Journal of Physical Mathematics
ISSN: 2090-0902

Makanae, J Phys Math 2016, 7:3
DOI: 10.4172/2090-0902.1000197

Research Article Open Access

# Mathematical Issue in Section 2 of 'On the Electrodynamics of Moving Bodies'

**Makanae M***

*Researcher, Representative Free Web College, Japan*

**Abstract**

In Albert Einstein's first published work, 'On the Electrodynamics of Moving Bodies', he introduced a set of equations, namely $t_B - t_A = \frac{\gamma_{AB}}{c-v}$ and $t'_A - t_B = \frac{\gamma_{AB}}{c+v}$ which have the form 'time=distance/velocity', as a part of the conclusion of Section 2. In these equations, Einstein implied that an event in a moving system viewed from within that moving system differs from the same event viewed from a reference stationary system. This perspective became the fundamental basis of the special theory of relativity (STR).

However, considering Sections 1 and 2 of Einstein's paper using practical examples and numerical values, we find that an inconsistency is caused by using 'relative speed' as 'velocity' in the universal equation 'time=distance/velocity'. In the conventional mathematics, only 'mobile speed' is admitted as 'velocity' in 'time=distance/velocity'. This is a pure mathematical issue that should be solved if we continue to use the STR, under the premise that Sections 1 and 2 of 'On the Electrodynamics of Moving Bodies' are correct.

**Keywords**: Special theory of relativity; Mathematics; Lorentz factor

## 1. Introduction

In 1905, Albert Einstein's first published work, 'On the Electrodynamics of Moving Bodies' [1], was released. The theory presented therein was later termed the special theory of relativity (STR). In Section 1 of his paper [1], Einstein defines the concept of 'time'. He then describes a method of confirming the synchronization of two clocks placed at points *A* and *B*.

In Section 2 of his paper [1], based on '*the principle of relativity*' and '*the principle of the constancy of the velocity of light*', Einstein considers the relationships between a moving system and a reference stationary system with the concepts of 'length' and 'time', using the form 'velocity=distance/time'. Both systems have a relationship under the special condition that an observer at rest in the reference stationary system observes an event in the moving system, which moves under parallel translation with uniform velocity with respect to the reference stationary system. Through the consideration in Section 2 of the paper [1], Einstein implies that an event in a moving system viewed within that moving system differs from the same event viewed from a reference stationary system. This perspective became the fundamental basis of STR.

In Section 3 of the paper [1], Einstein develops the expression $\beta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ which was later called the 'Lorentz factor'. This expression denotes the ratio of the value of 'length' or 'time' between the moving system and the reference stationary system when both systems are under the same special conditions described above.

The expression $\sqrt{1-\frac{v^2}{c^2}}$ on the right side of the Lorentz factor denotes the condition of the length or time in the moving system viewed from the reference stationary system, when the reference value of length or time in the stationary system is defined to be unity. In this expression, *c* denotes the velocity of light, and *v* denotes the velocity of the moving system. The effectiveness of the Lorentz factor for length and time is the starting point for studying STR.

From the above, we can say that, in Section 3 of the paper [1], the Lorentz factor was developed as the core relationship in the STR, while Sections 1 and 2 of the paper [1] were provided as the premises of Lorentz factor.

However; if examining the equations provided in Section 1 and 2 of the paper [1] by using numerical values in practical examples with Einstein's description in the paper [1], problems are caused by using equation 'time=distance/velocity' in a method that had never been considered in the field of mathematics.

## 2. Confirming Equations Provided in Sections 1 and 2 of the Paper [1]

At the beginning of his study in Section 1 of the paper [1], Einstein emphasized that the concept of 'time' is important in the study of physics. Then, he provided a method for confirming the synchronization of two clocks placed at points *A* and *B* using light traveling between the two points:

$$t_B - t_A = t'_A - t_B \quad (1)$$

Here, *t* represents time, $t_A$ is the point in time at which light is emitted from the light source placed at *A*, $t_B$ is the point in time at which the light is reflected by a mirror placed at *B*, and $t'_A$ is the point in time at which the light returns to *A*. The left-hand side of equation (1) represents the time required for the light to 'Go' (from *A* to *B*), and the right-hand side represents the time required for the light to 'Return'

