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Confidence regions for regression parameter in measurement error model

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Abstract. In linear regression analysis, estimation of the regression parameters in measurement error model is considered in this paper. Methods of estimation have been proposed and improved in many literature. We give an approximated confidence region of regression parameters and examine the accuracy of approximation by simulation.

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§1. Introduction

The linear model is widely used in data analysis, responses are observed along with covariates, which may have measurement errors in some practical situations. Sutradhar (2013) reviewed on analysis in linear and generalized linear models with measurement errors for longitudinal data. Fan, Sutradhar and Rao (2012) gave a bias corrected generalized quasi-likelihood (BCGQL) estimator and a bias corrected generalized method of moment (BCGMM) estimator in the linear measurement error model. These estimators are extension of the generalized quasi-likelihood estimator and the generalized method of moment estimator by Rao, Sutradhar and Pandit (2012). They compare these estimators by simulation, in which the BCGQL estimator is more efficient than the BCGMM estimator and prove that the BCGQL is a consistent estimator.

Let $\boldsymbol{y}_i = (y_{i1}, ..., y_{it})'$ denote the response variable for the ith individual and $X_i = (x_{ij\ell})$ be the associated $t \times p$ $(t > p)$ covariate matrix with normal measurement errors $(i = 1, ..., n;\ j = 1, ..., t;\ \ell = 1, ..., p)$. Let $Z_i = (z_{ij\ell})$ and $V_i = (\nu_{ij\ell})$ be the unobserved true $t \times p$ covariate matrix of and the $t \times p$

measurement error matrix, respectively, then we assume $X_i = Z_i + V_i$, and consider the model

$$\boldsymbol{y}_i = Z_i\boldsymbol{\beta} + \boldsymbol{\varepsilon}_i \quad (i = 1, ..., n), \tag{1}$$

where $\boldsymbol{\beta}$ is the p-dimensional regression vector and $\boldsymbol{\varepsilon}_i = (\varepsilon_{i1}, ..., \varepsilon_{it})'$ is the model error. Let $\boldsymbol{\varepsilon}_1, ..., \boldsymbol{\varepsilon}_n$ be independent and has the t-variate distribution with mean $\mathbf{0}$ and covariance matrix Σ_e, $N_t(\mathbf{0}, \Sigma_e)$, and the measurements be observed repeatedly, that is $z_{1j\ell} = ... = z_{nj\ell}$ for each j ($Z_1 = ... = Z_n \equiv Z$). Since Z is not observed, one may consider the inferences through the usual regression model

$$\boldsymbol{y}_i = X_i\boldsymbol{\beta} + \boldsymbol{\varepsilon}_i \quad (i = 1, ..., n). \tag{2}$$

Even if the model is the simple linear model $y = \beta_0 + \beta_1 x + \varepsilon$, it is impossible to give estimators of regression parameters and variances uniquely, when $n = 1$. Under model (2), let the measurement error $\nu_{ij\ell}$'s are independent of the model error $\boldsymbol{\varepsilon}_i$ and are independent and identically distributed as $N(0, \sigma_v^2)$, then $E(\boldsymbol{y}_i) = Z\boldsymbol{\beta}$ and $Var(\boldsymbol{y}_i) = \Sigma = \Sigma_e + \sigma_v^2 \text{diag}(\beta_1^2, ..., \beta_p^2)$. Since the covariate matrix X_i is a random matrix, rank$X_i = p$ with probability 1.

The BCGQL given by Fan, Strudhar and Rao (2012) is

$$\hat{\boldsymbol{\beta}}_B = \Big[\sum_{i=1}^{n} \frac{\partial \boldsymbol{\psi}_i'}{\partial \boldsymbol{\beta}} D_i^{-1} \frac{\partial \boldsymbol{\psi}_i}{\partial \boldsymbol{\beta}'}\Big]^{-1} \Big[\sum_{i=1}^{n} \frac{\partial \boldsymbol{\psi}_i'}{\partial \boldsymbol{\beta}} D_i^{-1} X_i' \Sigma^{-1} \boldsymbol{y}_i\Big], \tag{3}$$

where

$$\boldsymbol{\psi}_i = X_i'\Sigma^{-1}\boldsymbol{y}_i - [X_i'\Sigma^{-1}X_i - \sigma_v^2(\text{tr}\Sigma^{-1})I_p],$$

and $D_i = Var(\boldsymbol{\psi}_i)$, in which $E(\boldsymbol{\psi}_i) = \mathbf{0}$. However it is difficult to derive the distribution of (3), even if the model error variance and the measurement error variance are known. That is, we cannot give a confidence region of $\boldsymbol{\beta}$.

