Journal of Physical Mathematics

Mishra et al., J Phys Math 2016, 7:3
http://dx.doi.org/10.4172/2090-0902.1000183

Research Article Open Access

The Generalized Triple Difference Lacunary Statistical on Γ^3 Over P-Metric Spaces Defined by Musielak Orlicz Function

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Abstract

We introduce the generalized triple sequence spaces of entire difference lacunary statistical convergence and discuss general topological properties also inclusion theorems are with respect to a sequence of Musielak-Orlicz function.

Keywords: Analytic sequence; Triple sequences; Difference sequence; Γ^3 space; Musielak-Orlicz function; Lacunary sequence; Statistical convergence

Introduction

A triple sequence (real or complex) can be defined as a function $x:\mathbb{N}\times\mathbb{N}\times\mathbb{N}\to\mathbb{R}$ ($\mathbb{C}$) where $\mathbb{N},\mathbb{R}$ and $\mathbb{C}$ denote the set of natural numbers, real numbers and complex numbers respectively. The different types of notions of triple sequence was introduced and investigated at the initial by Sahiner [1,2], Esi [3-5], Datta [6], Subramanian [7], Debnath [8] and many others.

A triple sequence $x=(x_{mk})$ is said to be triple analytic if

$$sup_{m,n,k}\left|x_{mnk}\right|^{\frac{1}{m+n+k}}<\infty.$$

The space of all triple analytic sequences are usually denoted by Λ^3. A triple sequence $x=(x_{mk})$ is called triple entire sequence if

$$\left|x_{mnk}\right|^{\frac{1}{m+n+k}}\to 0 \text{ as } m,n,k\to\infty.$$

The notion of difference sequence spaces (for single sequences) was introduced by Kizmaz [9] as follows

$$Z(\Delta)=\{x=(x_k)\in w:(\Delta x_k)\in Z\}$$

For $Z=c,c_0$ and ℓ_∞ where $\Delta x_k=x_k-x_{k+1}$ for all $k\in\mathbb{N}$

The difference triple sequence space was introduced by Debnath et al. (see [8]) and is defined as

$$\Delta x_k = \begin{aligned}\Delta x_{mnk} &= x_{mnk}-x_{m,n+1,k}-x_{m,n,k+1}+x_{m,n+1,k+1}\\ &-x_{m+1,n,k}+x_{m+1,n+1,k}+x_{m+1,n,k+1}-x_{m+1,n+1,k+1}\end{aligned}$$

and $\Delta^0 x_{mnk}=\langle x_{mnk}\rangle$.

Definitions and Preliminaries

Throughout the article $w^3,\Gamma^3(\Delta)$, $\Lambda^3(\Delta)$ denote the spaces of all, triple entire difference sequence spaces and triple analytic difference sequence spaces respectively.

Subramanian introduced by a triple entire sequence spaces, triple analytic sequences spaces and triple gai sequence spaces [7]. The triple sequence spaces of $\Gamma^3(\Delta)$, $\Lambda^3(\Delta)$ are defined as follows:

$$\Gamma^3(\Delta)=\left\{x\in w^3:\left|\Delta x_{mnk}\right|^{1/m+n+k}\to 0 \quad as \quad m,n,k\to\infty\right\},$$

$$\Lambda^3(\Delta)=\left\{x\in w^3:sup_{m,n,k}\left|\Delta x_{mnk}\right|^{1/m+n+k}<\infty\right\}.$$

1. Definition

An Orlicz function is a function $M:[0,\infty)\to[0,\infty)$ which is continuous, non-decreasing and convex with $M(0)=M(x)>0$ for $M(x)>0$ $x>0$ and $M(x)\to\infty$ as $x\to\infty$ [10]. If convexity of Orlicz function M is replaced by $M(x+y)\leq M(x)+M(y)$ then this function is called modulus function. $M:[0,\infty)\to[0,\infty)$

Lindenstrauss and Tzafriri [11] used the idea of Orlicz function to construct Orlicz sequence space. A sequence $g=(g_{mn})$ defined by

$$g_{mn}(v)=sup\{|v|u-(f_{mnk})(u):u\geq 0\},m,n,k=1,2,\cdots$$

is called the complementary function of a Musielak-Orlicz function f. For a given Musielak-Orlicz function f the Musielak-Orlicz sequence space t_f is defined as follows [12]

$$t_f=\left\{x\in w^3:I_f\left(\left|x_{mnk}\right|\right)^{1/m+n+k}\to 0 \quad as \quad m,n,k\to\infty\right\},$$

Where I_f is a convex modular defined by

$$I_f(x)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}f_{mnk}\left(\left|x_{mnk}\right|\right)^{1/m+n+k},x=(x_{mnk})\in t_f.$$

We consider t_f equipped with the Luxemburg metric

$$d(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}f_{mnk}\left(\frac{\left|x_{mnk}\right|^{1/m+n+k}}{mnk}\right)$$

is an exteneded real number.

