

On super mean labeling of some graphs

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Abstract. Let G be a (p, q) -graph and $f : V(G) \rightarrow \{k, k+1, k+2, k+3, \dots, p+q+k-1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. Then f is called a k -super mean labeling if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{k, k+1, k+2, \dots, p+q+k-1\}$. A graph that admits a k -super mean labeling is called k -super mean graph. In this paper, we present k -super mean labeling of $C_{2n}(n \neq 2)$ and super mean labeling of Double cycle $C(m, n)$, Dumb bell graph $D(m, n)$ and Quadrilateral snake Q_n .

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§1. Introduction

By a graph we mean a finite, simple and undirected one. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. The disjoint union of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.

Let C_m and C_n be two disjoint cycles with $u \in V(C_m)$ and $v \in V(C_n)$. The double cycle, denoted by $C(m, n)$, is the graph obtained by identifying u and v . The dumb bell graph $D(m, n)$ is obtained by joining the two vertices u and v with an edge.

The antiprism graph G on $2n$ vertices has the vertex set $\{u_i, v_i : 1 \leq i \leq n\}$ and the edge set $\{u_i u_{i+1}, v_i v_{i+1}, u_1 u_n, v_1 v_n : 1 \leq i \leq n-1\} \cup \{u_i v_i : 1 \leq i \leq n\} \cup \{v_i u_{i-1}, v_1 u_n : 2 \leq i \leq n\}$.

Any quadrilateral snake Q_n is obtained from a path $u_1 u_2 u_3 \dots u_n$ by joining u_i and u_{i+1} to new vertices v_i and w_i ($1 \leq i \leq n-1$) respectively and joining v_i to w_i ($1 \leq i \leq n-1$). That is, every edge of the path is replaced by the cycle C_4 . $\lceil x \rceil$ denotes the smallest integer greater than or equal to x . For notations and terminology we follow [2].

§2. Preliminary Results

The concept of super mean labeling was introduced in [6] and further discussed in [3, 4, 5]. B. Gayathri et al. extended the notion of k -super mean labeling of graphs [1]. Let G be a (p, q) -graph and $f : V(G) \rightarrow \{k, k + 1, k + 2, k + 3, \dots, p + q + k - 1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \left\lceil \frac{f(u) + f(v)}{2} \right\rceil$. Then f is called a k -super mean labeling if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{k, k + 1, k + 2, \dots, p + q + k - 1\}$. A graph that admits a k -super mean labeling is called k -super mean graph. We use the following results in the subsequent theorems.

Theorem 2.1. [6] *Any path P_n is a super mean graph.*

Theorem 2.2. [6] *Let $G_1 = (p_1, q_1)$ and $G_2 = (p_2, q_2)$ be two super mean graphs with super mean labeling f and g respectively. Let $f(u) = p_1 + q_1$ and $g(v) = 1$. Then the graph $(G_1)_{f^*}(G_2)_g$ obtained from G_1 and G_2 by identifying the vertices u and v is also a super mean graph.*

Theorem 2.3. [6] *Any odd cycle C_{2n+1} is a super mean graph.*

Remark 2.4. [6] C_4 is not a super mean graph.

§3. k -Super Mean Graph

In this section we establish k -super mean labeling of the graphs such as even cycle (except C_4), antiprism on $2n$ vertices ($n > 4$), the generalized prism $C_n \times P_m$ (n is odd) and the grid $P_m \times P_n$ with one random crossing edge in every square.

Theorem 3.1. *Any even cycle $C_{2n}(n \neq 2)$ is a k -super mean graph.*

Proof. Let $V(C_{2n}) = \{u_1, u_2, u_3, \dots, u_{2n}\}$.