In this paper, an exact confidence region of $\boldsymbol{\beta}$ by extending Brown (1957) is given in Section 2, when $\Sigma_e = \sigma_e^2 I_t$ and the variances σ_e^2, σ_v^2 are known. A conservative confidence region is also given for known variances. An approximated confidence region is given by plug-in the estimators of variances, when the variances are unknown. In Section 3, the accuracy of approximation is examined by simulation.

§2. Confidence region

Brown (1957) gave an exact confidence region of regression parameters in the simple linear regression with measurement errors, when the model error and the measurement error variances are known. Cheng and Van Ness (1999) reviewed on confidence intervals of the slope parameter in the simple linear regression model with measurement error. When the variances are unknown, the asymptotic confidence interval of the slope parameter is given in Fuller (1987). When the variances are unknown and the ratio of the model error variance and and the measurement error variance σ_e^2/σ_v^2 is known, an exact confidence interval of arctangent of the slope parameter is given by Creasy (1956).

We give a confidence region of regression parameters in the multiple linear regression model with measurement error by extending Brown (1957) in this section. First of all, we assume that $\Sigma_e = \sigma_e^2 I_t$. Since $\boldsymbol{y}_i - X_i\boldsymbol{\beta} = \boldsymbol{\varepsilon}_i - V_i\boldsymbol{\beta}$ is distributed as $N_t(\mathbf{0}, \{\sigma_e^2 + \sigma_v^2\boldsymbol{\beta}'\boldsymbol{\beta}\}I_t)$ from the model (1),

$$w = \sum_{i=1}^{n} \frac{(\boldsymbol{y}_i - X_i\boldsymbol{\beta})'(\boldsymbol{y}_i - X_i\boldsymbol{\beta})}{\sigma_e^2 + \sigma_v^2\boldsymbol{\beta}'\boldsymbol{\beta}}$$

has the chi-square distribution with nt degrees of freedom, χ^2_{nt}. Hence $P\{w < \chi^2_{nt}(\alpha)\} = 1-\alpha$, where $\chi^2_{nt}(\alpha)$ is the upper $100\alpha\%$ point of χ^2_{nt}. If the variances σ_e^2 and σ_v^2 are known, then a $100(1-\alpha)\%$ confidence region of $\boldsymbol{\beta}$ is given by

$$\begin{aligned} &(\boldsymbol{\beta} - \Xi^{-1}\textstyle\sum_{i=1}^n X_i'\boldsymbol{y}_i)'\Xi(\boldsymbol{\beta} - \Xi^{-1}\textstyle\sum_{i=1}^n X_i'\boldsymbol{y}_i) \\ < \;& \chi^2_{nt}(\alpha)\sigma_e^2 - \textstyle\sum_{i=1}^n \boldsymbol{y}_i'\boldsymbol{y}_i + (\textstyle\sum_{i=1}^n X_i'\boldsymbol{y}_i)'\Xi^{-1}(\textstyle\sum_{i=1}^n X_i'\boldsymbol{y}_i), \end{aligned} \tag{4}$$

which is equivalent to $w < \chi^2_{nt}(\alpha)$, where $\Xi = \sum_{i=1}^{n} X_i'X_i - \chi^2_{nt}(\alpha)\sigma_v^2 I_p$. Since X_i is a random matrix, Ξ is a nonsingular matrix with probability 1. When Ξ is the positive definite and the right hand side of (4) is positive, the confidence region (4) is the ellipsoidal region and the center of the ellipsoidal region

$$\hat{\boldsymbol{\beta}}_{c'} = \Xi^{-1}\sum_{i=1}^{n} X_i'\boldsymbol{y}_i \tag{5}$$

may be considered as an estimator of $\boldsymbol{\beta}$. It is not easy to investigate properties of $\hat{\boldsymbol{\beta}}_{c'}$, because this estimator depends on the confidence coefficient.