2. Definition

Let $n\in\mathbb{N}$ and X be a real vector space of dimension w where nm. A real valued function $d_p(x_1,\ldots,x_n)=\|(d_1(x_1,0),\ldots,d_n(x_n,0))\|_p$ on X satisfying the following four conditions:

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Received May 30, 2016; **Accepted** July 01, 2016; **Published** July 04, 2016

Citation: Mishra LN, Deepmala, Subramanian N (2016) The Generalized Triple Difference Lacunary Statistical on Γ^3 Over P-Metric Spaces Defined by Musielak Orlicz Function. J Phys Math 7: 183. doi:10.4172/2090-0902.1000183

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(i) $\|(d_1(x_1,0),\ldots,d_n(x_n,0))\|_p = 0$ if and and only if $d_1(x_1,0),\ldots d_n(x_n,0)$ are linearly dependent,

(ii) $\|(d_1(x_1,0),\ldots,d_n(x_n,0))\|_p$ is invariant under permutation,

(iii) $\|(\alpha d_1(x_1,0),\ldots,d_n(x_n,0))\|_p = |\alpha| \|(d_1(x_1,0),\ldots,d_n(x_n,0))\|_p, \alpha \in \mathbb{R}$

(iv) $d_p((x_1,y_1),(x_2,y_2)\cdots(x_n,y_n)) = (d_X(x_1,x_2,\cdots x_n)^p + d_Y(y_1,y_2,\cdots y_n)^p)^{1/p}$ $for 1 \le p < \infty$; (or)

(v) $d((x_1,y_1),(x_2,y_2),\cdots(x_n,y_n)) := \sup\{d_X(x_1,x_2,\cdots x_n), d_Y(y_1,y_2,\cdots y_n)\}$,

For $x_1, x_2,\ldots x_n \in X$, $y_1, y_2,\ldots y_n \in Y$ is called the p product metric of the Cartesian product of n metric spaces [13].

3. Definition

Let X be a linear metric space. A function $\rho: X \to \mathbb{R}$ is called paranorm, if

(1) $\rho(x) \ge 0$ for all $x \in X$;

(2) $\rho(-x) = \rho(x)$ for all $x \in X$;

(3) $\rho(x+y+z) \le \rho(x)+\rho(y)+\rho(z)$, for all $x,y,z \in X$,

(4) If (σ_{mnk}) is a sequence of scalars with $\sigma_{mnk} \to \sigma$ as $m,n,k \to \infty$ and $x=(x_{mk})$ is a sequence of vectors with

$\rho(x_{mk}-x) \to 0$ as $m,n,k \to \infty$ then $\rho(\sigma_{mnk} x_{mk} - \sigma x) \to 0$ as $m,n,k \to \infty$

4. Definition

The triple sequence $\theta_{i,\ell,j} = \{(m_i, n_\ell, k_j)\}$ is called triple lacunary if there exist three increasing sequences of integers such that

$m_0 = 0, h_i = m_i - m_{r-1} \to \infty$ as $i \to \infty$ and

$n_0 = 0, \overline{h_\ell} = n_\ell - n_{\ell-1} \to \infty$ as $\ell \to \infty$

$k_0 = 0, \overline{h_j} = k_j - k_{j-1} \to \infty$ as $j \to \infty$

Let $m_{i,\ell,j} = m_i n_\ell k_j, h_{i,\ell,j} = h_i \overline{h_\ell h_j}$, and $\theta_{i,\ell,j}$ is determine by

$I_{i,\ell,j} = \{(m,n,k) : m_{i-1} < m < m_i \quad and \quad n_{\ell-1} < n \le n_\ell \quad and \quad k_{j-1} < k \le k_j\}$,

$q_k = \frac{m_k}{m_{k-1}}, \overline{q_\ell} = \frac{n_\ell}{n_{\ell-1}}, \overline{q_j} = \frac{k_j}{k_{j-1}}$.