***Corresponding author:** Makanae M, Researcher, Representative Free Web College, Japan, Tel: +81-(0)3-3878-9542; E-mail: edit@free-web-college.com

**Received** August 03, 2016; **Accepted** September 26, 2016; **Published** September 28, 2016

**Citation:** Makanae M (2016) Mathematical Issue in Section 2 of 'On the Electrodynamics of Moving Bodies'. J Phys Math 7: 197. doi: 10.4172/2090-0902.1000197

(from *B* to *A*). Based on the premise that the velocity of light is constant in a vacuum, the right- and left-hand sides of equation (1) are equal. Thus, we can confirm that the synchronization of the two clocks, placed at *A* and *B*, is satisfied. This method became the predominant basis of the thought experiments described in the paper [1]. In the same section of the paper [1], Einstein provided

$$\frac{2\overline{AB}}{t'_A - t_A} = c \tag{2}$$

The term $\overline{AB}$ was described as the distance between the points *A* and *B*; therefore, $2\overline{AB}$ denotes the distance of the round trip of light between the clock positions at points *A* and *B*. The term $t'_A$ - $t_A$ is described as the time required for the same round trip of light, and c is the velocity of light. In other words, equation (2) corresponds to the form of equation 'distance/time=velocity'. In the first half of Section 2 of the paper [1], the following expression was provided with the explanation that '*where time interval is to be taken in the sense of the definition in Section* 1'.

$$velocity = \frac{light\ path}{time\ interval} \tag{3}$$

Equation (3) corresponds to the form 'velocity=distance/time'; in other words, the form of equation (2) and equation (3) is established to have the same order. Therefore, the $2\overline{AB}$ term of equation (2) corresponds to the 'light path' of equation (3), while $t'_A$-$t_A$ of equation (2) corresponds to the 'time interval' of equation (3).

Immediately after equation (3), Einstein included the following description:

'*Let there be given a stationary rigid rod; and let its length be l as measured by a measuring-rod which is also stationary. We now imagine the axis of the rod lying along the axis of x of the stationary system of co-ordinates and that a uniform motion of parallel translation with velocity v along the axis of x in the direction of increasing x is then imparted to the rod*'.

Then, the two different '*operations*' were provided to '*inquire as to the length of the moving rod*'.

Operation (a): '*The observer moves together with the given measuring-rod and the rod to be measured, and measures the length of the rod directly by superposing the measuring-rod, in just the same way as if all three were at rest*'.

Operation (b): '*By means of stationary clocks set up in the stationary system and synchronizing in accordance with section 1, the observer ascertains at what points of the stationary system the two ends of the rod to be measured are located at a definite time. The distance between these two points, measured by the measuring-rod already employed, which in this case is at rest, is also a length which may be designated 'the length of the rod*'. Then, Einstein predicted the length of the moving rod as follows:

*'In accordance with the principle of relativity the length to be discovered by the operation (a)—we will call it 'the length of the rod in the moving system'—must be equal to the length l of the stationary rod. The length to be discovered by the operation (b) we will call 'the length of the (moving) rod in the stationary system.' This we shall determine on the basis of our two principles, and we shall find that it differs from l'.* Based on the above conditions, Einstein introduced the equations:

$$t_B - t_A = \frac{\gamma_{AB}}{c - v} \text{ and } t'_A - t_B = \frac{\gamma_{AB}}{c + v} \tag{4}$$

The form of both equations corresponds to the form 'time=distance /velocity,' and the premises were implied for equation (4) as follows:

1. Two coordinate systems: the moving system is described as a moving rigid rod, and the stationary system is described as the reference frame.

2. The rigid rod travels at a uniform velocity, undergoing parallel translation with respect to the stationary system, along the positive direction of the x-axis.

3. *A* is the point of the end rod, closest to the origin of x-axis, and *B* is the point of the end of the rod, at a distance *l* from *A*.

4. A light source is placed at *A*, and a mirror is placed at *B* to reflect the light in the opposite direction.

5. A clock is placed at each of the points *A* and *B*.

6. The round trip of a ray of light between *A* and *B* is performed according to the formalism contained in equation (1).

