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ASYMPTOTIC DIMENSION AND THE INTEGRAL K-THEORETIC NOVIKOV CONJECTURE FOR ARITHMETIC GROUPS

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Abstract

We prove that the integral K-theoretic Novikov conjecture holds for torsion free arithmetic subgroups of linear algebraic groups.

1. Introduction

The original Novikov conjecture concerns the homotopy invariance of higher signatures and is equivalent to the rational injectivity of the assembly map in surgery (i.e., L-) theory (see [**11**], [**17**, Section 24]),

$$A : H_*(B\Gamma, \mathbb{L}_\bullet(\mathbb{Z})) \to L_*(\mathbb{Z}[\Gamma]),$$

where $B\Gamma$ is the classifying space of a discrete group Γ, $\mathbb{L}_\bullet(\mathbb{Z})$ the algebraic surgery spectrum, and $L_*(\mathbb{Z}[\Gamma])$ is the L-group for the group ring $\mathbb{Z}[\Gamma]$. There are also assembly maps in algebraic K-theory

$$A : H_*(B\Gamma, \mathbb{K}(R)) \to K_*(R[\Gamma]),$$

where R is an associative ring with unit, and in C^*-algebra theory (see [**11**]). The integral injectivity of the assembly map is called the integral Novikov conjecture in that theory and is an important step in understanding the groups on the right-hand side. The version in the C^*-algebras is often called the strong (or analytic) Novikov conjecture since it implies the original Novikov conjecture. But each version is important for its own reasons.

For discrete subgroups of Lie groups, the analytic Novikov conjecture was proved by Kasparov (see [**11**]), and the integral Novikov conjecture in L-theory was proved by Farrell–Hsiang, Ferry–Weinberger, Farrell–Jones (see [**10**, pp. 217, 220]). On the other hand, the integral K-theoretic Novikov conjecture was known only for the following classes of discrete subgroups of Lie groups: (1) co-compact lattices in connected Lie groups due to Carlsson [**5**] (see also [**9**, **2**]), (2) arithmetic subgroups of semisimple linear algebraic groups $\mathbf{G}$ defined over $\mathbb{Q}$ of $\mathbb{R}$-rank equal

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to 1 due to Goldfarb [12], and more generally when $\mathbf{G}$ is semisimple and the $\mathbb{Q}$-rank of $\mathbf{G}$ is equal to the $\mathbb{R}$-rank of $\mathbf{G}$ in [13], (3) arithmetic groups of $\mathbf{G} = SL(3)$ due to Goldfarb [14].

In this note, we combine several simple observations, probably well-known to experts, to prove

Theorem 1.1. *Let $\mathbf{G}$ be a connected linear algebraic group $\mathbf{G}$ defined over $\mathbb{Q}$, not necessarily semisimple or reductive. Then the integral K-theoretic Novikov conjecture holds for any torsion free arithmetic subgroup $\Gamma \subset \mathbf{G}(\mathbb{Q})$.*

The method also gives an alternative proof of the integral Novikov conjecture in L-theory and C^*-algebra theory for torsion free arithmetic subgroups.

Recently, Bartels [1, Theorems 1.1 and 7.2] (for both K-theory and L-theory), Carlsson and Goldfarb [6, Main Theorem] (for K-theory) proved the following result, which is crucial to this paper.

Proposition 1.2. *If a finitely generated group Γ has finite asymptotic dimension and a finite CW-complex as its classifying space, then the integral Novikov conjecture in K-theory and L-theory holds for Γ.*

Note that if Γ has a finite classifying space, then Γ is torsion free. In fact, the existence of a finite classifying space implies that the cohomological dimension of Γ is finite, which in turn implies that Γ is torsion free. Proposition 1.2 was suggested by the result of Yu in [19] that under the same assumption on Γ, the analytic Novikov conjecture holds for Γ.