Main Results

The notion of λ-triple entire and triple analytic sequences as follows: Let $\lambda = (\lambda_{mnk})_{m,n,k=0}^\infty$ be a strictly increasing sequences of positive real numbers tending to infinity, that is

$0 < \lambda_{000} < \lambda_{111} < \ldots$ and $\lambda_{mnk} \to \infty$ as $m,n,k \to \infty$

and said that a sequence $x=(x_{mnk}) \in w^3$

is λ-convergent to 0, called a the λ-limit of x if $\mu_{mnk}(x) \to 0$ as $m,n,k \to \infty$ where

$$\mu_{mnk}(x) = \frac{1}{\varphi_{rst}} \sum_{m\in I_{rst}} \sum_{n\in I_{rst}} \sum_{k\in I_{rst}} \left(\Delta^{m-1}\lambda_{m,n,k}\right) - \left(\Delta^{m-1}\lambda_{m,n+1,k}\right) - \left(\Delta^{m-1}\lambda_{m,n,k+1}\right) + \left(\Delta^{m-1}\lambda_{m,n+1,k+1}\right) - \left(\Delta^{m-1}\lambda_{m+1,n,k}\right) + \left(\Delta^{m-1}\lambda_{m+1,n+1,k}\right) + \left(\Delta^{m-1}\lambda_{m+1,n,k+1}\right) - \left(\Delta^{m-1}\lambda_{m+1,n+1,k+1}\right) \left|\Delta^{m+1} x_{mnk}\right|^{1/m+n+k}.$$

The sequence $x=(x_{mnk}) \in w^3$ is λ-triple difference analytic if $sup_{uvs} |\mu_{mnk}(x)| < \infty$. If $lim_{xnk} x_{mnk}(x) = 0$ in the ordinary sense of convergence, then

$$lim_{mnk} \frac{1}{\varphi_{rst}} \sum_{m\in I_{rst}} \sum_{n\in I_{rst}} \sum_{k\in I_{rst}} \left(\Delta^{m-1}\lambda_{m,n,k}\right) - \left(\Delta^{m-1}\lambda_{m,n+1,k}\right) - \left(\Delta^{m-1}\lambda_{m,n,k+1}\right) + \left(\Delta^{m-1}\lambda_{m,n+1,k+1}\right) - \left(\Delta^{m-1}\lambda_{m+1,n,k}\right) + \left(\Delta^{m-1}\lambda_{m+1,n+1,k}\right) + \left(\Delta^{m-1}\lambda_{m+1,n,k+1}\right) - \left(\Delta^{m-1}\lambda_{m+1,n+1,k+1}\right) \left|\Delta^{m+1} x_{mnk}\right|^{1/m+n+k} = 0$$

This implies that

$$lim_{mnk} |\mu_{mnk}(x) - 0| = lim_{mnk} \frac{1}{\varphi_{rst}} \sum_{m\in I_{rst}} \sum_{n\in I_{rst}} \sum_{k\in I_{rst}} \left(\Delta^{m-1}\lambda_{m,n,k}\right) - \left(\Delta^{m-1}\lambda_{m,n+1,k}\right) - \left(\Delta^{m-1}\lambda_{m,n,k+1}\right) + \left(\Delta^{m-1}\lambda_{m,n+1,k+1}\right) - \left(\Delta^{m-1}\lambda_{m+1,n,k}\right) + \left(\Delta^{m-1}\lambda_{m+1,n+1,k}\right) + \left(\Delta^{m-1}\lambda_{m+1,n,k+1}\right) - \left(\Delta^{m-1}\lambda_{m+1,n+1,k+1}\right) \left|\Delta^{m+1} x_{mnk} - 0\right|^{1/m+n+k} = 0$$

which yields that $lim_{uvs} \mu_{mnk}(x) = 0$ and hence $x=(x_{mnk}) \in w^3$ is λ-convergent to 0. Let I^3- be an admissible ideal of $3^{\mathbb{N}\times\mathbb{N}\times\mathbb{N}}$, θ_{rst} be a triple difference lacunary sequence, $f = f_{mnk}$ be a Musielak-Orlicz function and $\left(X, \left\|\left(d(x_1,0), d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)$ be a p-metric space, $q=(q_{mnk})$ be triple difference analytic sequence of strictly positive real numbers. By $w^3(p\text{-}X)$ we denote the space of all sequences defined over

$\left(X, \left\|\left(d(x_1,0), d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)$.