7. The velocity of the moving rigid rod is *v*, and *c* is the velocity of light.

8. The point in time at which the light is emitted by the light source is $t_A$.

9. The point in time at which the light is reflected by the mirror is $t_B$.

10. The point in time at which the light returns to *A* is $t'_A$.

11. '$\gamma_{AB}$ *denotes the length of the moving rigid rod - measured in the stationary system*'.

Conditions 4, 5, 8, 9, and 10 are the same conditions as those in equation (1).

When an observer in the reference stationary system observes this round trip of the light, equation (4) holds. The left-hand side of equation (4) corresponds to the Go condition (light leaving *A*), and the right-hand side corresponds to the Return condition (light returning to *A*). Unlike equation (1), the structure of equation representing go and return is different in equation (4).

In total, four equations were provided in Section 1 and 2 of the paper [1]:

$$t_B - t_A = t'_A - t_B \tag{1}$$

$$\frac{2\overline{AB}}{t'_A - t_A} = c \tag{2}$$

$$velocity = \frac{light\ path}{time\ interval} \tag{3}$$

$$t_B - t_A = \frac{\gamma_{AB}}{c - v} \text{ and } t'_A - t_B = \frac{\gamma_{AB}}{c + v} \tag{4}$$

[Note] We should call (3) an 'expression,' but, for convenience, we refer to it as 'equation.'

## 3. Examining Each Equation Using Practical Numerical Values

We now examine these four equations using numerical values from practical examples. First, we provide the mutual conditions in the four equations. Regarding the velocity of light, we employ 'the principle of the constancy of the velocity of light' as Einstein did, i.e., the velocity of light is always *c*. We assume the velocity of the moving system *v* is half of velocity of light, i.e., 0.5 *c*. The distance between points *A* and *B* is

given by *l*. We further assume 1*l* is the distance that light can move in 1s when the system containing points *A* and *B* is stationary. Regarding the time required for light to travel from point *A* (or *B*) to point *B* (or *A*), this value is given as 1s based on above assumptions when the system containing points *A* and *B* is stationary.

### 3.1 Examining equation(1)

The examination of equation (1), namely, $t_B - t_A = t'_A - t_B$, is the simplest.

$$1s - 0s = 2s - 1s \tag{5}$$

### 3.2 Examining equation (2)

The examination of equation (2), namely $\frac{2\overline{AB}}{t'_A - t_A} = c$, which corresponds to the universal form 'velocity=distance/time,' is also simple. This equation is established by the round trip of light between points *A* and *B* viewed within the stationary system that contains points *A* and *B*. The distance *AB* is 1*l*, so that $2\overline{AB}$ is 2*l*. The round trip travel time of $t'_A - t_A$=2 s. Therefore, equation (2) is given by

$$2l/2s = c \tag{6}$$

### 3.3 Examining equation (3)

Assuming that the system containing points *A* and *B* is stationary and the observer at rest in the same system observes the round trip of light between *A* and *B*, the observer will obtain the simple results in equation (3), $velocity = \frac{light\ path}{time\ interval}$

- Go interval (the trip of light from *A* to *B*): $c = 1l/1\ s$ (7)
- Return interval (the trip of light from *B* to *A*): $c = 1l/1\ s$ (8)

On the other hand, assuming that the system containing points *A* and *B* moves at half of the velocity of light and that the observer at rest in the reference stationary system observes the round trip of light between *A* and *B*, we have to assume tangible conditions.

First, we consider the time sequence. In this case, viewed from the reference stationary system, the points *A* and *B*, corresponding to the locations of the light source and the mirror, respectively, are in motion. In other words, the positions of *A* or *B* are variable with the progress of time when viewed from the reference stationary system; thus, we need to specify the time sequence. For this purpose, we describe each position using the following references to time data:

- The position of *A* at time $t_A$ is denoted as $At_A$.
- The position of *B* at time $t_B$ is denoted as $Bt_B$.
- The position of *A* at time $t'_A$ is denoted as $At'_A$.

Using this notation, the description of the Go and Return intervals of the trip of light, viewed from the reference stationary system, can be described as follows:

- Go interval: from $At_A$ to $Bt_B$
- Return interval: from $Bt_B$ to $At`_A$

To show that every element of equation (3) is concerned with the motion of light between *A* and *B*, we describe equation (3) $velocity = \frac{light\ path}{time\ interval}$ as velocity of light=light path from $At_A$ (or $Bt_B$) to $Bt_B$ (or $At'_A$)/time interval required for light to travel between $At_A$ (or $Bt_B$) and $Bt_B$ (or $At'_A$).