To prove Theorem 1.1, it suffices to prove that torsion free arithmetic subgroups have both finite asymptotic dimension and a finite classifying space. A lot of work has been done to prove finite asymptotic dimensionality for various groups: hyperbolic groups [15], Coxeter groups, cocompact discrete subgroups of connected Lie groups [7], fundamental groups of graphs of groups and of complexes of groups under certain conditions (see the survey paper [8] and its references). The finiteness of the asymptotic dimension is inherited by any finitely generated subgroup, but this is not the case with the property of having a finite classifying space. Therefore, it is a natural problem to find classes of groups which have both finite asymptotic dimension and finite classifying spaces. The point of this note is that the class of torsion free arithmetic subgroups enjoys both properties. Sharp bounds on the asymptotic dimension are also given for arithmetic groups (3.5).

2. Asymptotic dimension

In this section, we recall some basic facts about asymptotic dimension. The asymptotic dimension of a metric space (M, d) is defined as the smallest integer n, which could be ∞, such that for every $r > 0$, there exists a cover $\mathcal{C} = \{U_i\}_{i \in I}$ of M by uniformly bounded sets U_i with the r-multiplicity less than or equal to $n + 1$, i.e., every ball in M of radius r intersects at most $n + 1$ sets in $\mathcal{C}$.

Clearly, if (N, d) is a subspace of (M, d) endowed with the induced subspace metric, then asdim $N \leq$ asdim M. Two metric spaces (M_1, d_1), (M_2, d_2) are *quasi-isometric* if there exists a map $f : M_1 \to M_2$ and a constant $C > 1$ such that (1) for every pair of points $x, y \in M_1$,

$$\frac{1}{C} d_1(x, y) - C \leq d_2(f(x), f(y)) \leq C d_1(x, y) + C,$$

and (2) every point of M_2 is in a C-neighborhood of the image $f(M_1)$. It is known (and easy to see) that quasi-isometric spaces have the same asymptotic dimension.

For any finitely generated group Γ, a choice of a symmetric generating set S, i.e., $S^{-1} = S$, defines a word metric d_S on Γ, which is left Γ-invariant. Define asdim Γ = asdim (Γ, d_S). It is known that different choices of symmetric generating sets S lead to quasi-isometric metrics on Γ (see [**16**, Example 21 in Chap. IV]), and hence asdim Γ is well-defined.

Recall [**16**, p. 84] that a metric space (M, d) is called *proper* if for any $x_0 \in M$ and $r > 0$, the closed ball $B(x_0, r) = \{x \in M \mid d(x, x_0) \leq r\}$ is compact. It is known that a proper metric space is complete and locally compact, and that a complete Riemannian manifold with the induced distance function is a proper metric space. A metric space is called *geodesic* if any two points can be joined by at least one (minimizing) geodesic segment.

The following result is well-known (see [**16**, Chap. IV, Theorem 23]).

Proposition 2.1. *If (M, d) is a geodesic proper metric space and if Γ acts properly and isometrically on M with a compact quotient $\Gamma \backslash M$, then Γ is finitely generated and* asdim Γ = asdim M.

It follows from [**19**, Proposition 6.2] that asdim $\Gamma_1 \leq$ asdim Γ for any finitely generated subgroup Γ_1 of Γ. A corollary of the above proposition is the following result.

Lemma 2.2. *If Γ_1 is a subgroup of finite index in Γ, then* asdim Γ_1 = asdim Γ. *In particular, two commensurable subgroups have the same asymptotic dimension.*

Proof. Let S_1 be a symmetric generating set of Γ_1, and S a generating set of Γ. By [**16**, Corollary 24 in Chap. IV], the metric spaces