For $n \neq 2$, define $f : V(C_{2n}) \rightarrow \{k, k + 1, k + 2, k + 3, \dots, p + q + k - 1 = 4n + k - 1\}$ by

$$\begin{aligned} f(u_1) &= k, \\ f(u_2) &= k + 2, \\ f(u_3) &= k + 6, \\ f(u_4) &= k + 11, \\ f(u_{4+i}) &= k + 11 + 4i \text{ for } 1 \leq i \leq n - 3, \\ f(u_{n+1+i}) &= 4(n - i)k \text{ for } 1 \leq i \leq n - 3, \\ f(u_{2n-1}) &= k + 8, \\ f(u_{2n}) &= k + 5. \end{aligned}$$

Then $f(V) = \{k, k+2, k+5, k+6, k+8, k+11, k+12, k+15, k+16, \dots, k+4n-9, k+4n-8, k+4n-5, k+4n-4, k+4n-1\}$ and $\{f^*(e) : e \in E(C_{2n})\} = \{k+1, k+3, k+4, k+7, k+9, k+13, k+14, \dots, k+4n-7, k+4n-6, \dots, k+4n-3, k+4n-2\}$. Clearly $f(V) \cup \{f^*(e) : e \in E(C_{2n})\} = \{k, k+1, k+2, \dots, k+4n-1\}$. So f is a k -super mean labeling. Hence $C_{2n}(n \neq 2)$ is a k -super mean graph. \square

Example 3.2. The 5-super mean labeling of C_8 is given in Figure 1.

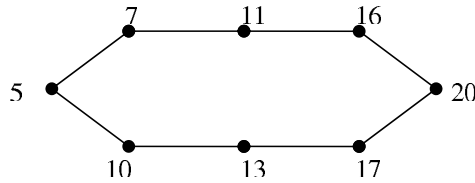


Figure 1

Theorem 3.3. An antiprism G on $2n$ vertices ($n > 4$) is a k -super mean graph.

Proof. Let $\{u_i, v_i : 1 \leq i \leq n\}$ be the $2n$ vertices of the antiprism graph G .

Case (i) n is odd. Take $n = 2s + 1$.

Define $f : V(G) \rightarrow \{k, k+1, k+2, k+3, \dots, p+q+k-1 = 6n+k-1\}$ by

$$\begin{aligned} f(u_1) &= k; \\ f(u_2) &= k+5; \\ f(u_{2+i}) &= k+5+4i \text{ for } 1 \leq i \leq s-1; \\ f(u_{s+2}) &= k+4s-2; \\ f(u_{s+2+i}) &= k+4s-2-4i \text{ for } 1 \leq i \leq s-1; \\ f(v_1) &= k+8s+4; \\ f(v_2) &= k+8s+9; \\ f(v_{2+i}) &= k+8s+9+4i \text{ for } 1 \leq i \leq s-1; \\ f(v_{s+2}) &= k+12s+2; \\ f(v_{s+2+i}) &= k+12s+2-4i \text{ for } 1 \leq i \leq s-1. \end{aligned}$$

It can be verified that f is a k -super mean labeling of G .

Case (ii) n is even. Take $n = 2s$.

Define $f : V(G) \rightarrow \{k, k+1, k+2, k+3, \dots, p+q+k-1 = 6n+k-1\}$ by

$$\begin{aligned} f(u_1) &= k; \\ f(u_2) &= k+2; \\ f(u_3) &= k+6; \\ f(u_4) &= k+11; \\ f(u_{4+i}) &= k+11+4i \text{ for } 1 \leq i \leq s-3; \\ f(u_{s+2}) &= k+4s-4; \\ f(u_{s+2+i}) &= k+4s-4-4i \text{ for } 1 \leq i \leq s-3; \\ f(u_{2s}) &= k+5; \\ f(v_1) &= k+8s+5; \\ f(v_2) &= k+8s; \\ f(v_3) &= k+8s+2; \\ f(v_4) &= k+8s+6; \\ f(v_5) &= k+8s+11; \\ f(v_{5+i}) &= k+8s+11+4i \text{ for } 1 \leq i \leq s-3; \\ f(v_{s+3}) &= k+12s-4; \\ f(v_{s+3+i}) &= k+12s-4-4i \text{ for } 1 \leq i \leq s-3. \end{aligned}$$

Clearly the induced edge labels are distinct. Therefore f is a k -super mean labeling of G . Hence G is a k -super mean graph. \square

Example 3.4. The 3-super mean labeling of antiprism on 12 vertices is given in Figure 2.