In the present paper we define the following sequence spaces:

$$\left[\Gamma^{3\Delta^m q}_{f\mu}, \left\|\left(d(x_1,0), d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^\varphi\right]^{I^3}_{\theta_{rst}} = \left\{r,s,t \in I_{rst} : \left[f_{mnk}\left(\left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}} \ge \varepsilon\right\} \in I^3,$$

$$\left[\Lambda^{3\Delta^m q}_{f\mu}, \left\|\left(d(x_1,0), d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^\varphi\right]^{I^3}_{\theta_{rst}} = \left\{r,s,t \in I_{rst} : \left[f_{mnk}\left(\left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}} \ge K\right\} \in I^3$$

If we take $f_{mnk}(x) = x$ we get

$$\left[\Gamma^{3\Delta^m q}_{f\mu}, \left\|\left(d(x_1,0), d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^\varphi\right]^{I^3}_{\theta_{rst}} = \left\{r,s,t \in I_{rst} : \left[\left(\left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}} \ge \varepsilon\right\} \in I^3,$$

$$\left[\Lambda^{3\Delta^m q}_{f\mu}, \left\|\left(d(x_1,0), d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^\varphi\right]^{I^3}_{\theta_{rst}} = \left\{r,s,t \in I_{rst} : \left[\left(\left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}} \ge K\right\} \in I^3,$$

If we take $q=(q_{mnk})=1$ we get

$$\left[\Gamma^{3\Delta^m}_{f\mu}, \left\|\left(d(x_1,0), d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^\varphi\right]^{I}_{\theta_{rst}} = \left\{r,s,t \in I_{rst} : \left[f_{mnk}\left(\left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right] \ge \varepsilon\right\} \in I^3,$$

$$\left[\Lambda^{3\Delta^m}_{f\mu}, \left\|\left(d(x_1,0), d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^\varphi\right]^{I^3}_{\theta_{rst}} = \left\{r,s,t \in I_{rst} : \left[f_{mnk}\left(\left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right] \ge K\right\} \in I^3$$

In the present paper we plan to study some topological properties and inclusion relation between the above defined sequence spaces. $\left[\Gamma^{3\Delta^m q}_{f\mu}, \left\|\left(d(x_1,0), d(x_2,0),\cdots,d(x_{n-1})\right)\right\|_p^\varphi\right]^{I^2}_{\theta_{rst}}$ and $\left[\Lambda^{3\Delta^m q}_{f\mu}, \left\|\left(d(x_1,0), d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^\varphi\right]^{I^3}_{\theta_{rst}}$ which we shall discuss in this paper.

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1. Theorem

Let $f=f_{mnk}$ be a Musielak-Orlicz function, $q=(q_{mnk})$be a triple analytic difference sequence of strictly positive real numbers, the sequence spaces

$$\left[\Gamma^{3\Delta^m q}_{f\mu}, \left\|\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^{\varphi}\right]^{I^3}_{\theta_{rst}} \text{ and}$$

$$\left[\Lambda^{3\Delta^m q}_{f\mu}, \left\|\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^{\varphi}\right]^{I^3}_{\theta_{rst}} \text{ are linear spaces.}$$

Proof: It is routine verification. Therefore the proof is omitted.

2. Theorem

Let $f=f_{mnk}$ be a Musielak-Orlicz function, $q=(q_{mnk})$be a triple analytic difference sequence of strictly positive real numbers, the sequence space $\left[\Gamma^{3\Delta^m q}_{f\mu}, \left\|\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^{\varphi}\right]^{I^3}_{\theta_{rst}}$ is a paranormed space with respect to the paranorm defined by

$$g(x)=\inf\left\{\left[f_{mnk}\left(\left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\le 1\right\}.$$

Proof: Clearly $g(x)\ge 0$ for

$$x=(x_{mnk})\in\left[\Gamma^{3\Delta^m q}_{f\mu}, \left\|\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^{\varphi}\right]^{I^3}_{\theta_{rst}}$$

Since $f_{mnk}(0)=0$ we get $g(0)=0$

Conversely, suppose that $g(x)$ then

$$\inf\left\{\left[f_{mnk}\left(\left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\le 1\right\}=0.$$

Suppose that $\mu_{mnk}(x)\neq 0$. for each $m,n,k\in\mathbb{N}$ Then

$\left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^{\varphi}\to\infty$. It follows that

$$\left(\left[f_{mnk}\left(\left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\right)^{1/H}\to\infty$$

which is a contradiction. Therefore $\mu_{mnk}(x)=0$. Let

$$\left(\left[f_{mnk}\left(\left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\right)^{1/H}\le 1$$

and

$$\left(\left[f_{mnk}\left(\left\|\mu_{mnk}(y),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\right)^{1/H}\le 1$$

