Journal of Physical Mathematics
ISSN: 2090-0902
Kalinowski, J Phys Math 2016, 7:1
http://dx.doi.org/10.4172/2090-0902.1000152
Research Article
Open Access

A Hierarchy of Symmetry Breaking in the Nonsymmetric Kaluza-Klein (Jordan-Thiry) Theory

Kalinowski MW*

Bioinformatics Laboratory, Medical Research Centre, Polish Academy of Sciences, Poland

Abstract

The paper is devoted to the hierarchy of a symmetr y breaking in the Non symmetric Kaluza–Klein (Jordan–Thiry) Theory. The basic idea consists in a deformation of a vacuum states manifold to the cartesian product of vacuum states manifolds of every stage of a symmetry breaking .In the paper we consider a pattern of a spontaneous symmetry breaking including a hierarchy in the Non symmetr ic Kaluza–Klein (Jordan–Thiry) Theory.

Introduction

In this paper we consider hierarchy of symmetry breaking in the Nonsymmetric Kaluza–Klein Theory and the Nonsymmetric Kaluza–Klein Theory with a spontaneous symmetry breaking and Higgs' mechanism . In the second section we consider a Nonsymmetric Kaluza–Klein Theory and the Nonsymmetric Kaluza–Klein Theory with a spontaneous symmetry breaking and Higgs' mechanism [1-6]. In the third section we develop a hierarchy of the symmetry breaking in our theory. For further development of the nonsymmetric Kaluza–Klein (Jordan–Thiry) Theory [7-10].

Elements of the Nonsymmetric Kaluza–Klein Theory in general non-Abelian case and with spontaneous symmetr y breaking and Higgs' mechanism

Let $\underline{P}$ be a principal fiber bundle over a space-time E with a structural group G which is a semisimple Lie group. On a space-time E we define a nonsymmetric tensor $g_{\mu\nu} = g_{\mu\nu)} + g_{\mu\nu]}$ such that

$$g = \det(g\mu\nu) \neq 0$$

$$\tilde{g} = \det(g_{(\mu\nu)}) \neq 0 \qquad (2.1)$$

$g_{[\mu\nu]}$ is called as usual a skewon field (e.g., in NGT, [6,11-13] We define on E a nonsymmetric connection compatible with $g_{\mu\nu}$ such that

$$\overline{D}_{g\alpha\beta} = g\alpha\delta\overline{Q}^{\delta}{}_{\beta\gamma}(\overline{\Gamma})\overline{\theta}^{\gamma} \qquad (2.2)$$

where $\overline{D}$ is an exterior covariant derivative for a connection $\overline{\omega}^{\alpha}{}_{\beta} = \overline{\Gamma}^{\alpha}{}_{\beta\gamma}\overline{\theta}^{\gamma}$ and $\overline{Q}^{\alpha}{}_{\beta\delta}$ is its torsion. We suppose also

$$\overline{Q}^{\alpha}{}_{\beta\alpha}(\overline{\Gamma}) = 0 \qquad (2.3)$$

We introduce on E a second connection

$$\overline{W}^{\alpha}\beta = \overline{W}^{\alpha}{}_{\beta\gamma}\overline{\theta}^{\gamma} \qquad (2.4)$$

such that

$$\overline{W}^{\alpha}\beta = \overline{\omega}^{\alpha}{}_{\beta} - \frac{2}{3}\delta^{\alpha}_{\beta}\overline{W} \qquad (2.5)$$

$$\overline{W} = \overline{W}\gamma\overline{\theta}^{\gamma} = \frac{1}{2}(\overline{W}^{\sigma}_{\gamma\sigma} - \overline{W}^{\sigma}_{\gamma\sigma})\overline{\theta}^{\gamma} \qquad (2.6)$$

Now we turn to nonsymmetric metrization of a bundle $\underline{P}$. We define a nonsymmetric tensor γ on a bundle manifold P such that

$$\gamma = \pi^{*}g \oplus \ell_{ab}\theta^{\alpha} \otimes \theta^{b} \qquad (2.7)$$

where π is a projection from P to E . On $\underline{P}$ we define a connection ω (a 1-form with values in a Lie algebra g of G. In this way we can introduce on P (a bundle manifold) a frame $\theta^{A} = (\pi^{*}(\overline{\theta}^{\alpha}), \theta^{a})$ such that

$$\theta^{a} = \lambda\omega^{a}, \quad \omega = \omega^{a}X_{a}, \quad a = 5,6,\ldots,n+4, \quad n = \dim G = \dim \mathfrak{g}, \quad \lambda = const.$$

Thus our nonsymmetric tensor looks like

$$\gamma = \gamma AB\theta^{A} \otimes \theta^{B}, \text{ A,B=1,2…..,n+4,} \qquad (2.8)$$

$$l_{ab} = h_{ab} + \mu K_{ab} \qquad (2.9)$$

where h_{ab} is a bi invarian t Killing–Cartan tensor on G and k_{ab} is a right-invariant t skew- symmetric tensor on G, μ=*const*

We have

$$h_{ab} = C^{c}_{ad}C^{d}_{bc} = h_{ab}$$
$$K_{ab} = -K_{ba} \qquad (2.10)$$

Thus we can write $\overline{\gamma}(X,Y) = \overline{g}(\pi'X, \pi'Y) + \lambda^{2}h(\omega(X),\omega(Y))$ (2.11)

$$\overline{\gamma}(X,Y) = \underline{g}(\pi'X, \pi'Y) + \lambda^{2}k(\omega(X),\omega(Y)) \qquad (2.12)$$

($C^{a}{}_{bc}$ are structural constant s of the Lie algebra g).