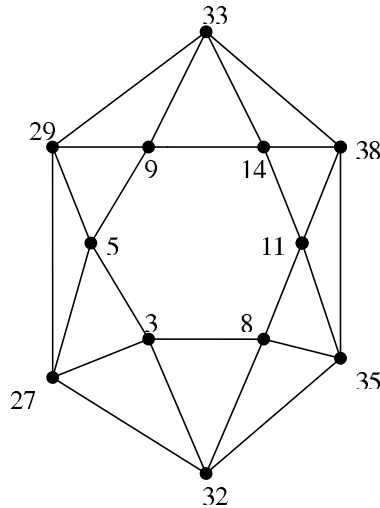


Figure 2

Theorem 3.5. *The graph $C_n \times P_m$ is a k -super mean graph where n is an odd integer and m is any integer.*

Proof. Let $\{u_j^i : 1 \leq j \leq n, 1 \leq i \leq m\}$ be the vertices of $C_n \times P_m$. Take $n = 2s + 1$.

Define $f : V(C_n \times P_m) \rightarrow \{k, k + 1, k + 2, k + 3, \dots, p + q + k - 1 = n(3m - 1) + k - 1\}$ by

$$\begin{aligned} f(u_j^1) &= k + 2j - 2 \text{ for } 1 \leq j \leq s + 1; \\ f(u_{s+2}^1) &= k + 2s + 3; \\ f(u_{s+2+j}^1) &= k + 2s + 3 + 2j \text{ for } 1 \leq j \leq s - 1; \\ f(u_1^2) &= k + 8s + 3; \\ f(u_{1+j}^2) &= k + 8s + 4 + 2j \text{ for } 1 \leq j \leq s; \\ f(u_{s+2}^2) &= k + 6s + 3; \\ f(u_{s+2+j}^2) &= k + 6s + 3 + 2j \text{ for } 1 \leq j \leq s - 1. \end{aligned}$$

For $m > 2$, $f(u_j^m) = f(u_j^{m-2}) + 6n$ for $1 \leq j \leq n$. One can prove that f is a k -super mean labeling of $C_n \times P_m$. Hence the theorem. \square

Example 3.6. *The 4-super mean labeling of $C_7 \times P_4$ is give in Figure 3.*

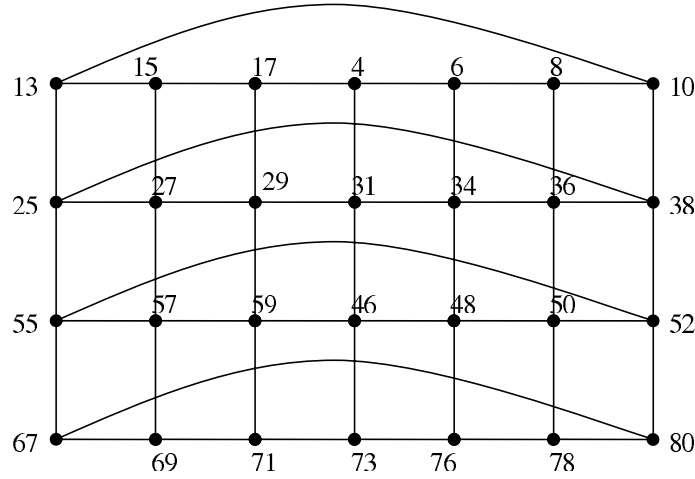


Figure 3

Theorem 3.7. *The grid $P_m \times P_n$ with one random crossing edge in every square is a k -super mean graph.*

Proof. Let $\{u_i^j : 1 \leq j \leq m, 1 \leq i \leq n\}$ be the vertices of $P_m \times P_n$. Define f as follows: $f(u_i^j) = k + 2j - 2 + (2i - 2)(2m - 1)$ for all $1 \leq j \leq m, 1 \leq i \leq n$. Hence

the edges $u_i^j u_{i+1}^j$ will get the label $k + 2j - 2 + (2i - 1)(2m - 1)$ and the edge $u_i^j u_i^{j+1}$ will get the label $k + 2j - 1 + (2i - 2)(2m - 1)$. A crossing edge is either $u_i^j u_{i+1}^{j+1}$ or $u_{i+1}^j u_i^{j+1}$ and both will get the label $k + 2j - 1 + (2i - 1)(2m - 1)$. Clearly f is a k -super mean labeling. Hence the grid $P_m \times P_n$ with one random crossing edge in every square is a k -super mean graph. \square

Example 3.8. The 2-super mean labeling obtained from $P_3 \times P_4$ is given in Figure 4.