Then by using Minkowski's inequality, we have

$$\left(\left[f_{mnk}\left(\left\|\mu_{mnk}(x+y),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\right)^{1/H}$$

$$\le\left(\left[f_{mnk}\left(\left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\right)^{1/H}$$

$$+\left(\left[f_{mnk}\left(\left\|\mu_{mnk}(y),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\right)^{1/H}.$$

So we have

$$g(x+y)=\inf\left\{\left[f_{mnk}\left(\left\|\mu_{mnk}(x+y),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\le 1\right\}$$

$$\le\inf\left\{\left[f_{mnk}\left(\left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\le 1\right\}+\inf$$

$$\left\{\left[f_{mnk}\left(\left\|\mu_{mnk}(y),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\le 1\right\}$$

Therefore,

$$g(x+y)\le g(x)+g(y).$$

Finally, to prove that the scalar multiplication is continuous. Let λ be any complex number. By definition,

$$g(\lambda x)=\inf\left\{\left[f_{mnk}\left(\left\|\mu_{mnk}(\lambda x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\le 1\right\}.$$

Then

$$g(\lambda\ x)=\inf\left\{((|\lambda| t)^{q_{mnk}/H}:\left[f_{mnk}\left(\left\|\mu_{mnk}(\lambda x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\le 1\right\}$$

where $t=\frac{1}{|\lambda|}$. Since $|\lambda|^{q_{mnk}}\le \max\left(1,|\lambda|^{\sup\ q_{mnk}}\right)$, we have

$$g(\lambda\ x)\le\max\left(1,|\lambda|^{\sup q_{mnk}}\right)\inf$$

$$\left\{t^{q_{mnk}/H}:\left[f_{mnk}\left(\left\|\mu_{mnk}(\lambda x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\le 1\right\}$$

This completes the proof.

3. Theorem

(i) If the Musielak Orlicz function (f_{mnk}) satisfies Δ_2- condition, then

$$\left[\Gamma^{3\Delta^m q}_{f\mu}, \left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^{\varphi}\right]^{I^{3\alpha}}_{\theta_{rst}}=$$

$$\left[\Gamma^{3\Delta^m q\mu}_{g}, \left\|\mu_{uvs}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^{\varphi}\right]^{I^3}_{\theta_{rst}}.$$

(ii) If the Musielak Orlicz function (g_{mnk}) satisfies Δ_2- condition, then

$$\left[\Gamma^{3\Delta^m q\mu}_{g}, \left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^{\varphi}\right]^{I^{3\alpha}}_{\theta_{rst}}=$$

$$\left[\Gamma^{3\Delta^m q}_{f\mu}, \left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^{\varphi}\right]^{I^3}_{\theta_{rst}}$$

Proof: Let the Musielak Orlicz function (f_{mk}) satisfies Δ_2-condition, we get

$$\left[\Gamma^{3\Delta^m q\mu}_{g}, \left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^{\varphi}\right]^{I^3}_{\theta_{rst}}$$

$$\subset\left[\Gamma^{3\Delta^m q}_{f\mu}, \left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^{\varphi}\right]^{I^{3\alpha}}_{\theta_{rst}}\cdots\ \cdots \quad (1)$$

To prove the inclusion

$$\left[\Gamma^{3\Delta^m q}_{f\mu}, \left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^{\varphi}\right]^{I^{3\alpha}}_{\theta_{rst}}$$

$$\subset\left[\Gamma^{3\Delta^m q\mu}_{g}, \left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^{\varphi}\right]^{I^3}_{\theta_{rst}},$$

let $a\in\left[\Gamma^{3\Delta^m q}_{f\mu}, \left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^{\varphi}\right]^{I^{3\alpha}}_{\theta rst}$.