The velocity of light of the above expression is c, because we already employed '*the principle of the constancy of the velocity of light*.' Therefore, using the practical example assumed at the beginning of this section of this paper, we obtain the following results:

Go interval

$$c = \frac{2l}{2s} \tag{9}$$

Return interval $c = \frac{l \times 2/3}{2/3s}$ (10)

The process of calculating equations (9) and (10) is given below.

We assume that, when *t*=0 s, the light source placed at point *A* of the moving system emits light. The time intervals, which are required for the trip of light in the Go interval and in the Return interval, viewed from the reference stationary system, are calculated as follows.

In the Go interval, when $l + vt = ct$, the tip of the light emitted by the light source placed at point *A* reaches the mirror placed at point *B*. We can transpose $l + vt = ct$ to

$$t = \frac{l}{c - v} \tag{11}$$

In the Return interval, temporarily assuming that the point in time at which the light is reflected by the mirror is 0 s, when $l - vt = ct$, the tip of the light reflected by the mirror reaches point *A*. We can transpose $l - vt = ct$ to

$$t = \frac{l}{c + v} \tag{12}$$

If we use the numerical values for our example in these equations, equation (11) becomes $t = \frac{1l}{0.5c}$, and equation (12) becomes $t = \frac{1l}{1.5c}$. With these calculations, we obtain the time in the Go interval to be 2s, and the time in the Return interval is 2/3 s. From these results, each point in time becomes clear:

- $t_A$ (the point in time at which light is emitted by the light source): 0 s
- $t_B$ (the point in time at which light is reflected by the mirror): 2 s
- $t'_A$ (the point of time at which light returns to *A*): 2/3 s past 2 s

Under '*the principle of the constancy of the velocity of light*,' the light moves at velocity *c* (i.e., 1*c*) in the Go and Return conditions, even if observing from the stationary reference system. Therefore, the light path from $At_A$ to $Bt_B$ (i.e., in the Go interval) becomes 2*l*; the light path from $Bt_B$ to $At'_A$ (i.e., in the Return interval) becomes 2/3*l*.

From the above, we can obtain the numerical values in equation (3) as shown in the equations (9) $c = \frac{l \times 2}{2s}$ (in Go interval) and (10) $2/3s = \frac{l \times 2/3}{c}$ (in Return interval), when the system, which contains the light source and the mirror, is in motion at the velocity 0.5*c* viewed from the reference stationary system.

We can transpose equations (9) and (10) to:

Go interval: $2s = \frac{l \times 2}{c}$ (13)

Return interval: $2/3s = \frac{l \times 2/3}{c}$ (14)

Now we should remember the following facts for later discussion in our study.

- Equations (4), (13), and (14) are established under '*the principle of the constancy of the velocity of light*'.
- Einstein's equation (4) $t_B - t_A = \frac{\gamma_{AB}}{c-v}$ and $t'_A - t_B = \frac{\gamma_{AB}}{c+v}$, corresponds to the form 'time=distance/velocity'.
- Equations (13) and (14) correspond to the form 'time=distance/velocity'.
- Equations (13) and (14) are established under conditions similar to that of equation (4).
- The velocity of equation (4) is '$c - v$' (in Go interval) or '$c + v$' (in Return interval).
- The velocity of equation (13) or (14) is just '$c$' in both intervals.

### 3.4 Examining equation (4)

In Einstein's equation (4) $t_B - t_A = \frac{\gamma_{AB}}{c-v}$ and $t'_A - t_B = \frac{\gamma_{AB}}{c+v}$ which correspond to the form 'time=distance/velocity,' the numerical values in the practical example are clear for the time case, because these are the same as the results using equation (11) $t = \frac{l}{c-v}$ and equation (12) $t = \frac{l}{c+v}$.

Therefore, the Go interval $t_B - t_A$ is 2s, and the Return interval $t'_A - t_B$ is 2/3 s as viewed from the stationary reference system.

Regarding the numerical value of $c-v$ or $c+v$, which provided as 'velocity' of 'time=distance/velocity' by Einstein in Go or Return interval, the former becomes $1c-0.5c=0.5c$, and the latter becomes $1c+0.5c=1.5c$, under the assumption that $v$ is $0.5c$.

