 (resp. in λ_{00}) and a natural number m such that $L \subset U + \cdots + U$ (m summands). In particular the spaces are not locally pseudo-convex, and their duals are poor.

(b) Every infinite-dimensional locally bounded closed subspace of λ_{00} contains a copy of c_0 that is extendable to a copy of ℓ^∞ in λ_0, and λ_{00} is not complemented in λ_0.

(c) The quotient λ_0/λ_{00} is such that for every neighborhood U there is an $m \in \mathbb{N}$ such that $\lambda_0/\lambda_{00} \subset U + \cdots + U$ (m-summands) and contains copy of ℓ^∞/c_0.

(d) λ_0 contains a copy of the space ℓ^p for each $p \in (0, 2]$.

(e) λ_{00} contains a closed Schwartz subspace of infinite dimension that is non-isomorphic to $\mathbb{R}^{\mathbb{N}}$.

(f) λ_0 and λ_0/λ_{00} have the *bounded multiplier property* (i.e., if a series $\sum_n x_n$ is unconditionally convergent, then $\sum_n t_n x_n$ converges for every bounded sequence (t_n) of scalars).

It is well known that the algebraic sum of two closed linear subspaces in a Banach space need not be closed. A situation is even worse because, as G.W. Mackey showed in [93], if Y is a closed linear subspace of infinite dimension and infinite codimension in a Banach space X then there exists a closed linear subspace $G \subset X$ such that $Y \cap G = \{0\}$ and $Y + G \neq \overline{Y + G}$. On the other hand H.P. Lotz, and independently E.B. Davies, proved (see [92], [21]) that if F_1, F_2 are closed ideals in a Banach lattice then $F_1 + F_2$ is closed. The quoted statement remains true in the class of F-lattices (see [97] the proof of Proposition 1.2.2). The assumption of a (topological) completeness is crucial because in [133] there was constructed a normed lattice and a closed ideal I in it such that $I + I^{\mathrm{d}}$ is not closed (I^{d} is the orthogonal complement of I and so it is possible that the algebraic sum of a band and a closed ideal is not closed). The paper [D115] contains a nice generalization of the Lotz-Davies result:

Theorem 3.13. *Let E be an F-lattice, and let $(I_\gamma)_{\gamma\in\Gamma}$ be a family of closed ideals in E. Define $I = \sum_{\gamma\in\Gamma} I_\gamma$ to be the set of all elements $z \in E$ that are of the form $z = \sum_{n=1}^{\infty} x_n$, where $x_n \in I_{\gamma_n}$ for every n, and (γ_n) is a sequence in Γ. Then I is the smallest closed ideal in E that contains all the ideals I_γ. Moreover, every element $z \in E$ can be represented so that also the series $\sum_{n=1}^{\infty} |x_n|$ converges in E.*

3.8. The Orlicz-Pettis type theorems

Classical Orlicz-Pettis theorem says that if a series $\sum_n x_n$ in a normed space X is weakly subseries convergent, then it is norm subseries (and so unconditionally) convergent. This can be reformulated in the following equivalent form: if an additive set function m mapping a σ-algebra into X is weakly countably additive (i.e., $x^* \circ m$ is countably additive for each $x^* \in X^*$), then m is norm countably additive (clearly it is enough to consider only one σ-algebra, namely the family $\mathcal{P} = \mathcal{P}(\mathbb{N})$, i.e., the power set of $\mathbb{N}$). The Orlicz-Pettis theorem has been a subject of many investigations leading to various generalizations (there exists several surveys discussing progress in these investigations, e.g., [75], [46], [D96]). Drewnowski joined to this trend of research and obtained a lot of valuable results. He gave simpler, more direct proofs of two Kalton's theorems ([72]) concerning group valued set functions ([D13],[D18]):

Theorem 3.14. *Let α, β be two Hausdorff group topologies on an abelian group G and suppose $m : \mathcal{P} \to G$ is α-countably additive.*

(a) *If β has a base of sequentially α-closed neighborhoods of 0 and $m(\mathcal{P})$ is β-separable, then m is β-countably additive.*
(b) *The condition $\alpha \leqslant \beta$ implies that m is β-countably additive if and only if $m(\mathcal{P})$ is β-separable and the topology induced by β on $m(\mathcal{P})$ is metrizable by a complete metric.*

It was shown in [D18] that Kalton-Drewnowski theorems can be extended to exhaustive additive set functions. It turns out that the Orlicz-Pettis type theorem works in some spaces X of measurable functions, i.e., X, equipped with suitable Hausdorff locally solid topologies, are order ideals in the Riesz space of all (equivalence classes of) measurable functions with respect to a semi-finite measure λ. Indeed, the paper [D93] contains the following result.

Theorem. *If τ is σ-order continuous and $m : \mathcal{P} \to X$ is σ-additive (exhaustive) with respect to the topology of convergence in measure λ on all sets of finite measure, then it is also τ-countably additive (exhaustive).*

An extension of the above theorem to spaces $L(\eta, E)$ of vector-valued Bochner measurable functions (mentioned in the Section 3.7) was obtained in [D101]. It is worth to recall a surprising version of the Orlicz-Pettis theorem for general topological Riesz spaces obtained in [D92]:

Theorem 3.15. *If an additive set function m maps $\mathcal{P}$ into a Riesz space E and m is countably additive (exhaustive) with respect to some Hausdorff order continuous topology on E, then it is countably additive (exhaustive) with respect to every Hausdorff order continuous topology on E.*

3.9. Series in topological vector spaces

Drewnowski's investigations devoted to vector series referred to natural classical problems concerning various types of convergence (e.g., subseries convergence) and maps preserving convergence or transferring one type of series to another (e.g., convergent series to series whose set of partial sums is bounded). In an early paper [D16] he considered the set P of all permutations on $\mathbb{N}$ endowed with the topology of pointwise convergence on $\mathbb{N}$ and proved two facts: P is of the second Baire category (but it is not complete) and if (x_n) is a sequence in a sequentially complete topological vector space (or more generally in a sequentially complete topological group), then the set $\{p \in P : \sum_{n=1}^{\infty} x_{p(n)}$ converges$\}$ as well as the set $\{p \in P :$ the sequence $(\sum_{n=1}^{k} x_{p(n)})_{k=1}^{\infty}$ is bounded$\}$ is either identical with P or of the first category. Many years latter Drewnowski came back to this topic but he investigated it in broader and more general context (see [D103]). Namely, every map ϱ of $\mathbb{N}$ into itself induces a transformation of a sequence (x_n) (of scalars or vectors) and the corresponding series $\sum_n x_n$ to the sequence $(x_{\varrho(n)})$ and the (formal) series $\sum_n x_{\varrho(n)}$. A number of authors have been interested in characterizing functions ϱ that the following conditions are satisfied:

- (bb) if the set of partial sums of a series $\sum_n x_n$ is bounded, then the same holds for the series $\sum_n x_{\varrho(n)}$,
- (cb) if $\sum_n x_n$ converges, then the set of partial sums of the series $\sum_n x_{\varrho(n)}$ is bounded,
- (cc) if $\sum_n x_n$ converges, then $\sum_n x_{\varrho(n)}$ is also convergent.