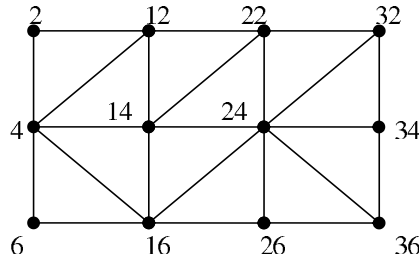


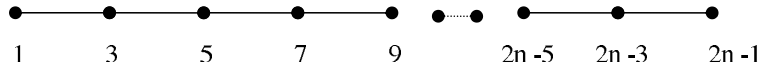
Figure 4

Note 3.9. The k -super mean labeling of the graph G is the generalization of super mean labeling of G .

§4. Super Mean Graph

Theorem 4.1. Let $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ be two super mean graphs with $u \in V(G_1)$ has the label $p_1 + q_1$ and $v \in V(G_2)$ has the label 1. Then the graph G which is obtained by joining u to v by any path P_n is a super mean graph.

Proof. Let f and h be the super mean labelings of G_1 and G_2 respectively. Let $u_1, u_2, u_3, \dots, u_n$ be vertices of path P_n . By Theorem 2.1, P_n is a super mean graph. Let g be the super mean labeling of P_n as follows.



Then $g(u_1) = 1$ and $g(u_n) = 2n - 1$. By Theorem 2.2, $(G_1)_f * (P_n)_g = G_3$ (say) is a super mean graph. Let k be the super mean labeling of G_3 . Again by Theorem 2.2, $(G_3)_k * (G_2)_h = G$ is a super mean graph. Hence G is a super mean graph. \square

Theorem 4.2. The double cycle $C(m, n)$ is a super mean graph for all $m \geq 3$ and $n \geq 3$.

Proof. **Case (i)** $m \neq 4$ and $n \neq 4$.

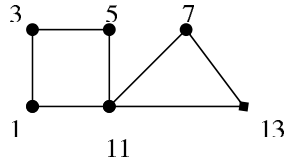
Since all cycles except C_4 are super mean graphs, by Theorem 2.2, $C(m, n)$ is a super mean graph.

Case (ii) At least one of m, n is 4. Assume $m = 4$.

Let u_1, u_2, u_3, u_4 be the vertices of C_4 and $V(C_n) = \{v_i : 1 \leq i \leq n\}$. Identify u_4 and v_1 . Then $V(C(m, n)) = \{u_i, v_j : 1 \leq i \leq 4, 1 \leq j \leq n \text{ with } u_4 = v_1\}$.

Subcase (i) n is odd. Take $n = 2s + 1$.

A super mean labeling of $C(4, 3)$ is given by



For $n > 3$, define $f : V(C(4, n)) \rightarrow \{1, 2, 3, \dots, p + q = 2n + 7 = 4s + 9\}$ by

$$\begin{aligned} f(u_1) &= 1; \\ f(u_2) &= 3; \\ f(u_3) &= 5; \\ f(u_4) &= f(v_1) = 11; \\ f(v_2) &= 7; \\ f(v_3) &= 12; \\ f(v_4) &= 4s + 9; \\ f(v_{4+i}) &= 2(2s - i) + 9 \text{ for } 1 \leq i \leq s - 2; \\ f(v_{s+2+i}) &= 2(4 - i) + n + 3 \text{ for } 1 \leq i \leq s - 1. \end{aligned}$$

It can be established that f is a super mean labeling.

Subcase (ii) n is even. Take $n = 2s$.