(Γ_1, d_{S_1}) and (Γ, d_S) are quasi-isometric. In fact, let $Cay(\Gamma, S)$ be the Cayley graph of Γ associated with the set of generators S. It has a natural distance function such that the embedding $(\Gamma, d_S) \hookrightarrow Cay(\Gamma, S)$ is isometric and gives a quasi-isometry between $Cay(\Gamma, S)$ and (Γ, d_S). The left multiplication of Γ on Γ extends to an isometric action on $Cay(\Gamma, S)$. Since Γ_1 is of finite index in Γ, the quotient $\Gamma_1\backslash Cay(\Gamma, S)$ is a finite graph. Since $Cay(\Gamma, S)$ is a geodesic, proper metric space, it follows from Proposition 2.1 that (Γ_1, d_{S_1}) is quasi-isometric to the Cayley graph $Cay(\Gamma, S)$ and hence to (Γ, d_S). Therefore, asdim $\Gamma_1 =$ asdim Γ, and the first statement is proved. By definition, two subgroups are called commensurable if their intersection is of finite index in each of them. Then, the second statement follows from the first one. q.e.d.

Given two metric spaces $(M_1, d_1), (M_2, d_2)$, a proper map $\varphi : (M_1, d_1) \to (M_2, d_2)$ is called *coarsely uniform* if there exists a positive increasing function $f(r)$ such that

$$d_2(\varphi(x_1), \varphi(x_2)) \leq f(d_1(x_1, x_2)).$$

Two maps $\varphi_1, \varphi_2 : (M_1, d_1) \to (M_2, d_2)$ are called *coarsely equivalent* if there exists a constant C such that $d_1(\varphi_1(x), \varphi_2(x)) \leq C$, for all $x \in M_1$. Two metric spaces (M_1, d_1), (M_2, d_2) are called coarsely equivalent if there exist coarsely uniform maps $\varphi : (M_1, d_1) \to (M_2, d_2)$, $\psi : (M_2, d_2) \to (M_1, d_1)$ such that $\psi\varphi : M_1 \to M_1, \varphi\psi : M_2 \to M_2$ are coarsely equivalent to the identity map on M_1, M_2 respectively. It is also known (and easy to see) that two coarsely equivalent metric spaces have the same asymptotic dimension.

Proposition 2.3. *Let (M, d) be any proper metric space. If a finitely generated group Γ acts isometrically and properly on M, then for any point $x_0 \in M$, the map $(\Gamma, d_S) \to (\Gamma x_0, d), \gamma \mapsto \gamma x_0$, is a coarse equivalence, and hence* asdim $\Gamma \leq$ asdim M.

Proof. First, we note that there exists a constant $C > 0$ such that for any $\gamma, \delta \in \Gamma$,

$$d(\gamma x_0, \delta x_0) \leq C d_S(\gamma, \delta).$$

In fact, let $S = S^{-1}$ be a generating set of Γ. Write $\gamma^{-1}\delta = g_1 \cdots g_n$, where $g_1, \cdots, g_n \in S$, and $n = d_S(\gamma, \delta)$. Then,

$$\begin{aligned} d(\gamma x_0, \delta x_0) =& d(x_0, \gamma^{-1}\delta x_0) = d(x_0, g_1 \cdots g_n x_0) \\ &\leq d(x_0, g_1 x_0) + d(g_1 x_0, g_1 g_2 x_0) + \cdots \\ &\quad + d(g_1 \cdots g_{n-1} x_0, g_1 \cdots g_n x_0) \\ =& d(x_0, g_1 x_0) + d(x_0, g_2 x_0) + \cdots + d(x_0, g_n x_0) \\ &\leq Cn = C d_S(\gamma, \delta), \end{aligned}$$

where $C = \max\{d(x_0, gx_0) \mid g \in S\}$.

On the other hand, since (M, d) is a proper metric space, for any $r \geq 0$, the ball $B(x_0, r) = \{x \in M \mid d(x, x_0) \leq r\}$ is compact. By assumption, Γ acts properly on M, and hence the set

$$I_r = \{\gamma \in \Gamma \mid d(\gamma x_0, x_0) \leq r\} = \{\gamma \in \Gamma \mid \gamma\{x_0\} \cap B(x_0, r) \neq \emptyset\}$$

is finite. Define

$$f(r) = 1 + \max\{d_S(\gamma, e) \mid \gamma \in I_r\}.$$

Clearly $f(r)$ is a finite positive increasing function. We claim that for any $\gamma, \delta \in \Gamma$,