It is shown in [D103] how one can apply elementary methods of functional analysis to cope with problems formulated above. This approach was used to prove that for series in arbitrary topological vector space properties (bb), (cb), (cc) are mutually equivalent for an arbitrary ϱ and additionally equivalent to an intrinsic property of ϱ. Moreover, similar relationships hold for more general transformations of $\sum_n x_n$ onto $\sum_n T_{\varrho(n)}(x_{\varrho(n)})$ where (T_n) is a sequence of some continuous and linear operators.

A subject of two papers mentioned above was also developed in [D99]. For broad classes of (Hausdorff) topological vector spaces X and Y there are given complete characterizations of those maps $f : X \to Y$ for which the induced transformation of series $\sum_n x_n \to \sum_n f(x_n)$ preserve properties such as convergence, boundedness of the set of partial sums, absolute convergence and unconditional convergence. A special part is played by locally additive maps f, i.e., there exists a neighborhood U of zero in X such that $f(u+v) = f(u) + f(v)$ whenever $u, v \in U$. It is interesting that every continuous at zero locally additive $f : X \to Y$ is uniquely determined by a continuous $\mathbb{R}$-linear operator $T : X \to Y$, i.e., f and T coincide in a neighborhood of zero in X (this observation was also made in [D99]). It turns out (see [D99] Theorem 1) that:

Theorem 3.16. *For a metrizable topological vector space X and a topological vector space Y which has no subspace isomorphic to the countable product of one dimensional spaces, the following statements are equivalent.*

(a) *A function $f : X \to Y$ is continuous at zero and locally additive.*
(b) *f sends convergent series in X to convergent series in Y.*

Maps preserving absolute convergence are characterized in the following way ([D99] Theorem 3).

Theorem 3.17. *A map f between F-normed spaces $(X, \|\cdot\|_X)$ and $(Y, \|\cdot\|_Y)$ sends absolutely convergent series to absolutely convergent series if and only if there exists a constant K such that $\|f(x)\|_Y \leqslant K\|x\|_X$ in a neighborhood of zero in X.*

A description of functions preserving unconditional convergence is more complicated. Namely (see [D99] Theorem 7):

Theorem 3.18. *For a function f mapping an F-space $(X, \|\cdot\|_X)$ into a normed space $(Y, \|\cdot\|_Y)$ the following are equivalent.*

(a) *f sends unconditionally Cauchy series to unconditionally Cauchy series.*
(b) *f sends unconditionally Cauchy series to series with bounded set of all partial sums.*
(c) *There are constants c and K such that*

$$\left\|\sum_{i=1}^{n} f(x_i)\right\|_Y \leqslant K \max\nolimits_{F\subset\{1,\dots,n\}} \left\|\sum_{k\in F} x_k\right\|_X$$

whenever $\max_{F\subset\{1,\dots,n\}} \|\sum_{k\in F} x_k\|_X \leqslant c$.

Drewnowski studied also *rare subseries*, i.e., lacunary and zero-density subseries. This area of research goes back to a 1930 result due to H. Auerbach: a scalar series is absolutely convergent if all its zero-density subseries converge. Let us recall that a subseries $\sum_k x_{n_k}$ of a given series $\sum_n x_n$ in a topological vector space is *zero-density* (resp. *lacunary*) if $\lim_{k\to\infty} k/n_k = 0$ (resp. $n_{k+1} - n_k \xrightarrow[k\to\infty]{} \infty$).

In [D95] there were investigated topological vector spaces with the *lacunary convergence property* (= if each lacunary subseries of a series $\sum_n x_n$ is convergent, then $\sum_n x_n$ converges) and with the *zero-density convergence property* (which is defined analogously: if each zero density subseries of a series $\sum_n x_n$ is convergent, then $\sum_n x_n$ converges). It was already known from [D91] that the space $L^0(\lambda)$ (the space of all equivalence classes of measurable functions with respect to a measure λ) with the topology of convergence in measure and finite λ has the lacunary convergence property (this fact was also proved, in a simpler manner, in [D97] and was generalized to semi-finite λ's providing some completeness property of $L^0(\lambda)$). The most difficult result of [D95] is that if a Banach space E contains no copy of c_0 and λ is finite, then the space $L^0(\lambda, E)$ of Bochner measurable functions has the lacunary convergence property. The same holds in a more general situation, i.e., for some vast class of spaces continuously included in $L^0(\lambda, E)$ (see [D95] Theorems 10.3 and 10.4).

An absence of a copy of c_0 is necessary for the lacunary convergence property and sometimes a lack of c_0 is also sufficient for the property. Indeed, as it was noticed in [D95] for sequentially complete locally pseudo-convex spaces the lacunary convergence property as well as the zero-density convergence property are equivalent to the condition that a space contains no isomorphic copy of c_0. However an F-space was constructed that contains no copy of c_0 but lacks the lacunary convergence property. Another example presented there shows that in general the zero-density convergence property $\not\Rightarrow$ lacunary convergence property. Another interesting observation made in [D95] is the following: if every lacunary subseries of a series $\sum_n x_n$ converges than the set $\{\sum_{n\in F} x_n : F \subset \mathbb{N} \text{ is finite}\}$ need not be bounded but it is always additively bounded, i.e., for every neighborhood U of zero the set of finite sums is included in $U^m = U + \cdots + U$ where the number m of summands depends on U.