Define $f : V(C(4, n)) \rightarrow \{1, 2, 3, \dots, p + q = 2n + 7 = 4s + 7\}$ by

$$\begin{aligned} f(u_1) &= 1; \\ f(u_2) &= 3; \\ f(u_3) &= 5; \\ f(u_4) &= f(v_1) = 11; \\ f(v_2) &= 7; \\ f(v_3) &= 12; \\ f(v_{3+i}) &= 12 + 2i \text{ for } 1 \leq i \leq s - 2; \\ f(v_{s+1+i}) &= 2s + 2i + 9 \text{ for } 1 \leq i \leq s - 1. \end{aligned}$$

It can be verified that f is a super mean labeling. Hence the double cycles $C(m, n)$ are super mean graphs for all $m \geq 3$ and $n \geq 3$. \square

Example 4.3. *The super mean labeling of $C(4, 8)$ is given in Figure 5.*

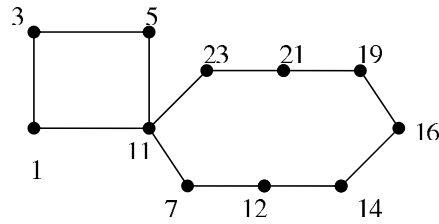


Figure 5

Theorem 4.4. *The dumb bell graph $D(m, n)$ is a super mean graph for all $m \geq 3$ and $n \geq 3$.*

Proof. We consider the following two cases.

Case (i) $m \neq 4$ and $n \neq 4$.

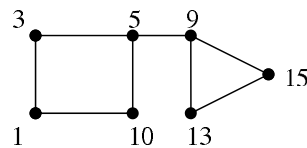
The proof follows from fact that all cycles except C_4 are super mean graphs and by Theorem 4.1.

Case (ii) At least one of m, n is 4. Let $m = 4$.

Let $V(C_m) = \{u_i : i = 1, 2, 3, 4\}$ and $V(C_n) = \{v_i : 1 \leq i \leq n\}$.

Subcase (i) n is odd. Take $n = 2s + 1$.

Join u_3 and v_3 by an edge. Then $V(D(m, n)) = V(C_m) \cup V(C_n)$ and $E(D(m, n)) = E(C_m) \cup E(C_n) \cup \{u_3v_3\}$. A super mean labeling of $D(4, 3)$ is given below:



For $n > 3$, define $f : V(D(m, n)) \rightarrow \{1, 2, 3, \dots, p + q = 2n + 9 = 4s + 11\}$ by

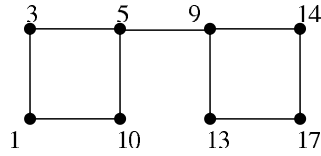
$$\begin{aligned} f(u_1) &= 1; \\ f(u_2) &= 3; \\ f(u_3) &= 5; \\ f(u_4) &= 10; \\ f(v_1) &= 15; \\ f(v_2) &= 12; \\ f(v_3) &= 9; \\ f(v_4) &= 16; \\ f(v_{4+i}) &= 16 + 2i \text{ for } 1 \leq i \leq s - 2; \\ f(v_{s+3}) &= 2s + 15; \end{aligned}$$

$$f(v_{s+3+i}) = 2s + 15 + 2i \text{ for } 1 \leq i \leq s - 2.$$

One can verify that f is a super mean labeling.

Subcase (ii) n is even. Take $n = 2s$.

Join u_3 and v_2 with an edge. Then $V(D(m, n)) = V(C_m) \cup V(C_n)$ and $E(D(m, n)) = E(C_m) \cup E(C_n) \cup \{u_3v_2\}$. For $n = 4$, a super mean labeling of $D(4, n)$ is given by



For $n > 4$, define $f : V(D(m, n)) \rightarrow \{1, 2, 3, \dots, p + q = 2n + 9 = 4s + 9\}$ by

$$\begin{aligned} f(u_1) &= 1; \\ f(u_2) &= 3; \\ f(u_3) &= 5; \\ f(u_4) &= 10; \\ f(v_1) &= 13; \\ f(v_2) &= 9; \\ f(v_3) &= 14; \\ f(v_{3+i}) &= 14 + 2i \text{ for } 1 \leq i \leq s - 2; \\ f(v_{s+2}) &= 2s + 13; \\ f(v_{s+2+i}) &= 2s + 13 + 2i \text{ for } 1 \leq i \leq s - 2. \end{aligned}$$

It can be established that f is a super mean labeling. Hence the dumb bell graphs $D(m, n)$ are super mean graphs for all $m \geq 3$ and $n \geq 3$. \square

Example 4.5. The super mean labeling of $D(4, 7)$ is given in Figure 6.