$$d_S(\gamma, \delta) \leq f(d(\gamma x_0, \delta x_0)).$$

In fact, let $r = d(\gamma x_0, \delta x_0)$. Then, $d(\delta^{-1}\gamma x_0, x_0) = r$, and hence $\delta^{-1}\gamma \in I_r$. By definition, $d_S(\delta^{-1}\gamma, e) \leq f(r)$, which is equivalent to

$$d_S(\gamma, \delta) \leq f(d(\gamma x_0, \delta x_0)).$$

Since the map $\Gamma \to \Gamma x_0$ is proper, this proves that the map $(\Gamma, d_S) \to (\Gamma x_0, d)$ is a coarse equivalence. Hence, asdim Γ = asdim $(\Gamma x_0, d) \leq$ asdim M. q.e.d.

3. Arithmetic groups

Let $\mathbf{G} \subset GL(n, \mathbb{C})$ be a linear algebraic group defined over $\mathbb{Q}$. A subgroup Γ of $\mathbf{G}(\mathbb{Q})$ is called *arithmetic* if Γ is commensurable with $\mathbf{G} \cap GL(n, \mathbb{Z})$. Let $G = \mathbf{G}(\mathbb{R})$, $K \subset G$ a maximal compact subgroup, and $X = G/K$ the associated homogeneous space with a G-invariant metric. Then X is diffeomorphic to an Euclidean space. If Γ is torsion free, then Γ acts properly and freely on X, and the quotient $\Gamma\backslash X$ is a manifold and a $K(\Gamma, 1)$-space.

Lemma 3.1. *Let* $\mathbf{G}$ *be a connected linear algebraic group* $\mathbf{G}$ *defined over* $\mathbb{Q}$*, not necessarily semisimple or reductive. Any torsion free arithmetic subgroup* $\Gamma \subset \mathbf{G}(\mathbb{Q})$ *has a finite classifying space given by a compact manifold with corners.*

Proof. First note that we can assume Γ is a lattice in $G = \mathbf{G}(\mathbb{R})$; otherwise, replace $\mathbf{G}$ by the algebraic subgroup ${}^0\mathbf{G}$ (see [**4**, Section 1.1]) over $\mathbb{Q}$ defined by ${}^0\mathbf{G} = \cap_{a \in X(\mathbf{G})_{\mathbb{Q}}} \ker a^2$, where $X(\mathbf{G})_{\mathbb{Q}}$ is the group of $\mathbb{Q}$-morphisms of $\mathbf{G}$ into GL_1 and note that $\Gamma \subset {}^0\mathbf{G}(\mathbb{Q})$ and has cofinite volume in ${}^0\mathbf{G}(\mathbb{R})$. If Γ is co-compact, then $\Gamma\backslash X$ is compact and hence a finite classifying space of Γ. Otherwise, $\Gamma\backslash X$ is non-compact. If $\mathbf{G}$ is semisimple, then X is a symmetric space of non-compact type, and it is well-known that the locally symmetric space $\Gamma\backslash X$ admits the Borel–Serre compactification $\overline{\Gamma\backslash X}^{BS}$ in [4], a compact manifold with corners $\overline{\Gamma\backslash X}^{BS}$, which is homotopic to $\Gamma\backslash X$ and hence gives a desired finite classifying space of Γ. When $\mathbf{G}$ is not semisimple, the compactification in [4] also applies. In fact, the compactification in [4] was developed for spaces of type S. Since the isotropy groups of points in G/K are conjugates of K and hence reductive, [4, Remark 2.4.(2)] implies that $X = G/K$ is a space of type S. Since Γ is a lattice in G, $\mathbf{G}$ does not contain any non-trivial trivial split torus, and hence the $\mathbb{Q}$-split radical of $\mathbf{G}$ is equal to its unipotent radical. Hence, maximal compact subgroups of G are exactly the stabilizers used in [4] from Section 4 on (see [4, Section 2.3, p. 446]). By [4, Theorem 9.3], the quotient $\Gamma\backslash\overline{X}^{BS}$ is a compact manifold with corners, which is a finite classifying space for Γ. q.e.d.