4. Barreledness for non-complete spaces

Drewnowski's interest in barreledness was born in early nineties along to his collaboration with Sevilla group especially with Pedro Paúl. The root seems to go back to his interest in the Nikodým Boundedness Theorem already in seventies (see Section 1.1 above). Let us recall that the theorem says that for every σ-algebra Σ of sets every family of bounded finitely additive scalar set functions on Σ which is pointwisely bounded is automatically uniformly bounded on Σ. This, of course, reminds the famous Principle of Uniform Boundedness of Banach and Steinhaus for linear operators on Banach spaces or F-spaces. In fact, the class of barrelled spaces is a "good" class of domain spaces for the Principle to hold (the same is true for the Closed Graph Theorem). Let us recall that the locally convex space E is barreled if every pointwisely bounded set of continuous functionals on E is equicontinuous. Coming back to Nikodým Theorem, we call rings of sets Σ for which the Nikodým Boundedness Theorem holds to have *Nikodým Property* and as shown by Schachermayer [111] this is equivalent with barreledness of the space $S(\Sigma)$ of Σ-simple functions with the sup-norm.

Already in seventies and eighties there was a huge literature on barreledness. Several authors tried to find some weaker or stronger versions of barreledness related to weaker or stronger versions of the Principle of Uniform Boundedness and they compared these properties as well as various versions of Baire property (crucial in the proof of barrelledness/Banach-Steinhaus Theorem). These efforts are well documented in the literature (see the monographs of Jarchow [69] or Perrez-Carreras and Bonet [104]). Drewnowski went in a completely other direction: the Principle of Uniform Boundedness is so useful that he wanted to know which spaces satisfy the Principle as it is even though they are not complete — like spaces of simple (vector-valued) functions or spaces of Pettis integrable functions. For a relatively new survey on the Nikodým property and barreledness of spaces of simple and measurable functions see [44].

4.1. Nikodým property

The first result in this area we have to mention is Drewnowski's vector valued version of Nikodým Boundedness Theorem [D11]:

Theorem 4.1. *Let Σ be a σ-ring of sets, let X be a barreled locally convex space and let $\mathcal{M}$ be a family of additive X-valued set functions on Σ with bounded range. If the family $\mathcal{M}$ is pointwise bounded then it is uniformly bounded on Σ.*

A generalization of Drewnowski's theorem is proved in [83].

In [D78] Drewnowski with his coauthors proved that the ring $\mathcal{Z}$ of density zero subsets of integers has the Nikodým property. Let us recall that the set $A \subset \mathbb{N}$ has *density zero* if

$$d(A) := \limsup_{n\to\infty} \frac{\operatorname{card}\{k \in A : k \leqslant n\}}{n} = 0.$$

One can generalize the density taking instead of the fraction above the sum $\sum_j t_{nj}\chi_A(j)$, where $T = (t_{nj})$ is the non-negative matrix and χ_A denotes the characteristic function of the set A. In [D98] the authors proved that the ring $\mathcal{Z}_T$ of sets with the above generalized density zero has the Nikodým property if and only if it has the *absolute summability property*, i.e., for every sequence $(x_n)_{n\in\mathbb{N}}$ of complex scalars the convergence of subseries $\sum_{n\in A} x_n$ for each $A \in \mathcal{Z}_T$ implies that $\sum_{n=1}^{\infty} |x_n| < \infty$. This gives a surprising connection between the Nikodým boundedness circle of ideas and the Orlicz–Pettis type series convergence problems. This ideas were developed further in a series of papers of Boos and Leiger [13], [14], where the connection with the so-called Hahn property is discovered. More classes of rings with the Nikodým property are found in [D83]. For other sufficient conditions see [118], [119] and the survey [44] . Let us note that the characterization of rings with the Nikodým property is not known.

As we mentioned, a ring Σ has the Nikodým property if and only if the space $S(\Sigma)$ is barreled. In [D27] Drewnowski with his coauthors proved that if Σ is an infinite algebra then $S(\Sigma)$ is never ultrabornological (a property of domains D which gives for many range spaces R the Closed Graph Theorem for operators $T : D \to R$).

4.2. Barreledness of spaces of vector valued functions

It seems that the first space where Drewnowski and his collaborators want to study barreledness was the space of Pettis integrable functions.

Let us recall that the space of Pettis integrable functions $f : \Omega \to X$, X a normed space, over a measure space (Ω, Σ, μ) consists of functions f for which the map $t \mapsto \langle x', f(t) \rangle$ is an integrable function for every $x' \in X'$ and for every measurable set $A \in \Sigma$ there is $x \in X$ such that

$$\langle x', x \rangle = \int_A \langle x', f(t) \rangle d\mu(t) \qquad \text{for every } x' \in X',$$

of course, the functions g, f such that $x' \circ f = x' \circ g$ μ-a.e. for any $x' \in X'$ are identified. The space of Pettis integrable functions is denoted by $\mathcal{P}(\mu, X)$ and it is equipped with the norm:

$$\|f\| := \sup \left\{ \int_\Omega |\langle x', f(t) \rangle| d\mu(t) : x' \in X', \|x'\| \leqslant 1 \right\}.$$

Clearly, completeness of a normed space implies barreledness. Unfortunately, as shown by Pettis [105] (for infinite dimensional Hilbert space X) and independently, by Thomas [124] and Janicka and Kalton [68] (for arbitrary infinite dimensional Banach space X) the space of Pettis integrable functions is *never* complete. Nevertheless, the three collaborating authors proved in [D68] (another proof is contained in [D70]) the following theorem:

Theorem 4.2. *The space $\mathcal{P}(\mu, X)$ is barreled for every Banach space X and every measure μ.*

Moreover, as it was proved in [D81] (p. 9 and Theorem 5.6), the space $\mathcal{P}(\mu, X)$ is never ultrabarrelled if μ is atomless, and additionally $\mathcal{P}(\mu, X)$, for a separable X, is an $F_{\sigma,\delta}$- but not F_σ-set in the space of μ-continuous measures on Σ.

The above result as well as further results on barreledness of spaces of simple functions was a motivation for further search of natural non-complete normed spaces which are barreled. The taste of results they obtained is given by the following one (see [D69]). First let us recall that if (Ω, Σ, μ) is a finite measure space, Λ a complete, solid locally convex lattice of scalar measurable functions defined on Ω, and X a normed space then we define the space $\Lambda(X)$ of strongly measurable functions $f : \Omega \to X$ such that $\|f(\cdot)\| \in \Lambda$.