It is difficult to derive the confidence region of $\boldsymbol{\beta}$, when the model error and the measurement error variances are unknown. So we consider that estimators of variances are substituted to the confidence region (4). The moment estimator of σ_v^2 is given by

$$\hat{\sigma}_v^2 = \sum_{i=1}^{n}\sum_{j=1}^{t}(x_{ij\ell} - \bar{x}_{i\cdot\ell})^2/nt$$

which has consistency, where $\bar{x}_{i\cdot\ell} = \sum_j x_{ij\ell}$. Let $s(\boldsymbol{\beta}) = \sum_i(\boldsymbol{y}_i - X_i\boldsymbol{\beta})'(\boldsymbol{y}_i - X_i\boldsymbol{\beta})$, then $E[s(\boldsymbol{\beta})] = nt(\sigma_e^2 + \sigma_v^2\boldsymbol{\beta}'\boldsymbol{\beta})$. Hence

$$\hat{\sigma}_e^2 = s(\hat{\boldsymbol{\beta}})/nt - \hat{\sigma}_v^2\hat{\boldsymbol{\beta}}'\hat{\boldsymbol{\beta}}$$

is an estimator of σ_e^2, where $\hat{\boldsymbol{\beta}}$ is the ordinary least squares estimator of $\boldsymbol{\beta}$. Both of $\hat{\sigma}_v^2$ and $\hat{\sigma}_e^2$ have the consistency. By substituting $\hat{\sigma}_e^2$ and $\hat{\sigma}_v^2$ into (4), the $100(1-\alpha)\%$ confidence region of $\boldsymbol{\beta}$ is approximated by

$$\begin{aligned} &(\boldsymbol{\beta} - \hat{\Xi}^{-1}\textstyle\sum_{i=1}^n X_i'\boldsymbol{y}_i)'\hat{\Xi}(\boldsymbol{\beta} - \hat{\Xi}^{-1}\textstyle\sum_{i=1}^n X_i'\boldsymbol{y}_i) \\ < \ &\chi^2_{nt}(\alpha)\hat{\sigma}_e^2 - \textstyle\sum_{i=1}^n \boldsymbol{y}_i'\boldsymbol{y}_i + (\textstyle\sum_{i=1}^n X_i'\boldsymbol{y}_i)'\hat{\Xi}^{-1}(\textstyle\sum_{i=1}^n X_i'\boldsymbol{y}_i), \end{aligned} \tag{6}$$

where $\hat{\Xi} = \displaystyle\sum_{i=1}^n X_i'X_i - \chi^2_{nt}(\alpha)\hat{\sigma}_v^2 I_p$. Then the estimator of the regression parameter is

$$\hat{\boldsymbol{\beta}}_c = \hat{\Xi}^{-1}\sum_{i=1}^n X_i'\boldsymbol{y}_i. \tag{7}$$

Next, we consider the confidence region of $\boldsymbol{\beta}$ in a general covariance structure of Σ_e, which is the positive definite. Let λ is the maximum characteristic root of Σ_e, then

$$\sum_{i=1}^n(\boldsymbol{y}_i - X_i\boldsymbol{\beta})'(\Sigma_e + \sigma_v^2\boldsymbol{\beta}'\boldsymbol{\beta}I_t)^{-1}(\boldsymbol{y}_i - X_i\boldsymbol{\beta}) \geq \sum_{i=1}^n \frac{(\boldsymbol{y}_i - X_i\boldsymbol{\beta})'(\boldsymbol{y}_i - X_i\boldsymbol{\beta})}{\lambda + \sigma_v^2\boldsymbol{\beta}'\boldsymbol{\beta}}. \tag{8}$$

The left hand side of (8) is distributed as χ^2_{nt}, because the distribution of $\boldsymbol{y}_i - X_i\boldsymbol{\beta}$ is $N_t(\mathbf{0}, \Sigma_e + \sigma_v\boldsymbol{\beta}'\boldsymbol{\beta}I_t)$. Hence we have

$$P\Big\{\sum_{i=1}^n \frac{(\boldsymbol{y}_i - X_i\boldsymbol{\beta})'(\boldsymbol{y}_i - X_i\boldsymbol{\beta})}{\lambda + \sigma_v^2\boldsymbol{\beta}'\boldsymbol{\beta}} < \chi^2_{nt}(\alpha)\Big\} \geq 1 - \alpha.$$