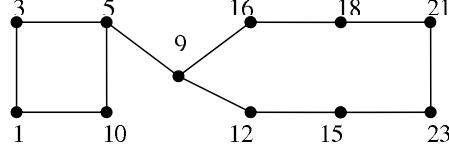


Figure 6

Theorem 4.6. Let $C_n (n \geq 3)$ be an odd cycle. Consider n copies of an odd cycle $C_m (m \geq 3)$. If G is a graph obtained by identifying a vertex of each cycle C_m with a vertex of the cycle C_n is a super mean graph.

Proof. Let $u_1, u_2, u_3, \dots, u_n$ be the vertices of the cycle C_n . Let $u_{1j}, u_{2j}, u_{3j}, \dots, u_{nj}, 1 \leq j \leq m$, be the vertices of the cycles $C_m^{(1)}, C_m^{(2)}, C_m^{(3)}, \dots, C_m^{(n)}$ respectively, identified at each vertex of C_n such that $u_1 = u_{1m}, u_2 = u_{21}, u_3 = u_{3m}, \dots, u_{n-1} = u_{n-1,1}$ and $u_n = u_{nm}$ which means that $u_{1m}, u_{21}, u_{3m}, u_{41}, \dots, u_{n-1,1}, u_{nm}$ are the vertices of the cycle C_n .

Take $n = 2s + 1$ and $m = 2t + 1$.

Define $f : V(G) \rightarrow \{1, 2, 3, \dots, (2m + 1)n = 8st + 6s + 4t + 3\}$ as follows:

$$\text{For the cycle } C_m^{(1)}, f(u_{1j}) = \begin{cases} 2j - 1 & \text{for } 1 \leq j \leq t + 1 \\ 2j & \text{for } t + 2 \leq j \leq m. \end{cases}$$

For the cycle $C_m^{(k)}$, where $2 \leq k \leq s + 1$,

$$f(u_{kj}) = \begin{cases} 2(k - 1)m + 2(j - 1) + k & \text{for } 1 \leq j \leq t + 1 \\ 2(k - 1)m + 2(j - 1) + k + 1 & \text{for } t + 2 \leq j \leq m. \end{cases}$$

For the cycle $C_m^{(k)}$, where $s + 2 \leq k \leq n$.

$$f(u_{kj}) = \begin{cases} 2(k - 1)m + 2(j - 1) + k + 1 & \text{for } 1 \leq j \leq t + 1 \\ 2(k - 1)m + 2(j - 1) + k + 2 & \text{for } t + 2 \leq j \leq m. \end{cases}$$

Now we have $\bigcup_{i=1}^n \{f(V(C_m^{(i)})) \cup f^*(E(C_m^{(i)}))\} = \{1, 2, 3, \dots, 2m\} \cup \{2m + 2, 2m + 3, \dots, 4m + 1\} \cup \{4m + 3, 4m + 4, \dots, 6m + 2\} \cup \dots \cup \{(2m + 1)s + 1, (2m + 1)s + 2, \dots, (2m + 1)s + 2m\} \cup \{(2m + 1)(s + 1) + 2, (2m + 1)(s + 1) + 3, \dots, (2m + 1)(s + 2)\} \cup \dots \cup \{(2m + 1)(n - 1) + 2, \dots, (2m + 1)n\}$. Clearly these labels are all distinct. Further the labels of the edges $u_1u_2, u_2u_3, u_3u_4, \dots, u_{s+1}u_{s+2}, u_{s+2}u_{s+3}, \dots, u_nu_1$ of the cycle C_n are $2m + 1, 4m + 2, 6m + 3, \dots, (2m + 1)(s + 1) + 1, (2m + 1)(s + 2) + 1, \dots, (2m + 1)(s + 1)$ respectively. It can be easily verified that $f(V) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \dots, n(2m + 1)\}$. Hence G is a super mean graph. \square

Corollary 4.7. *The graph $C_{2n+1} \odot K_2$ is a super mean graph for all n .*

Example 4.8. *The super mean labeling of G obtained from C_3 by identifying a vertex of the cycle C_5 with each vertex of the cycle C_3 is given in Figure 7.*

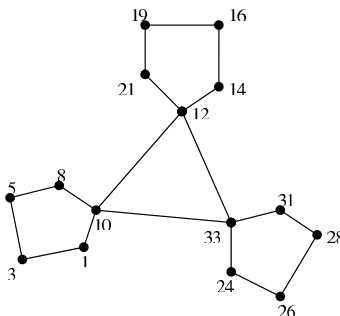


Figure 7

The graph Q_2 is C_4 , and hence it is not a super mean graph [6]. Next we prove Q_n is a super mean graph for all odd values of n .