Theorem 4.3. *If Λ is order-continuous and barreled and the measure is atomless, then $\Lambda(X)$ is barreled even for a non-barreled space X. If Λ is quasibarreled, then so is $\Lambda(X)$.*

Freniche [49] proved that the space of vector valued simple functions $S(\Sigma, X)$ for infinite algebra of sets Σ is barreled if and only if both X and $S(\Sigma)$ are barreled and X is nuclear. Drewnowski and his coauthors [D72] prove the analogue of this result for the space $S(\mu, X)$ for a finitely additive positive set function μ, where we identify functions μ-equal a.e. They also proved that $S(\mu, X)$ is not ultrabornological.

Finally, they showed [D80] that:

Theorem 4.4. *Let Ω be a set and X a barreled normed space. If* $\operatorname{card}\Omega$ *or* $\operatorname{card}X$ *is nonmeasurable, then the space of bounded X-valued functions $\ell_\infty(\Omega, X)$ is barrelled.*

Let us note that continuing this line of research Ferrando and Lüdkovsky proved [43] that the space $c_0(\Omega, X)$ of vanishing at infinity functions on an arbitrary set Ω with values in a locally convex space X is barreled if and only if X is barreled. Similar results for other subspaces of $\ell_\infty(\Omega, X)$ are found in [45].

4.3. Boolean algebras of projections

Drewnowski with his collaborators from Sevilla developed a method of proving barreledness via constructing of a good Boolean algebra of projections. Let us recall that for a ring of sets Σ the family of linear continuous projections $(P_S)_{S\in\Sigma}$ on a locally convex space X is called a Boolean ring of projections if $P_{S_1\cup S_2} = P_{S_1} + P_{S_2}$ and $P_{S_1\cap S_2} = P_{S_1}\circ P_{S_2}$. In particular, in [D68] barreledness of the space of Pettis integrable functions is proved by applying the following result:

Theorem 4.5. *Let a locally convex space X admit a Boolean algebra of projections $(P_S)_{S\in\Sigma}$ where (Ω, Σ, μ) is a measure space with Σ a σ-algebra, μ a σ-additive atomless measure, such that*

1. *for every $x \in X$ holds $\lim_{\mu(S)\to 0} P_S(x) = 0$;*
2. *for every family of pairwise disjoint sets $(S_n) \subset \Sigma$ whenever $P(S_n)(x_n) = x_n$ for every $n \in N$ then $(x_n)_{n\in\mathbb{N}}$ contains a subsequence which produces a convergent series.*

If X is quasibarreled then it is barreled.

In [D73] a similar methods were applied to a larger variety of spaces. In [D78] another result of a similar spirit was proved:

Theorem 4.6. *Let X be a barreled locally convex space with an equicontinuous ring of projections $(P_S)_{S\in\Sigma}$, where the ring of sets Σ has the Nikodým property. Then also the subspace $\Sigma(X) := \bigcup_{S\in\Sigma} P_S(X)$ of X is barreled.*

A survey of this ideas is presented in [D87]. Another approach to prove barreledness was developed in [D70].

4.4. Other results

In a [22] De Wilde and Tsirulnikov have shown that if Y is a dense barrelled subspace of a B-complete locally convex space X, then there is an order-reversing correspondence between the closed subspaces M of X *transversal* to Y (i.e. satisfying $M \cap Y = \{0\}$) and the weaker barreled topologies on Y. In the paper

[D39] this result was generalized to topological vector spaces (i.e., local convexity was removed). In the paper [D37] Drewnowski solved in the negative the follwing problem posed in the paper [22]:

Problem 4.7. *Suppose Y is a dense barrelled subspace of uncountable codimension in a Banach space X. Does there always exist an infinite dimensional closed subspace M of X transversal to Y?*

5. Spaces of operators

Again in nineties Drewnowski worked intensively on various properties of spaces of vector valued measures (like $ba(\Sigma, X)$ the space of finitely additive bounded X-valued set functions on a σ-algebra Σ, its subspace $ca(\Sigma, X)$ of countable additive elements (i.e., measures) and a further subspace of countable additive measures with compact range $cca(\Sigma, X)$), spaces of vector valued continuous functions (like the space $C(K, X)$ of continuous X-valued functions and its superspaces $C(K, X, w)$ of weakly continuous functions $f : K \to X$ and $C(K, X', w^*)$ the space of weak* continuous functions — all of them equipped with the topology of uniform convergence), spaces of operators (like the space of all linear continuous operators $L(X, Y)$ and of compact operators $K(X, Y)$ — all of them equipped with the topology of uniform convergence on bounded sets). All these topics can be unified in the language of tensor products, more precisely ε-product of Schwartz.

Let us recall that for any Banach spaces X, Y its ε-product of Schwartz is by definition the space $K_{w^*}(X', Y)$ of compact weak*-weak continuous linear operators $T : X' \to Y$. It is naturally contained in the space $L_{w^*}(X', Y)$ of all weak*-weak continuous operators $T : X' \to Y$. Let us observe that the space of compact operators $K(X, Y)$ can be naturally identified with $K_{w^*}(X'', Y)$ while the space of weakly compact operators $W(X, Y)$ identifies with $L_{w^*}(X'', Y)$. Similarly, $ca(\Sigma, X)$ is isomorphic to $L_{w^*}(X', ca(\Sigma)) \cong L_{w^*}(ca(\Sigma)', X)$ while $cca(\Sigma, X)$ is isomorphic to $K_{w^*}(X', ca(\Sigma)) = K_{w^*}(ca(\Sigma)', X)$. Finally, it is known that $C(K, X', w^*)$, $C(K, X', w)$ and $C(K, X')$ are naturally isomorphic to spaces $L(X, C(K))$, $W(X, C(K)) = L_{w^*}(X'', C(K))$ and $K(X, C(K)) = K_{w^*}(X'', C(K))$, respectively.

There are three main types of results Drewnowski obtained for the above kind of spaces. First, he studied the so-called Gelfand-Phillips property connected with the Josefson-Nissenzweig theorem. Then he asked when the spaces in question contain isomorphic copies of c_0 and ℓ_∞ — here again the Josefson-Nissenzweig theorem is a tool for producing a copy of c_0 and Rosenthal's theorem becomes a tool to produce a copy of ℓ_∞ out of an existing copy of c_0. Finally, Drewnowski asked if one of the considered spaces is complemented in the bigger one (a prototypic question is if $K(X, Y)$ can be complemented in $L(X, Y)$) and the main tool to contradict complementation is to produce a complemented copy of c_0 in the smaller space such that it extends to a copy of ℓ_∞ in the bigger space via a variant of the Rosenthal theorem (of course, c_0 is never complemented in ℓ_∞). This considerations also lead to results about injectivity of studied spaces.