Then for all $\{x_{mnk}\}$ with

$$(x_{mnk})\in\left[\Gamma^{3\Delta^m q}_{f\mu}, \left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^{\varphi}\right]^{I^3}_{\theta_{rst}}$$

we have

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\left|\Delta^m x_{mnk}a_{mnk}\right|<\infty. \quad (1)$$

Since the Musielak Orlicz function (f_{mk}) satisfies condition, then

$(y_{mnk})\in\left[\Gamma^{3\Delta^m q}_{f\mu}, \left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p^{\varphi}\right]^{I^3}_{\theta_{rst}}$, we get

$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\left|\frac{\varphi_{rst}y_{mnk}a_{mnk}}{\Delta^m\lambda_{mnk}}\right|<\infty.$ by (1). Thus

$(\varphi_{rst}a_{mnk})\in\left[\Gamma_{f\mu}^{3\Delta^m q},\left\|\mu_{mnk}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]_{\theta_{rst}}^{I^3}=$ $\left[\Gamma_{g}^{3\Delta^m q\mu},\left\|\mu_{mnk}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]_{\theta_{rst}}^{I^3}$ and hence

$(a_{mnk})\in\left[\Gamma_{g}^{3\Delta^m q\mu},\left\|\mu_{mnk}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]_{\theta_{rst}}^{I^3}$. This gives that

$$\left[\Gamma_{f\mu}^{3\Delta^m q},\left\|\mu_{mnk}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]_{\theta_{rst}}^{I^{3\alpha}}$$

$$\subset\left[\Gamma_{g}^{3\Delta^m q\mu},\left\|\mu_{mnk}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]_{\theta_{rs}}^{I^3}\cdots\cdots\cdots(2)$$

we are granted with (1) and (2)

$$\left[\Gamma_{f\mu}^{3\Delta^m q},\left\|\mu_{mnk}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]_{\theta_{rst}}^{I^{3\alpha}}=$$

$$\left[\Gamma_{g}^{3\Delta^m q\mu},\left\|\mu_{mnk}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]_{\theta_{rst}}^{I^3}$$

(ii)Similarly,onecanprovethat

$$\left[\Gamma_{g}^{3\Delta^m q\mu},\left\|\mu_{mnk}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]_{\theta_{rst}}^{I^{3\alpha}}$$

$$\subset\left[\Gamma_{f\mu}^{3\Delta^m q},\left\|\mu_{mnk}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]_{\theta_{rst}}^{I^3}$$

if the Musielak Orlicz function (g_{mk}) satisfies Δ_2-condition.

1. Proposition

The sequence space

$\left[\Gamma_{f\mu}^{3\Delta^m q},\left\|\mu_{mnk}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]_{\theta_{rst}}^{I^3}$ is not solid

Proof: The result follows from the following example.

Example: Consider

$\Delta^m x=(\Delta^m x_{mnk})=\begin{bmatrix}1&1&\ldots&1\\1&1&\ldots&1\\ .\\ .\\ .\end{bmatrix}\in\left[\Gamma_{f\mu}^{3\Delta^m q},\left\|\mu_{mnk}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]_{\theta_{rst}}^{I^3}$. Let

$$\Delta^m\alpha_{mnk}=\begin{bmatrix}-1^{m+n+k}&-1^{m+n+k}&\ldots&-1^{m+n+k}\\-1^{m+n+k}&-1^{m+n+k}&\ldots&-1^{m+n+k}\\ .\\ .\\ .\end{bmatrix},\text{ for all } m,n,k\in\mathbb{N}$$

Then $\Delta^m\alpha_{mnk}x_{mnk}\notin\left[\Gamma_{f\mu}^{3q},\left\|\mu_{mnk}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]_{\theta_{rst}}^{I^3}$. Hence

$\left[\Gamma_{f\mu}^{3\Delta^m q},\left\|\mu_{mnk}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]_{\theta_{rst}}^{I^3}$ is not solid.

2. Proposition

The sequence space

$\left[\Gamma_{f\mu}^{3\Delta^m q},\left\|\mu_{mnk}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]_{\theta_{rst}}^{I^3}$

is not monotone.

Proof: The proof follows from Proposition 3.4.

3. Proposition

The sequence space

$\left[\Lambda_{f\mu}^{3\Delta^m q},\left\|\mu_{mnk}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]_{\theta_{rst}}^{I^3}$ is not solid.

4. Proposition

The sequence space

$\left[\Lambda_{f\mu}^{3\Delta^m q},\left\|\mu_{mnk}(x),(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0))\right\|_p^{\varphi}\right]_{\theta_{rst}}^{I^3}$

is not monotone.

Conclusion

Through this paper we studied some topological properties and inclusion relation with respect to a sequence of Musielak-Orlicz function.

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Citation: Mishra LN, Deepmala N, Subramanian (2016) The Generalized Triple Difference Lacunary Statistical on Γ³ Over P-Metric Spaces Defined by Musielak Orlicz Function. J Phys Math 7: 183. doi:10.4172/2090-0902.1000183