Theorem 4.9. *The quadrilateral snake Q_n , where n is odd, is a super mean graph.*

Proof. Let $V(Q_n) = \{u_i, v_i, w_i, u_n : 1 \leq i \leq n - 1\}$. Define $f : V(Q_n) \rightarrow \{1, 2, 3, \dots, 7n - 6\}$ by

$$\begin{aligned} f(u_1) &= 1; \\ f(u_{2i}) &= f(u_{2i-1}) + 10 \text{ for } 1 \leq i \leq s; \\ f(u_{2i+1}) &= f(u_{2i}) + 4 \text{ for } 1 \leq i \leq s; \\ f(v_1) &= 3; \\ f(v_{2i}) &= f(v_{2i-1}) + 4 \text{ for } 1 \leq i \leq s; \\ f(v_{2i+1}) &= f(v_{2i}) + 10 \text{ for } 1 \leq i \leq s - 1; \\ f(w_1) &= 5; \\ f(w_{i+1}) &= f(w_i) + 7 \text{ for } 1 \leq i \leq n - 1. \end{aligned}$$

Clearly $f(V) \cup \{f^*(e) : e \in E(Q_n)\} = \{1, 2, 3, \dots, 7n - 6\}$. Hence, Q_n where n is odd, is a super mean graph. \square

Example 4.10. *The super mean labeling of Q_5 is given in Figure 8.*

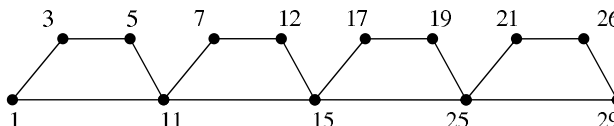


Figure 8

Theorem 4.11. *Let $C_n : u_1u_2u_3 \dots u_nu_1$ (n is odd) be a cycle. Let G be the graph with $V(G) = V(C_n) \cup \{v_i : 1 \leq i \leq n\}$, $E(G) = E(C_n) \cup \{u_iv_i, u_{i+1}v_i : 1 \leq i \leq n-1\} \cup \{u_nv_n, u_1v_n\}$. Then G is a super mean graph.*

Proof. Take $n = 2s + 1$. Define $f : V(G) \rightarrow \{1, 2, 3, \dots, p + q = 5n\}$ by

$$\begin{aligned} f(u_1) &= 1; \\ f(u_i) &= 5i - 4 \text{ for } 2 \leq i \leq s + 1; \\ f(u_{s+2}) &= 5s + 8; \\ f(u_{s+2+i}) &= 5s + 8 + 5i \text{ for } 1 \leq i \leq s - 1; \\ f(v_1) &= 3; \\ f(v_i) &= 5i - 2 \text{ for } 2 \leq i \leq s; \\ f(v_{s+1}) &= 5s + 6; \\ f(v_{s+2}) &= 5(s + 2); \\ f(v_{s+2+i}) &= 5(s + 2) + 5i \text{ for } 1 \leq i \leq s - 1. \end{aligned}$$

Clearly the vertex labels, the induced edge labels are distinct and $f(V) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \dots, 5n\}$. Hence G is a super mean graph. \square

Theorem 4.12. *Let $C_n : u_1u_2u_3 \dots u_nu_1$ (n is odd) be a cycle. Let G be the graph obtained from C_n by joining the vertices u_i and u_{i+1} by the path P_m^i (m is odd) $1 \leq i \leq n-1$ and joining the vertices u_n and u_1 by the path P_m^n . Then G is a super mean graph.*

Proof. By Theorem 4.11, the theorem is true when $m = 3$. We prove the theorem for $m > 3$. Let $v_1^j, v_2^j, v_3^j, \dots, v_m^j$ for $1 \leq j \leq m$ be the vertices of the path P_m^i ($1 \leq i \leq n$) such that $v_m^j = v_1^{j+1} = u_{j+1}$ for $1 \leq j \leq n-1$ and $v_m^n = v_1^1 = u_1$. Take $n = 2s + 1$ and $m = 2t + 1$.