5.1. Gelfand-Phillips property

Let us recall that a subset A in a Banach space X is called *limited* if for every weak-star null sequence $(x_n^*) \subseteq E'$ holds

$$x_n^*(x) \to 0 \qquad \text{uniformly on } A.$$

A Banach space X has the *Gelfand-Phillips property* if every limited subset of X is relatively compact. The limited sets are related to one of the very few deep results true for all infinite dimensional Banach spaces:

Theorem 5.1. *(Josefson-Nissenzweig 1975 [23, Ch. XII]) The unit ball in an infinite dimensional Banach space is never limited.*

Later the result was generalized to arbitrary Fréchet spaces by Bonet, Lindström and Valdivia [12]: they showed that a Fréchet space contains bounded non-limited sets if and only if it is not Montel (i.e., there are bounded non-relatively compact sets in the space).

The Gelfand-Phillips property was studied by Bourgain and Diestel in early eighties [16] and it still attracts some attention (see the following papers citing the results of Drewnowski [15], [55], [48], [70], [71]).

Drewnowski [D46] proved:

Theorem 5.2. *Let X and Y be Banach spaces. If X and Y have the Gelfand-Phillips property then the completed injective tensor product $X\tilde{\otimes}_\varepsilon Y$ has the Gelfand-Phillips property. If X' and Y have the Gelfand-Phillips property then the space of compact operators $K(X,Y)$ has the Gelfand-Phillips property.*

A little bit later Drewnowski and Emmanuele [D58] proved even:

Theorem 5.3. *Let X and Y be Banach spaces. If X and Y have the Gelfand--Phillips property then the space of compact weak*-weak continuous operators $K_{w^*}(X',Y)$ has the Gelfand-Phillips property.*

This result was generalized to arbitrary locally convex spaces by Lindström and Schlumprecht [91]. A more general result for Banach spaces from which the previous result follows was found in [53]. Recalling the following isomorphisms we get the Gelfand-Phillips property criteria for more spaces:

- $K(X,Y) \cong K_{w^*}(X'',Y)$;
- $C(K,X) \cong K_{w^*}(X',C(K))$;
- $cca(\Sigma,X) \cong K_{w^*}(X',ca(\Sigma))$;
- $X\tilde{\otimes}_\varepsilon Y \cong K_{w^*}(X',Y)$ under some approximation property assumption.

Since for every σ-algebra Σ the space of countable additive measures $ca(\Sigma)$ has the Gelfand-Phillips property [D58] so $cca(\Sigma,X)$ has the Gelfand-Phillips property for any Banach space X with the Gelfand-Phillips property ([D58] Corollary 3.3).

Quite recently Ghenciu and Lewis [55, Th. 26] proved that if a Banach space X has the Gelfand-Phillips property then the space $ca(\mathscr{P}(\mathbb{N}), X)$ has also the Gelfand-Phillips property (here $\mathscr{P}(\mathbb{N})$ denotes the algebra of all subsets of integers).

Emmanuele [28], [27] and very recently in [31] continued the line of research in [D46] and [D58]. In particular, he proved that if Banach spaces X' and Y have the so-called *Schur property* (i.e., every weakly convergent sequence in the space is norm convergent) then even the space of all continuous operators $L(X, Y)$ has the Gelfand-Phillips property. Again in [D58] the authors showed that the vector valued space $L_p(\mu, X)$, $1 \leqslant p < \infty$, has the Gelfand-Phillips property whenever the Banach space X has the same property.

For spaces of vector valued continuous functions it follows from the above results that $C(K, X)$ has the Gelfand-Phillips property if and only if both $C(K)$ and X have this property. For spaces of continuous functions the following was obtained in [D46]:

Theorem 5.4. *If a compact topological space K has a dense subset S which is conditionally sequentially compact, i.e., every sequence in S has a convergent subsequence then $C(K)$ has the Gelfand-Phillips property.*

Schlumprecht [113] later proved that this condition is not necessary as well as he proved that not necessary for the Gelfand-Pettis property in a Banach space X is the sufficient condition (discovered in [D46]): X' has a norming weak* conditionally sequentially compact subset.

5.2. Copies of ℓ_∞

The interest in this topic starts with the result of Rosenthal in [109]. Drewnowski proved in [D22] an analogous result for arbitrary linear topological spaces:

Theorem 5.5. *Let X be a subspace of $\ell_\infty(\Gamma)$ containing all unit vectors and let Y be an arbitrary topological vector space. If $T : X \to Y$ is a linear continuous operator and there is a 0-neighborhood U in Y such that for any $\gamma \in \Gamma$ and any unit vector e_γ holds $T(e_\gamma) \notin U$ then for some subset $\Gamma' \subset \Gamma$, $\operatorname{card}\Gamma = \operatorname{card}\Gamma'$, the operator T maps*

$$X(\Gamma') := \{x \in X : \operatorname{supp} x \subset \Gamma'\}$$

isomorphically.

The result was later modified and extended by Globevnik [56] to the setting of analytic maps.

Theorem 5.6. *Let X be a Banach space, and let $A : B_{c_0(\Gamma)} \to X$ be an analytic map, B_Y denotes here the unit ball of Y. If the density of the range of A is bigger than d then X contains an isomorphic copy of $c_0(\Gamma')$ for $\operatorname{card}(\Gamma') > d$.*

Of course, if A above is linear continuous then the result follows from the previous theorem.

In fact nowadays many authors prefer to cite Drewnowski's result instead of the Rosenthal's original paper because he presented in [D21] a beautiful and incredible simple proof of the result. The theorem was applied, for instance, in the work of Capon [19] where she proved that for any Banach space X with a symmetric basis not isomorphic to ℓ_1 and not contained isomorphically in c_0 the ℓ_∞ direct sum Y of X is *primary*, i.e., for any projection in Y either its image or its kernel is isomorphic to the whole space. Another place of application is the recent work of Koszmider [85] who constructed two non-isomorphic spaces of continuous functions $C(K_1)$ and $C(K_2)$ such that each one can be embedded complementably into the other one — this solves in the negative the Schroeder-Bernstein problem for Banach spaces also in the class of Banach spaces of continuous functions. For general Banach spaces the problem was solved earlier by Gowers [59].