Regarding the term '$\gamma_{AB}$', which is defined as '*the length of the moving rigid rod - measured in the stationary system*', we should consider the difference between the concepts of '$\gamma_{AB}$' and 'light path'. Usually, if two terms differ, the concept behind the two terms also differs, unless one term is rephrased to have the same meaning as the other term. However, it is difficult to say that '$\gamma_{AB}$' is a rephrasing of the 'light path', even though both terms were provided within the same context in the same section. This is because, in the first place, the rigid rod is a solid body but the light path is not, regardless of whether they belong to the moving system or whether observing them from the reference stationary system.

However, there is a light source placed at point $A$ of the 'light path' as well as at point $A$ of '$\gamma_{AB}$': similarly, a mirror is placed at point $B$ of the 'light path' as well as at point $B$ of '$\gamma_{AB}$'. In other words, the round trip of light between points $A$ and $B$ is incarnated not only in the concept of 'light path' but also in '$\gamma_{AB}$' under the same conditions of Section 2 [1].

Based on the above, there are two possibilities for the relationship between the terminologies '$\gamma_{AB}$' and 'light path':

Possibility 1: '$\gamma_{AB}$' and 'light path' are effectively the same concept with different names.

Possibility 2: '$\gamma_{AB}$' and 'light path' are different concepts, as given by the difference in names.

For the present, we choose Possibility 1 with the reasoning that the round trip of light between points $A$ and $B$ can be incarnated on a moving rod, i.e., the '*light path*' can be incarnated on the moving rod, and considering Einstein's equation (4) under the condition of Possibility 1 in Sections 3, 4, and 5 of our study. Regarding Possibility 2, we will consider in Section 6 of our study.

Under the above choice, we consider the numerical value of '$\gamma_{AB}$' in our practical example. The length of the rod measured by Einstein's operation (a) has a value of $1l$. However, in our examination, we are trying to clarify each value when the system containing points $A$ and $B$ is in motion and is observed from the stationary reference system; thus, we employ Einstein's operation (b). The numerical value of the length of the moving rod measured using operation (b) can be calculated using equations for the Go interval:

$$\gamma_{AB}=(t_B - t_A) \times (c - v) \tag{15}$$

and for the Return interval

$$\gamma_{AB}=(t'_A - t_B) \times (c + v). \tag{16}$$

With the numerical values of $t_B - t_A$=2 s and $t'_A - t_B$=2/3s from equations (11) and (12), in the Go interval, equation (15) becomes $\gamma_{AB}=2s \times 0.5c$; therefore, $\gamma_{AB}=1l$. Likewise, in the Return interval, equation (16) becomes $\gamma_{AB}=2/3s \times 1.5c$; therefore, $\gamma_{AB}=1l$.

From the results of these calculations, we can describe Einstein's equation (4), $t_B - t_A = \frac{\gamma_{AB}}{c-v}$ and $t'_A - t_B = \frac{\gamma_{AB}}{c+v}$ as

$$2\text{s} = \frac{1l}{0.5c} \text{ and } 2/3\text{s} = \frac{1l}{1.5c} \tag{17}$$

## 4. Validating the Numerical Values of Each Examination

Let us validate our examination of the four equations in Section 3 of our study. Regarding the equations (5), (6), (7), (8), (9) and (10), these calculations are correct.

Regarding the equation (17), which is the result of the calculation of Einstein's equation (4)

$t_B - t_A = \frac{\gamma_{AB}}{c-v}$ and $t'_A - t_B = \frac{\gamma_{AB}}{c+v}$ the numerical values $2\text{s} = \frac{1l}{0.5c}$ and $2/3\text{s} = \frac{1l}{1.5c}$ are clearly consistent.

However, considering this result with Einstein's prediction that '*The length to be discovered by the operation (b) we will call 'the length of the (moving) rod in the stationary system'. This we shall determine on the basis of our two principles, and we shall find that it differs from l*' as described in Section 2 of [1], we find an inconsistency. Because the numerical value of '$\gamma_{AB}$' is $1l$ in both the Go and Return intervals in the above results, these values do not '*differ from l*', even though equation (17) was examined using operation (b). Therefore, we should validate Einstein's equation (4) with equation (17) in more detail.