Define $f : V(G) \rightarrow \{1, 2, 3, \dots, p + q = n(2m - 1)\}$ by

$$\begin{aligned} f(v_i^1) &= 2i - 1 \text{ for } 1 \leq i \leq t + 1; \\ f(v_i^1) &= 2i \text{ for } t + 2 \leq i \leq 2t + 1; \\ f(v_i^j) &= f(v_i^{j-1}) + 2m - 1 \text{ for } 1 \leq i \leq 2t + 1 \text{ and } 2 \leq j \leq s; \\ f(v_1^{s+1}) &= f(v_m^s) = 1 + (2m - 1)s; \\ f(v_2^{s+1}) &= 4 + (2m - 1)s; \end{aligned}$$

$$\begin{aligned}
 f(v_{2+i}^{s+1}) &= 4 + (2m - 1)s + 2i \text{ for } 1 \leq i \leq t - 2; \\
 f(v_{t+1}^{s+1}) &= 2t(2s + 1) + s + 4; \\
 f(v_{t+1+i}^{s+1}) &= 2t(2s + 1) + s + 4 + 2i \text{ for } 1 \leq i \leq t; \\
 f(v_i^{s+2}) &= 4t(s + 1) + s + 2 + 2i \text{ for } 1 \leq i \leq t + 1; \\
 f(v_i^{s+2}) &= 4t(s + 1) + s + 3 + 2i \text{ for } t + 2 \leq i \leq 2t + 1; \\
 f(v_i^j) &= f(v_i^{j-1}) + 2m - 1 \text{ for } 1 \leq i \leq 2t + 1 \text{ and } s + 3 \leq j \leq 2s; \\
 f(v_{1+i}^{2s+1}) &= f(v_m^{2s}) + 2i \text{ for } 1 \leq i \leq 2t - 1.
 \end{aligned}$$

It can be verified that f is a super mean labeling of G . Hence G is a super mean graph. \square

Example 4.13. The super mean labeling of G with $m = 5$ and $n = 7$ is given in Figure 9.

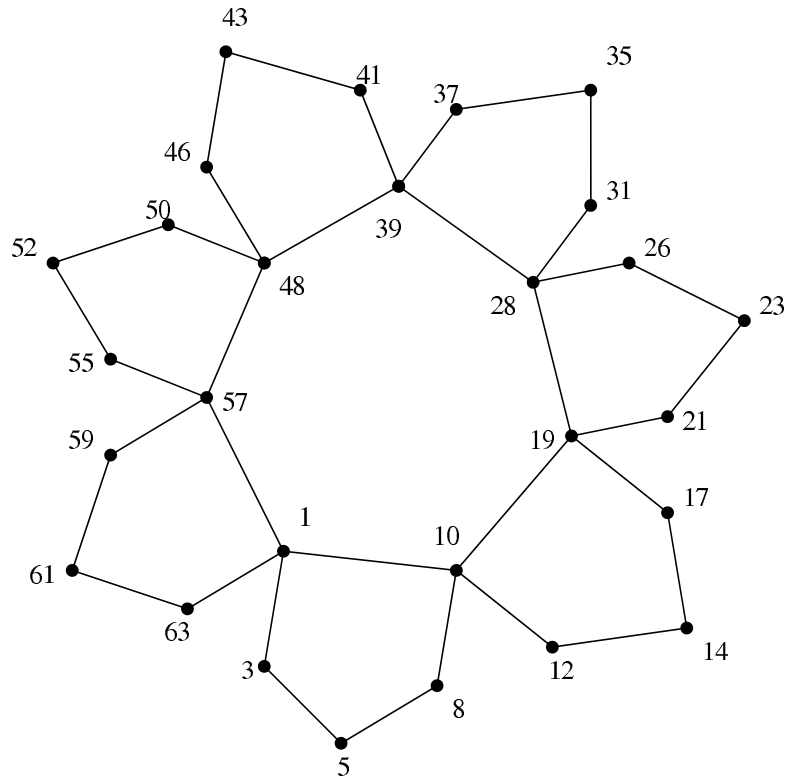


Figure 9

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