We suspect that the fact which attracted Drewnowski to Rosenthal's work was his lemma about families of finitely additive set functions — the part which connects Rosenthal's paper [109] with Drewnowski measure theoretic interests. From that point on the question of ℓ_∞ or c_0 subspaces is a constant theme in Drewnowski's work — we will see it later also in the section on spaces of vector valued continuous functions.

The next step was a series of three papers of Drewnowski about containment of copies of ℓ_∞ and c_0 in some spaces of operators and in some spaces represented as spaces of operators. These papers are still regularly cited (in 2012 the paper [D62] was cited 5 times). The story started with the paper [D62] and its main theorem:

Theorem 5.7. *Let X and Y be Banach spaces. The space $K_{w^*}(X',Y)$ contains an isomorphic copy of ℓ_∞ if and only if either X or Y contains such a copy.*

This result is clearly inspired by the earlier result of Kalton [73] that the space of compact operators $K(X,Y)$ contains a copy of ℓ_∞ if and only if either X' or Y contains such a copy but Drewnowski's result is much more powerful. It implies Kalton's result but it also proves the following corollary:

Corollary 5.8. *Let X and Y be Banach spaces, K a compact topological space and Σ a σ-algebra of subsets of some set Ω. Then:*

- $X\tilde{\otimes}_\varepsilon Y \supset \ell_\infty$ *if and only if* $X \supset \ell_\infty$ *or* $Y \supset \ell_\infty$.
- $C(K,X) \supset \ell_\infty$ *if and only if* $C(K) \supset \ell_\infty$ *or* $X \supset \ell_\infty$.
- $cca(\Sigma, X) \supset \ell_\infty$ *if and only if* $X \supset \ell_\infty$.

The corollary was generalized to the Fréchet or DF-space setting in [11]. The second part of the above corollary was applied by E. M. Galego [51] in order to show that every Banach space is isomorphic to a subspace of a Banach space Y such that $Y \not\simeq Y^2$ but $Y \simeq Y^3$. The main theorem was also applied by Ferrando [38] for deciding when the space of Pettis integrable functions contains a copy of c_0 or ℓ_∞. Recently Ferrando has obtained results on containment of c_0 and ℓ_∞ in the Musiał space of Pettis integrable functions [39], [40].

Slightly later Drewnowski tried to extend the third part of the corollary above to the space $ca(\Sigma, X)$ of countable additive X-valued measures on Σ. Then two cases has to be considered separately: the case of σ-algebra Σ admitting a nonzero atomless finite measure and the case when every non-zero measure on Σ is purely atomic. In [D64] it is proved that in the second case nothing unexpected happens:

Theorem 5.9. *Let every measure on Σ be purely atomic and X be a Banach space. Then*

- $ca(\Sigma, X) \supset \ell_\infty$ *if and only if* $X \supset \ell_\infty$,
- $ca(\Sigma, X) \supset c_0$ *if and only if* $X \supset c_0$.

The other case turned out to be the surprising one see [D64], [D90].

Theorem 5.10. *Let Σ admits a nonzero finite atomless measure m and X be a Banach space. Then the following assertions are equivalent:*

(a) $ca(\Sigma, X) \supset \ell_\infty$;
(b) $ca(\Sigma, X) \supset c_0$;
(c) $K(\ell_2, X) \neq L(\ell_2, X)$;
(d) *there exist an isomorphic embedding* $J : \ell_\infty \to ca(\Sigma, X)$ *such that* $J(c_0) = J(\ell_\infty) \cap cca(\Sigma, m, X)$ *and* $J(c_0)$ *is complemented in* $cca(\Sigma, X)$;
(e) $cca(\Sigma, X) \supset c_0$;
(f) $cca(\Sigma, X) \supset c_0$ *as a complemented copy.*

Here $cca(\Sigma, m, X)$ denotes the space of measures $\mu \in cca(\Sigma, X)$ which are m-absolutely continuous. The most surprising part is of course (c).

Thanks to the work of Drewnowski, Emmanuele and Ferrando we have now a pretty complete understanding when the spaces of vector measures $ca(\Sigma, X)$, $cca(\Sigma, X)$, $ba(\Sigma, X)$ contain ℓ_∞ or c_0 (complemented or uncomplemented). Apart from the results listed above we know that if $ca(\Sigma, X)$ contains a complemented copy of c_0 then X contains a copy of c_0 [29] (comp. also in [41, Th. 2.4]), $ba(\Sigma, X)$ contains a copy of c_0 if and only if it contains a copy of ℓ_∞ [11], if $ba(\Sigma, X)$ contains a complemented copy of c_0 then X contains a copy of c_0, on the contrary if X contains a copy of c_0 but not a copy of ℓ_∞ then $ba(\Sigma, X)$ contains a complemented copy of c_0 [35]. From the results of Drewnowski follows also [41, Th. 2.1]. The case of complemented copies of c_0 was generalized for many other spaces of vector measures in [106].

In the proof of the main result in [D64] important role plays an analogue of Phillips Lemma for vector measures (Lemma 3 in [D64]): for any sequence (μ_n) of measures in $ca(\mathcal{P}(\mathbb{N}), X)$ if $\mu_n(E) \to 0$ for any set $E \subset \mathbb{N}$ then $\|\mu_n\| \to 0$. This result was extended for more algebras of subsets of $\mathbb{N}$ in the paper [1].

Let us recall that the space of measures can be identified with some spaces of linear continuous operators and let us come back to spaces of operators as in the paper [D62]. It was known before [49] that if a Banach space X contains an isomorphic copy of c_0 and Y is an infinite dimensional Banach space then $K_{w^*}(X', Y)$ contains a complemented isomorphic copy of c_0.