Then the confidence coefficient of the confidence region

$$(\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}_{c'})'\Xi(\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}_{c'}) < \chi^2_{nt}(\alpha)\lambda - \sum_{i=1}^n \boldsymbol{y}_i'\boldsymbol{y}_i + (\sum_{i=1}^n X_i'\boldsymbol{y}_i)'\Xi^{-1}(\sum_{i=1}^n X_i'\boldsymbol{y}_i) \tag{9}$$

is greater than $100(1-\alpha)\%$, when Σ_e and σ_v^2 are known. Let $S(\boldsymbol{\beta}) = \sum_i(\boldsymbol{y}_i - X_i\boldsymbol{\beta})(\boldsymbol{y}_i - X_i\boldsymbol{\beta})'$, then $E[S(\boldsymbol{\beta})] = n(\Sigma_e + \sigma_v\boldsymbol{\beta}'\boldsymbol{\beta}I_t)$. So, we use the estimator

$$\hat{\Sigma}_e = S(\hat{\boldsymbol{\beta}})/n - \hat{\sigma}_v^2\hat{\boldsymbol{\beta}}'\hat{\boldsymbol{\beta}}I_t$$

and substitute $\hat{\lambda}$ and $\hat{\Xi}$ into (9), when the variances are unknown, where $\hat{\lambda}$ is the maximum characteristic root of $\hat{\Sigma}_e$. The $100(1-\alpha)\%$ confidence region of $\boldsymbol{\beta}$ is approximated by

$$(\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}_c)'\hat{\Xi}(\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}_c) < \chi^2_{nt}(\alpha)\hat{\lambda} - \sum_{i=1}^n \boldsymbol{y}_i'\boldsymbol{y}_i + (\sum_{i=1}^n X_i'\boldsymbol{y}_i)'\hat{\Xi}^{-1}(\sum_{i=1}^n X_i'\boldsymbol{y}_i). \quad (10)$$

When the covariance matrix of the model error has an intraclass correlation structure $\Sigma_e = \sigma_e^2\{(1-\rho)I_t + \rho\mathbf{1}_t\mathbf{1}_t'\}$, the characteristic roots of Σ are $\sigma_e^2\{1+(t-1)\rho\} + \sigma_v^2\boldsymbol{\beta}'\boldsymbol{\beta}$ and $\sigma_e^2(1-\rho) + \sigma_v^2\boldsymbol{\beta}'\boldsymbol{\beta}$, where $-1/(t-1) < \rho < 1$ and $\mathbf{1}_t$ is the t dimensional vector of ones. Hence λ in (9) is $\max(\tau_1 + \sigma_v^2\boldsymbol{\beta}'\boldsymbol{\beta},\ \tau_2 + \sigma_v^2\boldsymbol{\beta}'\boldsymbol{\beta})$, where $\tau_1 = \sigma_e^2\{1+(t-1)\rho\}$ and $\tau_2 = \sigma_e^2(1-\rho)$. Since $\tau_1 = \mathbf{1}_t'\Sigma_e\mathbf{1}_t/t$ and $\tau_2 = (\mathrm{tr}\Sigma_e - \tau_1)/(t-1)$, $\hat{\lambda} = \max(\hat{\tau}_1 + \hat{\sigma}_v^2\hat{\boldsymbol{\beta}}'\hat{\boldsymbol{\beta}},\ \hat{\tau}_2 + \hat{\sigma}_v^2\hat{\boldsymbol{\beta}}'\hat{\boldsymbol{\beta}})$ is substituted into (10) for unknown intraclass correlation structure, where $\hat{\tau}_1 = \mathbf{1}_t'\hat{\Sigma}_e\mathbf{1}_t/t$ and $\hat{\tau}_2 = (\mathrm{tr}\hat{\Sigma}_e - \hat{\tau}_1)/(t-1)$. Note that $\mathrm{tr}\hat{\Sigma}_e = s(\hat{\boldsymbol{\beta}}) - t\hat{\sigma}_v\hat{\boldsymbol{\beta}}'\hat{\boldsymbol{\beta}}$ by $\mathrm{tr}S(\boldsymbol{\beta}) = s(\boldsymbol{\beta})$, that is $\mathrm{tr}\hat{\Sigma}_e/t = \hat{\sigma}_e^2$. In the intraclass correlation structure, the left hand side of (8) is equivalent to

$$\sum_{i=1}^n(\boldsymbol{y}_i - X_i\boldsymbol{\beta})'(\boldsymbol{y}_i - X_i\boldsymbol{\beta}) - \frac{(\tau_1-\tau_2)/t}{\tau_1 + \sigma_v^2\boldsymbol{\beta}'\boldsymbol{\beta}}\mathbf{1}_t'S(\boldsymbol{\beta})\mathbf{1}_t \le \chi^2_{nt}(\alpha)(\tau_2 + \sigma_v^2\boldsymbol{\beta}'\boldsymbol{\beta}). \quad (11)$$