Remark 3.2. Though it was not mentioned there, [10, Corollary 0.3] applies to the Borel–Serre compactification $\Gamma\backslash\overline{X}^{BS}$ in Lemma 3.1 for *non-semisimple* algebraic groups $\mathbf{G}$ and shows that it is topologically rigid if $\dim \Gamma\backslash X \neq 3, 4$.

To show that arithmetic groups have finite asymptotic dimension, we use the following result of Carlsson–Goldfarb [7, Theorem 3.5], which was anticipated in [15, p. 32].

Proposition 3.3. *For a connected Lie group G and a maximal compact subgroup K, let $X = G/K$ be the associated homogeneous space endowed with a G-invariant Riemannian metric. Then,* $\operatorname{asdim} X = \dim X$*.*

Corollary 3.4. *Let G be a connected Lie group and $\Gamma \subset G$ any finitely generated discrete subgroup. Then* $\operatorname{asdim} \Gamma < +\infty$*.*

Proof. Let $X = G/K$ be the Riemannian homogeneous space in the above proposition. Since X is a complete Riemannian manifold and hence a proper metric space and Γ acts isometrically and properly on X, Propositions 2.3 and 3.3 imply that $\operatorname{asdim} \Gamma \leq \operatorname{asdim} X = \dim X$. q.e.d.

For arithmetic subgroups Γ, we can get a sharp lower bound on $\operatorname{asdim} \Gamma$, which is not needed in the proof of Theorem 1.1 but is interesting in itself. Recall that for a linear algebraic group $\mathbf{G}$ defined over $\mathbb{Q}$, its $\mathbb{Q}$-rank is equal to the maximal dimension of $\mathbb{Q}$-split tori in $\mathbf{G}$.

Proposition 3.5. *Let* $\mathbf{G}$ *be a connected linear algebraic group defined over* $\mathbb{Q}$ *with* $\mathbb{Q}$*-rank equal to* ρ*. Let* Γ *be an arithmetic subgroup of* $\mathbf{G}$ *which is assumed to be a lattice in* G*. Then,*

$$\dim X - \rho \leq \text{asdim } \Gamma \leq \dim X.$$

Proof. We only need to prove the lower bound. Assume first that the $\mathbb{Q}$-rank $\rho > 0$, i.e., Γ is not a co-compact lattice in G. In this case, there are non-trivial proper rational parabolic subgroups. Take a minimal rational parabolic subgroup $\mathbf{P}$ of $\mathbf{G}$. Let N_P be the unipotent radical of P, and $P = N_P A_P M_P$ be the rational Langlands decomposition with respect to the basepoint $K \in X = G/K$. Let $X_P = M_P/(K \cap M_P)$ be the associated boundary space, and let $X = N_P \times A_P \times X_P$ be the associated horospherical decomposition of X. Since $\mathbf{P}$ is minimal and Γ is a lattice in $\mathbf{G}$, $\dim A_P = \rho$, and hence $\dim N_P \times X_P = \dim X - \rho$. The subgroup $\Gamma \cap P$ of Γ acts properly and cocompactly on the homogeneous space $N_P M_P/(K \cap M_P) = N_P \times X_P$. For the compactness of the quotient, we have used the assumption that $\mathbf{P}$ is minimal. By Propositions 2.1 and 3.3,

$$\text{asdim } \Gamma \cap N_P = \text{asdim } N_P M_P/(K \cap M_P) = \dim N_P \times X_P = \dim X - \rho.$$

Hence, asdim $\Gamma \geq$ asdim $\Gamma \cap N_P = \dim X - \rho$. If $\rho = 0$, then $\Gamma \backslash X$ is compact, and Propositions 2.1 and 3.3 imply that asdim $\Gamma = \dim X$. q.e.d.