The results in [D62] clearly launched a long line of research which practically lasted till now. Emmanuele [29] proved that if c_0 embeds in the subspace $K_{w^*}(X', Y)$, then either $K_{w^*}(X', Y) = L_{w^*}(X', Y)$ or $K_{w^*}(X', Y)$ is not complemented in $L_{w^*}(X', Y)$. Moreover, the equality holds if and only if c_0 embeds in one of the spaces X, Y and the other has the Schur property. Then in [30] the same author proved that if $L_{w^*}(X', Y)$ contains a complemented copy of c_0 then either X or Y contains an isomorphic copy of c_0, the converse holds as well [36]. A little bit later Bonet, Domański, Lindström and Ramanujan [11, Cor.14] proved that $L_{w^*}(X', Y)$ contains c_0 if and only if either X or Y contains an isomorphic copy of c_0 or $L_{w^*}(X', Y)$ contains also a copy of ℓ_∞ (later the same theorem appears in [36]). It is worth noting that even if $L_{w^*}(X', Y)$ contains c_0 it need not contain ℓ_∞ as the example provided by [54] $X = Y = \ell_1$ shows. In general [11] contains several results on containment of c_0, ℓ_∞, complementability or equality of various spaces of operators clearly inspired by the papers of Drewnowski and by his method as well as generalizations to the Fréchet and dual Fréchet spaces. Moreover, Ferrando [36] proved that if both X and Y contain a copy of c_0 then $L_{w^*}(X', Y)$ contains a copy of ℓ_∞. For more results on $L_{w^*}(X', Y)$ and $K_{w^*}(X', Y)$ see [54]. The condition $K(\ell_2, X) \neq L(\ell_2, X)$ and the proof in [D64] and [D90] inspires, for instance, two papers. First the paper of Ansari [5] where it was proved that $K(C(K), X) = L(C(K), X)$ if and only if $K(\ell_2, X) = L(\ell_2, X)$ and operators $T : C(K) \to X$ factorize through a subspace in c_0. The second paper [10] contains among others the following two results together with their Fréchet space generalizations:

Theorem 5.11. *Let X, Y be Banach spaces. If*

$$L(\ell_q, X) \neq K(\ell_q, X), \quad L(\ell_p, Y) \neq K(\ell_p, Y), \qquad 1 < p, q < \infty, \quad \frac{1}{p} + \frac{1}{q} = 1,$$

then

$$\ell_\infty \subseteq L_{w^*}(X', Y) \qquad \text{and} \qquad c_0 \subseteq K_{w^*}(X', Y).$$

Theorem 5.12. *Let X be a Banach space and let Y be a p-Orlicz Banach space (i.e., unconditional convergence of $\sum x_n$ in Y implies $\sum \|x_n\|^p < \infty$), then*

$$L_{w^*}(E', F) \supset c_0 \Rightarrow K(\ell_q, E) \neq L(\ell_q, E),$$

again for $1 < p, q, < \infty$, $\frac{1}{p} + \frac{1}{q} = 1$.

There is also a series of results and consequences of the previous theorems clarifying when the space $L(X, Y)$ of all linear continuous operators contains a copy of c_0 see [11], [34], [42] and [37].

The whole research above is closely connected to the long standing open question: is it true that, for all Banach spaces X and Y, either $K(X, Y) = L(X, Y)$ or $K(X, Y)$ is an uncomplemented subspace of $L(X, Y)$? See [32] and the introduction containing a survey of results and [33] as well the introduction to the paper [10].

5.3. Spaces of vector valued continuous functions

In the beginning of 1990' Drewnowski together with the first named author of this survey started to study spaces of vector valued continuous functions. They studied spaces of the form $C(K, X, \tau)$ of functions $f : K \to X$ which are continuous with respect to a vector topology τ on X weaker than the original one. Here K is usually a compact topological space, X is usually a Banach space, and we equip $C(K, X, \tau)$ with the topology of uniform convergence with respect to the original topology of X. The most interesting cases are when τ is the weak topology w on a Banach space or a weak* topology w^* on a dual Banach space (see a recent paper [67] on extremal points of the unit ball in $C(K, X, w)$). Recall again that the considered spaces of vector valued functions can be identified with some spaces of linear continuous operators, so the research in this area is closely related to Drewnowski's work on spaces of operators and containment of c_0 and ℓ_∞ therein. The general still not-proved conjecture is that $C(K, X)$ is complemented in $C(K, X, \tau)$ if and only if both spaces coincide — several results supporting this conjecture were proved in [D66]. For instance:

Theorem 5.13. *For arbitrary Banach space X and arbitrary vector topology τ on X weaker than the norm topology the space $C(\beta\mathbb{N}, X)$ is complemented in $C(\beta\mathbb{N}, X, \tau)$ if and only if they coincide.*

Theorem 5.14. *If a Banach space X contains a τ-convergent sequence which is not norm convergent then for every infinite compact set K the Banach space $C(K, X)$ is uncomplemented in the Banach space $C(K, X, \tau)$.*

In particular, if X is an infinite dimensional Banach space then $C(K, X')$ is never complemented in $C(K, X', w^*)$. Similarly $C(K, X)$ is complemented in $C(K, X, w)$ if and only if X has the so-called Schur property (i.e., every weak null sequence is norm null) and, in particular, it contains copies of ℓ_1. More results supporting the conjecture are collected in [D77].

There is a standard example of an uncomplemented Banach subspace: c_0 in ℓ_∞. Cembranos [20] and Freniche [49], using the famous Josefson-Nissenzweig Theorem (Theorem 5.1), proved that for every Banach space X and any infinite compact set K the space $C(K, X)$ contains a complemented subspace isomorphic to c_0 so $C(K, X)$ is never injective and never contained as complemented subspace in a dual Banach space. Let us recall that a locally convex space is *injective* if and only if it is complemented in every locally convex space containing it. The problem of describing all injective (Banach, Fréchet, locally convex spaces) is still open. In [D71] the authors extended the result above to a Fréchet space context:

Theorem 5.15. *Let E be a completely regular Hausdorff space with an infinite compact subset and let X be a non-Montel Fréchet space. Then $C(E, X)$ contains a complemented subspace isomorphic to c_0. In particular, $C(E, X)$ is not injective.*

The authors went one step more and proved:

Theorem 5.16. *Let E be as above and let X be a locally convex space containing an isomorphic copy of ℓ_1. Then $C(E, X, w)$ contains a complemented isomorphic copy of ℓ_1, hence is not injective.*

The above methods lead to the following result:

Theorem 5.17. *Let E be a non-discrete locally compact topological space with a fundamental sequence of compact sets. Let X be an infinite dimensional Fréchet space. Then the following assertions are equivalent:*

(a) *$C(E, X)$ is injective;*
(b) *$C(E, X, w)$ is injective;*
(c) *X is isomorphic to the space of all sequences ω and $C(E)$ is injective.*

The collaboration of Drewnowski and the first named author on that subject produced later a long preprint *Injectivity of spaces of bounded vector sequences and spaces of operators* which pushes the method developed earlier to its limits. We get several results on complementability of various ideals of operators in bigger ideals as well several results on vector valued sequence spaces of the similar spirit as above and we connect even closer this research with Drewnowski's papers on c_0 and ℓ_∞ in spaces of operators. The preprint was never published — partly because the authors were never satisfied with the outcome. This is Drewnowski's school of looking for excellence and completeness of the results.