This is an exact confidence region of $\boldsymbol{\beta}$, but we can not see what shape is the region (11). By $\mathbf{1}_t'S(\hat{\boldsymbol{\beta}})\mathbf{1}_t/t = \hat{\tau}_1 + \hat{\sigma}_v^2\hat{\boldsymbol{\beta}}'\hat{\boldsymbol{\beta}}$, the confidence region would be approximated by

$$(\boldsymbol{\beta}-\hat{\boldsymbol{\beta}}_c)'\hat{\Xi}(\boldsymbol{\beta}-\hat{\boldsymbol{\beta}}_c) < \chi^2_{nt}(\alpha)\hat{\tau}_2 + n(\hat{\tau}_1-\hat{\tau}_2) - \sum_{i=1}^n \boldsymbol{y}_i'\boldsymbol{y}_i + (\sum_{i=1}^n X_i'\boldsymbol{y}_i)'\hat{\Xi}^{-1}(\sum_{i=1}^n X_i'\boldsymbol{y}_i), \quad (12)$$

when the variances are unknown. By the χ^2 table, $\chi_{nt}(\alpha) > nt$ in usual confidence level (say, $\alpha = 0.1, 0.05, 0.01$), then the right hand side of (12) is smaller than that of (10) for the case of intraclass correlation structure.

§3. Simulation

In this section, we examine the properties of $\hat{\boldsymbol{\beta}}_{c'}$ with known variances and examine the accuracy of approximation of (6) with unknown variances by simulation, when $\Sigma_e = \sigma_e^2 I_t$. In simulation study, we choose $p = 2;\ t = 4;\ n = 10, 20, 30$ and $\alpha = 0.10, 0.05, 0.01$. The covariates are in Table 1.

Table 1. Covariate

z_{i11}	z_{i21}	z_{i31}	z_{i41}	z_{i21}	z_{i22}	z_{i23}	z_{i24}
$U(0,2)$	$U(0,2)$	$U(1,4)$	$U(1,4)$	1.0	2.0	$N(0,0.8)$	$N(0,0.8)$

In Table 1, $U(a,b)$ is the random number generated from the uniform distribution over the interval (a,b) and $N(0,0.8)$ is the random number generated from the normal distribution, but these random numbers are treated as fixed. We choose the parameters as $\boldsymbol{\beta} = (3.0, 1.5)'$, $\sigma_e^2 = 1.0$ and $\sigma_v^2 = 0.1, 0.3$. The values of $z_{ij\ell}$'s chosen in this simulation are similar to the values chosen in Fan, Sutradhar and Rao (2012). In their simulation, BCGQL, BCGMM and other estimators are compared and confidence intervals of each component of $\boldsymbol{\beta}$ are computed by using BCGQL and BCGMM. However we can not use their confidence intervals for practical application.

First of all, we compute 5,000 estimators $\hat{\boldsymbol{\beta}}_B$ in (3) and $\hat{\boldsymbol{\beta}}_c$ in (5). When the variances are known, the mean and the mean square error (MSE) of each estimator are in Tables 2.1 and 2.2, in which $\hat{\boldsymbol{\beta}}_B = (\hat{\beta}_{B1}, \hat{\beta}_{B2})'$ and $\hat{\boldsymbol{\beta}}_{c'} = (\hat{\beta}_{c'1}, \hat{\beta}_{c'2})'$. The BCGQL estimator is not depend on the confidence coefficient, but the results are tabulated for each confidence coefficient for easily comparison.

Table 2.1. Mean and MSE ($\sigma_v^2 = 0.1$)

		$\hat{\boldsymbol{\beta}}_B$ mean		MSE	$\hat{\boldsymbol{\beta}}_{c'}$ mean		MSE
n	$1-\alpha$	$\hat{\beta}_{B1}$	$\hat{\beta}_{B2}$		$\hat{\beta}_{c'1}$	$\hat{\beta}_{c'2}$	
10	0.90	3.006	1.449	0.0248	3.021	1.532	0.0458
	0.95	3.003	1.452	0.0251	3.024	1.552	0.0498
	0.99	2.995	1.435	0.0242	3.033	1.565	0.0581
20	0.90	3.024	1.467	0.0203	3.007	1.526	0.0222
	0.95	3.028	1.465	0.0208	3.014	1.533	0.0241
	0.99	3.017	1.450	0.0217	3.018	1.540	0.0256
30	0.90	3.027	1.466	0.0169	3.008	1.513	0.0179
	0.95	3.032	1.456	0.0172	3.013	1.514	0.0186
	0.99	3.033	1.459	0.0173	3.020	1.527	0.0202