Hereafter, we call Einstein's prediction 'the prediction '*it differs from l*' ' for brevity.

## 5. Validating Equation (4) More Thoroughly with Equation (17)

If we interpret the prediction '*it differs from l*' as that the length of the moving rod viewed from the reference stationary system differ from $1l$, this prediction becomes correct by using the Lorentz factor. In particular, when calculating the length of the moving rod by using the

Lorentz factor, the value is $\sqrt{0.75}l$ when viewed from the reference stationary system in all cases if the velocity of the moving rod is constant at 0.5*c*; i.e., the value of $\gamma_{AB}$ is constant as $\sqrt{0.75}l$ in both interval of the round trip of light. Certainly, $\sqrt{0.75}l$ differs from 1*l*. However, the Lorentz factor was provided as a conclusion of its development process in Section 3 [1]; in other words, the prediction '*it differs from l*', which was provided in the first half of Section 2 [1], and the description of Lorentz factor are distantly discussed in the paper [1].

From the above conditions, it is not clear that '*it*' of '*it differs from l*', which corresponds to the value of the length of the moving rod viewed from the reference system, was controlled by Einstein's equation (4) or by the Lorentz factor.

Even if we neglect this issue, there is another issue which we discuss below.

The numerical values of $\gamma_{AB}$ in the Go interval and Return interval are the same value, namely, 1*l* (if it was controlled by Einstein's equation (4)) or $\sqrt{0.75}l$ (if it was controlled by Lorentz factor). However, the values of 'light path' in the Go interval and Return interval, which can be incarnated on the moving rod, are different when viewed from the stationary reference system, as shown in equation (9) as 2*l* and in equation (10) as 2/3*l*. If the measurement is performed by operation (a), the length of rod and the '*light path*' between points *A* and *B* are constant as 1*l* because the values are obtained '*directly by superposing the measuring-rod, in just the same way as if all three were at rest*'. However, the value of $\gamma_{AB}$ cannot be incarnated by operation (a) because the rod defined in $\gamma_{AB}$ is not at rest. Thus, Einstein's equation (4), which contains $\gamma_{AB}$, can only be determined using operation (b).

However, examining the description of operation (b) more thoroughly, we find that '*the observer ascertains at what points of the stationary system the two ends of the rod to be measured are located 'at a definite time*'. We can interpret this description as follows: the observer at rest in the stationary reference system measures the length of the moving rod '*at a definite time*'. The meaning of measuring the rod '*at a definite time*' is effectively the same as the meaning of measuring the moving rod when this rod temporarily stops. Therefore, if the observer at rest in the stationary reference system measures the length of the moving rod '*at a definite time*', the observer will obtain the value of the length of the rod as 1*l*.

Therefore, we can say that equation (17), which denotes the numerical values of $\gamma_{AB}$ in the Go and Return intervals as 1*l* using Einstein's equation (4) in operation (b), is correct. In reality, the numerical values of equation (17) were provided at a definite time, namely when the light reaches the mirror or when the reflected light reaches point *A*. However, the numerical values of the 'light path,' which can be incarnated on the moving rod in the Go interval and Return interval, are 2*l* and 2/3*l* when viewed from the stationary reference system, if employing '*the principle of the constancy of the velocity of light*'. Thus, Possibility 1, the prediction '*it differs from l*' and operation (b) cannot be assumed in the same context.

## 6. Considering Equation (4) in Possibility 2

Let us consider equation (4) using Possibility 2, in which $\gamma_{AB}$ and the 'light path' are different concepts. The condition of '*the principle of the constancy of the velocity of light*' is still employed.

First, we confirm the form of equation (4) $t_B - t_A = \frac{\gamma_{AB}}{c-v}$ and $t'_A - t_B = \frac{\gamma_{AB}}{c+v}$.

The term '$\gamma_{AB}$'(i.e., the length of the moving rod measured in the reference stationary system) was used as the distance in equation of the form 'time=distance/velocity'. Both $t_B - t_A$ and $t'_A - t_B$ were described as time and denote the time interval of the round trip of light between points *A* and *B*. *c*-*v* and *c*+*v* were provided as the velocity of 'time=distance/velocity'. Therefore, we can say that equation (4) denotes the time in which the light moves between points *A* and *B* is equal to the length of the moving rigid rod divided by *c*-*v* (in the Go interval) or *c*+*v* (in the Return interval).