Remark 3.6. Kleiner has pointed out that the lower bound on asdim Γ also follows from (a) the fact that asdim $\Gamma \geq$ cod(Γ), which is essentially contained in [**15**, p. 33], and (b) a result of Borel–Serre [**4**, Corollary 11.4] that the cohomological dimension of Γ, cod(Γ), is equal to $\dim X - \rho$.

The bounds in Proposition 3.5 are sharp. The upper bound is equal to the lower bound when Γ is cocompact. The lower bound can also be achieved for non-cocompact Γ.

Proposition 3.7. *For any arithmetic subgroup* Γ *of* $\mathbf{G} = SL(2)$*, asdim* $\Gamma = \dim X - \rho$*.*

Proof. By Lemma 2.2, it suffices to consider $\Gamma = SL(2, \mathbb{Z})$. By [**18**, Section 2.3], X can be $SL(2, \mathbb{Z})$-equivariantly deformation retracted to a tree T, and $SL(2, \mathbb{Z})$ acts properly and co-compactly on the tree. Note the induced subspace metric d on T is not a geodesic metric. Endow the tree with the induced length metric d_ℓ. Then, (T, d_ℓ) is a geodesic proper metric space. By [**15**, p. 29, Example (b)], the tree (T, d_ℓ) has asymptotic dimension equal to 1, and hence by Proposition 2.1, asdim $SL(2, \mathbb{Z}) = 1 = \dim X - \rho$. q.e.d.

Remark 3.8. It is natural to conjecture that the equality asdim $\Gamma = \dim X - \rho$ always holds. In fact, as mentioned in Remark 3.6, the cohomological dimension of Γ is equal to $\dim X - \rho$.

Proof of Theorem 1.1. It follows from Lemma 3.1 and Corollary 3.4 that Γ has a finite classifying space and finite asymptotic dimension. Hence, Proposition 1.2 implies that the integral Novikov conjecture in K-theory holds for Γ and hence proves Theorem 1.1.

Remark 3.9.

1) By Proposition 1.2 and [**19**], the integral Novikov conjecture in L-theory and C^*-algebras holds as well for groups Γ of finite asymptotic dimension and having a finite classifying space, and hence the above proof also shows that the integral Novikov conjecture in L-theory and C^*-algebra theory holds for torsion free arithmetic subgroups Γ. (Recently, the author learnt that Dranishnikov, Ferry and Weinberger gave an independent proof of Proposition 1.2 for the L-theoretic version of the integral Novikov conjecture.)
2) Some non-discrete subgroups of Lie groups can be realized as arithmetic groups of linear algebraic groups. For example, let F be a number field and $\mathcal{O}_F$ be its ring of integers. Let $\mathbf{G} \subset GL(n, \mathbb{C})$ a connected linear algebraic group defined over F. If F has some real embedding, then $\mathbf{G}(\mathcal{O}_F)$ can be realized as a subgroup of the Lie group $\mathbf{G}(\mathbb{R}) \subset GL(n, \mathbb{R})$. But the image is often a non-discrete subgroup, for example when $F = \mathbb{Q}(\sqrt{d})$, where d is a square free positive integer. On the other hand, by the functor of restriction of scalars, $\mathbf{G}(\mathcal{O}_F)$ or any subgroup Γ of $\mathbf{G}(F)$ commensurable with $\mathbf{G}(\mathcal{O}_F)$ can be realized as an arithmetic subgroup of the linear algebraic group $Res_{F/\mathbb{Q}}\mathbf{G}$ defined over $\mathbb{Q}$. Therefore, the integral Novikov conjecture in K-theory, L-theory and C^*-algebra theory holds for any such subgroup $\Gamma \subset \mathbf{G}(F)$ commensurable with $\mathbf{G}(\mathcal{O}_F)$, provided it is torsion free.

Carlsson and Goldfarb [**7**, Cor 1.9] introduced a notion of weakly coherent discrete groups, which is important in computing algebraic K-groups, and proved that discrete groups of finite asymptotic dimension are weakly coherent. Then, Corollary 3.4 implies that any finitely generated subgroup of a connected Lie group is weakly coherent.

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