Table 2.2. Mean and MSE ($\sigma_v^2 = 0.3$)

		$\hat{\boldsymbol{\beta}}_B$ mean		MSE	$\hat{\boldsymbol{\beta}}_{c'}$ mean		MSE
n	$1-\alpha$	$\hat{\beta}_{B1}$	$\hat{\beta}_{B2}$		$\hat{\beta}_{c'1}$	$\hat{\beta}_{c'2}$	
10	0.90	2.992	1.443	0.0355	3.044	1.617	0.1251
	0.95	2.986	1.456	0.0363	3.060	1.692	0.1929
	0.99	2.984	1.451	0.0368	3.104	1.744	0.2201
20	0.90	2.996	1.435	0.0322	3.031	1.591	0.0604
	0.95	2.998	1.434	0.0326	3.034	1.583	0.0699
	0.99	3.002	1.426	0.0346	3.066	1.642	0.0898
30	0.90	3.003	1.437	0.0299	3.026	1.552	0.0385
	0.95	3.003	1.433	0.0306	3.031	1.561	0.0406
	0.99	2.997	1.431	0.0304	3.045	1.609	0.0520

From Tables 2.1 and 2.2, the MSE of BCGQL estimator $\hat{\boldsymbol{\beta}}_B$ is smaller than that of $\hat{\boldsymbol{\beta}}_{c'}$ as stated in Section 1. The estimator $\hat{\boldsymbol{\beta}}_{c'}$ may not have unbiasedness, however the bias of $\hat{\boldsymbol{\beta}}_{c'}$ would be small. The bias of $\hat{\boldsymbol{\beta}}_{c'}$ would be small when the confidence coefficient and the measurement error variance are small. The MSE of $\hat{\boldsymbol{\beta}}_{c'}$ is also small, when $1-\alpha$ and σ_v^2 are small. The difference of the MSEs of $\hat{\boldsymbol{\beta}}_B$ and $\hat{\boldsymbol{\beta}}_{c'}$ is small, when n is large.

Next we examine the accuracy of approximation of (6) under the same situation as above. When the variances σ_e^2 and σ_v^2 are unknown, the proportion, that confidence regions include the true values $\boldsymbol{\beta} = (3.0, 1.5)$, is calculated. The mean and MSE of $\hat{\boldsymbol{\beta}}_c = (\hat{\beta}_{c1}, \hat{\beta}_{c2})'$ in (7) are also calculated. The results are in Tables 3.1 and 3.2.

Table 3.1. Coverage probability ($\sigma_v^2 = 0.1$)

		CP	mean		MSE
n	$1-\alpha$		$\hat{\beta}_{c1}$	$\hat{\beta}_{c2}$	
10	0.90	0.916	3.026	1.544	0.0489
	0.95	0.962	3.027	1.555	0.0512
	0.99	0.989	3.042	1.595	0.0633
20	0.90	0.952	3.019	1.532	0.0239
	0.95	0.970	3.020	1.538	0.0247
	0.99	0.997	3.022	1.554	0.0261
30	0.90	0.957	3.011	1.523	0.0176
	0.95	0.976	3.014	1.528	0.0182
	0.99	0.998	3.016	1.530	0.0189

Table 3.2. Coverage probability ($\sigma_v^2 = 0.3$)

n	$1-\alpha$	CP	mean $\hat{\beta}_{c1}$	$\hat{\beta}_{c2}$	MSE
10	0.90	0.874	3.073	1.665	0.1630
	0.95	0.925	3.096	1.708	0.1983
	0.99	0.968	3.132	1.778	0.2722
20	0.90	0.919	3.044	1.587	0.0681
	0.95	0.964	3.059	1.608	0.0772
	0.99	0.986	3.071	1.655	0.0924
30	0.90	0.939	3.036	1.563	0.0388
	0.95	0.972	3.044	1.585	0.0470
	0.99	0.995	3.056	1.618	0.0562

From Tables 3.1 and 3.2, the coverage probabilities are larger than the given confidence coefficient except for $\sigma_v^2 = 0.3$ and $n = 10$, in which the mean and the MSE of the estimator $\hat{\boldsymbol{\beta}}_c$ are not good in Table 3.2. However, the coverage probability, the mean and the MSE are good, when n is large. It seems that the proposed confidence region (6) is good approximation for large n and small σ_v^2.

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