6. Theory of locally convex spaces

In this section we collect some very nice results of Drewnowski related to locally convex spaces which do not belong to the main streams of his interest previously described. Three areas will be treated. First, we consider a result of primariness which is deeply rooted in the set theory (connected with independence to axioms questions). Secondly, universality questions — here simplicity of the used tools is a striking feature. Thirdly, some results on uncountable bases.

6.1. Primariness of ℓ_∞/c_0

A beautiful little gem is a paper [D65] where Drewnowski and Roberts proved under the Continuum Hypothesis (CH) that the space ℓ_∞/c_0 is primary (recall that it means that for any splitting of the space into a direct sum at least one summand is isomorphic to the whole space). Let us mention that the space ℓ_∞/c_0 is isomorphic to the space of continuous functions on the remainder of the Čech-Stone compactification of natural numbers, in particular, it is a C^*-algebra.

That paper heavily depends on set theoretical considerations. One of the facts obtained by Drewnowski and Roberts is that ℓ_∞ direct sum of copies of ℓ_∞/c_0 is isomorphic to ℓ_∞/c_0 provided CH. Grzech proved later [63] that under the assumptions of the so-called Open Coloring Axiom (OCA) and $2^\omega = \omega_2$, an isomorphism between $\ell_\infty(\ell_\infty/c_0)$ and ℓ_∞/c_0 does not exist! It is not known, however, whether the primariness of ℓ_∞/c_0 can be established within ZFC without CH. Another result, given by Drewnowski and Roberts in [D65] is that, under CH, the spaces $\ell_\infty(\ell_\infty/c_0)/c_0(\ell_\infty/c_0)$ and ℓ_∞/c_0 are isometric. Grzech proved also [63] that, assuming the OCA and $2^\omega = \omega_2$, an isometry between these spaces does not exist. Thus the considered problem seems to be an example of a problem about C^*-algebras which is solved under set-theoretical assumptions or which is independent to the ZFC axioms. A survey of such problems is provided by [130].

From the results of the paper [D65] it follows also (as P. Koszmider observed) that in the algebra of all linear continuous operators on ℓ_∞/c_0 there is the unique maximal ideal (the set of all operators T such that the identity does not factorize through T) see [81, p. 4832].

6.2. Universality questions

In the papers [D24] and [D45] the universality questions were considered — the negative answers were obtained via a simple count. For instance, in the former joint paper with Lohman the authors proved that there are exactly $2^{2^{\mathfrak{c}}}$ separable pairwise non-isomorphic locally convex spaces. Thus there cannot be a quotient universal space in the class of separable locally convex spaces (i.e., a separable locally convex space U such that every separable locally convex space is isomorphic to a quotient of U) since otherwise U has to have too many quotients. It is surprising that using such seemingly elementary tools one can solve question explicitly mentioned in the monograph of Rolewicz [107]. In a similar way they proved that there is no quotient universal spaces in the classes of separable metrizable locally convex spaces or separable normed spaces. In [D45] the author proved that there is no universal space in the class of locally convex spaces with a basis or with an unconditional basis (a space U is called *universal* in a class $\mathcal{C}$ if for any space in $\mathcal{C}$ the space U contains a subspace isomorphic to it).

6.3. Bases

In [D50] Drewnowski proved that if two nonseparable Banach spaces have symmetric bases and each is isomorphic to a subspace of the other then the bases are equivalent. In particular, every Banach space with an uncountable symmetric basis has a unique symmetric basis. This is not true in the countable case.

The paper [D54] is a kind of continuation of [D50]. The author showed that if a Banach space X has an (uncountable) unconditional basis and contains isomorphic copy of $\ell_1(A)$ for some uncountable set A then the basis contains “big” parts equivalent to $\ell_1(J)$ basis for “big” sets J. An analogous result was also proved for the c_0-case. This is a generalization of Troyanski's result [125] where the basis is assumed to be even symmetric. Of course, it follows that X contains complemented isomorphic copies of ℓ_1 or, respectively, c_0 spaces. Let us note that later on Finol and Wójtowicz [47] proved that if a Banach space X with (possibly uncountable) an unconditional basis contains an isomorphic copy Y of $\ell_1(A)$ then Y contains an isomorphic copy of $\ell_1(A)$ complemented in X.

7. Drewnowski as a teacher

Finishing our survey we want to add some more personal remarks concerning our collaboration with Lech Drewnowski. We both met him as students around forty years ago when we started our mathematical adventure at the Faculty of Mathematics, Physics and Chemistry. At that time he was already a star with a quickly progressing career and a fame of a very demanding professor. We both soon realized that first of all he demands from himself. His lectures were clear, lively, interesting, rich with examples, perfectly prepared – such a style of teaching is characteristic for him till now. Definitely most appealing to us as young people was (and still is) his never ending enthusiasm for mathematics. Then we wrote both master and Ph.D. theses under his supervision. Lech is perfectionist so we faced many remarks concerning mathematical improvements and editorial smoothness. Immediately, we also experienced his erudition – it was always much easier to ask Drewnowski for some explanations, references etc. instead of laborious look through the literature – the more so that Drewnowski's answer was usually more precise, correct and definitely faster. His "proof power ", connected with his erudition, solved many of difficult for us questions. Drewnowski listened to our problem, next lit up a cigarette, was silent for five to fifteen minutes and ... wrote a detailed solution on a piece of paper.

Drewnowski's attitude to his pupils was very democratic, he treated us as partners, saved us from loosing time. He stimulated his pupils to look for their own way in mathematics; this was a good strategy – by now five of his pupils have become professors. We really felt like partners and soon became, we dare to say, friends. His seminar was a true scientific school: each meeting took some five hours when we discussed mathematics, presented our new results and current papers published in journals. During breaks (sometimes they were long) we were used to argue about all aspects of life.

Both of the authors had pleasure to write joint papers with Lech Drewnowski – his approach is to go in depth rather than to look for generalizations. As you have seen above he has the talent to discover unexpected connections and find, behind a seemingly final solution, something deeper: an opening to the ultimate reason and perfect elegance of an argument.

Lech is far from a popular image of a mathematician: he is not the man living exclusively in a world of numbers and mathematical structures. He loves life, nature and animals, participates in all social events of the Faculty. He is fond of Tolkien's novels and Russian literature.

Dear Lech, our Master, we wish you many happy years to come, exciting mathematics, good health and all the best.

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