Fundamentally, when we want to determine the numerical value for the time in which the light moves between points *A* and *B* using 'time=distance/velocity,' the distance should be associated with the motion of light between points *A* and *B*. However, in equation (4), '$\gamma_{AB}$,' which corresponds to 'distance,' was defined by Einstein as the length of the moving rigid rod measured in the stationary system, i.e., this definition does not discuss the motion of light.

We can treat '$\gamma_{AB}$' as the 'light path' using the reason that a round trip of light can be incarnated between points *A* of $\gamma_{AB}$ and *B* of $\gamma_{AB}$; however, this assumption belongs to Possibility 1. Therefore, in Possibility 2, we ignore the motion of light when considering $\gamma_{AB}$, regardless of whether the 'light path' can be incarnated between points *A* of $\gamma_{AB}$ and *B* of $\gamma_{AB}$.

If we ignore the motion of light in '$\gamma_{AB}$,' equation (17), namely, $2\mathrm{s} = \frac{1l}{0.5c}$ and $2/3\mathrm{s} = \frac{1l}{1.5c}$ correct when employing the interpretation that the meaning of '*the two ends of the rod to be measured are located at a definite time*' using operation (b) is the same as measuring the length of the rod when it temporarily stops. Under this condition, the observer at rest in the reference stationary system measures the length of the rod as 1*l* in both the Go interval and Return interval. However, the same observer can also measure the length of the rod as 1*l* using operation (a). Thus, classifying the operation as (a) and (b) become meaningless.

Therefore, we can conclude that Section 2 of the paper [1], which provided '*the principle of the constancy of the velocity of light*,' operations (a) and (b), and equation (4), is inconsistent.

## 7. Considering the Factor that Caused the above Problems

Here, let us consider the factor that caused the problems introduced in Section 5 and 6 of our study. Primarily, 'time=distance/velocity' states that the time in which a certain object moves between two points is equal to the distance that a certain object moves between the two points divided by the velocity with which the object moves between the two points. In other words, if we consider the equation 'time=distance/velocity' as it is used for an object that moves from a certain point to another point, each element contained in this equation should be associated with the motion of the object. From this viewpoint, we call the velocity in this equation the 'mobile speed'. Therefore, if using 'time=distance/velocity' to perceive the motion of light, the above description can be arranged as follows: the time in which light moves between two points is equal to the distance that light moves between two points divided by the velocity with which the light moves between the two points (i.e., the mobile speed of light).

However, the terms *c-v* and *c+v*, which correspond to 'velocity' in Einstein's equation (4), are not the mobile speed because the mobile speed of light is always '*c*' if employing '*the principle of the constancy of the velocity of light*.' Einstein did not define to these terms; thus, let us denominate the concepts of '*c-v*' and '*c+v*' as 'relative speed' using the following reasoning.

Regarding *c-v*, under '*the principle of the constancy of the velocity of light*' in the Go interval, the light moves '*relatively*' to the point at which the light source is placed after light is emitted with velocity *c-v* in the positive direction of the x-axis, as viewed from the stationary reference system. In reality, Einstein used the term '*relatively*' and '*c-v*' in the following sentence: '*the ray moves relatively to the initial point of k, when measured in the stationary system, with the velocity c-v*' in Section 3 of the paper [1]. (Note that the term '*k*' corresponds to the moving system.) Therefore, we can say that the term *c-v* represents relative speed in the Go interval.

Regarding *c+v*; this term represents the reverse situation of the Go interval that the concept of relative speed *c-v* can be established, therefore we can say that the term *c+v* represents the relative speed between the moving system and the reflected light in the Return interval.

## 8. Considering the Factor Causing Einstein to use 'Relative Speed' in 'Time=Distance/Velocity'

From the earlier considerations in our study, we can determine that Einstein's equation (4) states that the time in which the light moves between points *A* and *B* is equal to the length of the moving rigid rod divided by the relative speed between the light and the moving system. However, the method, which uses 'time=distance/velocity' in the above form, had never been considered in the conventional law of mathematics. Because the velocity of 'time=distance/velocity' must be the mobile speed, the relative speed *c-v* or *c+v* cannot be used as the mobile speed of light.

Therefore, we trace the factor causing Einstein to use the relative speed *c-v* and *c+v* as 'velocity' in the universal equation 'time=distance/velocity'.

We first confirm the background facts.

1. In the Go interval: when $l + vt=ct$, the tip of the light emitted from the light source reaches the mirror (i.e., is reflected by the mirror). Moreover, $l + vt=ct$ can be transposed to the form in equation (11), $t=\frac{l}{c-v}$.

2. In the Return interval: when $l - vt=ct$, the tip of the light reflected by the mirror reaches point *A* if we temporarily assume that the point of time at which the mirror reflected the light is 0 s. Then, $l - vt=ct$ can be transposed to the form in equation (12), $t=\frac{l}{c+v}$.

3. Both equations (11) and (12) can be used to calculate the time interval between two points, such as $t_B - t_A$ or $t'_A - t_B$, under the condition that distance is *l* as constant.

4. The form of equations (11) and (12) corresponds to the form 'time=distance/velocity,' i.e., the term *t* of the equations (11) and (12) corresponds to 'time', the term *l* of equations (11) and (12) corresponds to 'distance,' and the terms *c-v* and *c+v* of equations (11) and (12) correspond to 'velocity'.

5. The examination of equations (11) and (12) is correct using practical examples under the condition of a time point at which the tip of light reaches the mirror or point *A*.

In Section 2 of the paper [1], equations (11) and (12) of our study are not provided. However, the conditions given in points 1 and 2 above are tacit facts in Section 2 of the paper [1]; therefore, we can conjecture that Einstein considered and examined equations (11) and (12) when he prepared the paper [1]. If this conjecture is correct, we can infer the factor causing Einstein to use *c-v* and *c+v* as 'velocity' in 'time=distance/velocity' is that he relied on equations (11) and (12) as the basis that *c-v* and *c+v* can be used as the 'velocity' in 'time=distance/velocity'.

## 9. Conclusion

The Lorentz factor $\beta=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, which is the core premise of the STR, requires '*v*' as one of its elements. The Lorentz factor was established based on the concept of Einstein's equations of time=distance/velocity provided in the paper [1]. However, the 'mobile speed' of light '*c*' is configured for the velocity in

'time=distance/velocity' even if we consider the round trip of light in the moving system viewed from the stationary reference system; in other words, the velocity of the moving system '*v*' does not always affect the velocity of light, regardless of whether the system containing the light source and the mirror moves, if '*the principle of the constancy of the velocity of light*' is employed.

If *v* does not always affect the velocity of light, the numerical value of *v* in the Lorentz factor is always zero; thus we can describe $\beta=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ $\beta=\frac{1}{\sqrt{1-\frac{0^2}{c^2}}}$ i.e., $\beta$=1/1, the ratio of length or time between the moving system and the reference system is always unity. This means that relativity itself does not affect the physical condition of length or time.

Einstein's STR is the one of the most important and respected theories of modern physics; therefore, all the more reason that the ambiguity that lurks in the premise of the theory should be solved. For this purpose, we need to establish a new law, which admits that the relative speed can be used instead of mobile speed as 'velocity' in the universal equation 'time=distance/velocity'. Had this law been already established, everybody should have learned it in youth because 'time=distance/velocity' is the subject taught in mathematics of high degree of elementary school or junior high school; however, nobody learns it. Therefore, we can declare that the new law has not yet been established. If this issue is neglected and we continue using the Lorentz factor, the STR will become less persuasive someday. This is a purely mathematical issue concerning the human wisdom; thus, we require a coalition of mathematicians to establish the new law.

### Acknowledgments

I would like to thank Editage and Elsevier for English language editing.

### References

1. Perrett,W., Jeffery, G.B., On the Electrodynamics of Moving Bodies, English translation of 'Zur Elektrodynamik bewegter Körper', German Das Relativatsprinzip, [4th ed] Tuebner, 1922

2. Einstein A (1905) On the electrodynamics of moving bodies. Annalen der Physik 17: 891-921.

**Citation:** Makanae M (2016) Mathematical Issue in Section 2 of 'On the Electrodynamics of Moving Bodies'. J Phys Math 7: 197. doi: 10.4172/2090-0902.1000197