$\overline{\gamma}$ is the symmetr ic part of γ and $\underline{\gamma}$ is the anti symmetr ic part of γ We have as usual

$$[X_{a}, X_{b}] = C^{c}_{ab}X_{c} \qquad (2.13)$$

and

$$\Omega = \frac{1}{2}H^{a}\mu\upsilon\theta^{\mu}\Lambda\theta^{\nu} \qquad (2.14)$$

is a curvature of the connection ω

$$\Omega = d\omega + \frac{1}{2}[\omega,\omega] \qquad (2.15)$$

The frame θ^{A} on P is partially nonholonomic. We have

$$d\theta^{a} = \frac{\lambda}{2}(H^{a}_{\mu\upsilon}\theta^{\mu}\Lambda\theta^{\upsilon} - \frac{1}{\lambda^{2}}C^{a}_{bc}\theta^{b}\Lambda\theta^{c}) \neq 0 \qquad (2.16)$$

Even if the bundle $\underline{P}$ is trivial, i.e. for Ω=0 This is different than

***Corresponding author:** Kalinowski MW, Bioinformatics Laboratory, Medical Research Centre, Polish Academy of Sciences, Poland, Tel: 61641643156; E-mail: markwkal@bioexploratorium.pl

Received September 05, 2015; **Accepted** January 13, 2016; **Published** January 20, 2016

Citation: Kalinowski MW (2016) A Hierarchy of Symmetry Breaking in the Nonsymmetric Kaluza–Klein (Jordan–Thiry) Theory. J Phys Math 7: 152. doi:10.4172/2090-0902.1000152

Copyright: © 2016 Kalinowski MW. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

Citation: Kalinowski MW (2016) A Hierarchy of Symmetry Breaking in the Nonsymmetric Kaluza–Klein (Jordan–Thiry) Theory. J Phys Math 7: 152. doi:10.4172/2090-0902.1000152

in an electromagnetic case explanied by Kalinowski MW [3]. Our nonsymmetric metrization of a principal fiber bundle gives us a right-invarian t structure on P with respect to an action of a group G on P [3]. Having P nonsymmetric ally metrized one defines two connection s on P right- invarian t with respect to an action of a group G on P. We have

$$\gamma AB = \begin{pmatrix} g_{\alpha\beta} & 0 \\ 0 & l_{ab} \end{pmatrix} \tag{2.17}$$

In our left horizontal frame θ^A

$$D_{\gamma AB} = \gamma ADQ^D BC^{(\Gamma)\theta^C} \tag{2.18}$$

$$Q^D BD^{(\Gamma)} = 0 \cdot \tag{2.19}$$

where D is an exterior covariant derivative with respect to a connection $\omega^A{}_B = \Gamma^A{}_B, \theta^C$ on P and $Q^A{}_B, (\Gamma)$ its torsion. One can solve Equation (2.18)– (2.19) getting the following results

$$\omega^A B = \left(\frac{\pi^*(\overline{\omega}^\alpha \beta) - \ell_{\mathrm{db}}\, \mathrm{g}^{\mu\alpha} \mathrm{L}^{\mathrm{d}}{}_{\mu\beta}\, \theta^{\mathrm{b}}\, \mathrm{L}^a{}_{\beta\gamma}\, \theta^\gamma}{\ell_{\mathrm{db}} g^{\alpha\beta} (2\mathrm{H}^d_{\gamma\beta} - \mathrm{L}^d_{\gamma\beta}) \theta^\gamma \tilde{\omega}^a_b} \right). \tag{2.20}$$

where $g^{\mu\alpha}$ is an inverse tensor of $g(\alpha)$

$$g_{\alpha\beta} g^{\gamma\beta} = g_{\beta\alpha} g^{\beta\gamma} = \delta^\gamma_\alpha \tag{2.21}$$

$L^d{}_\gamma, = -L^a{}_\beta,$ is an Ad-type tensor on P such that

$$\ell_{dc} g\mu\beta g^{\gamma\mu} L^d_{\gamma\alpha} + \ell_{cd} g\alpha\mu g^{\mu\gamma} L^d_{\beta\gamma} = 2\ell_{cd} g\alpha\mu g^{\mu\gamma} H^d_{\beta\gamma} \tag{2.22}$$

$\tilde{\omega}^a_b = \tilde{\Gamma}^a_{bc}\theta^c$ is a connection on an internal space (typical fiber) compatible with a metric ℓ_{ab} such that

$$\ell_{db}\tilde{\Gamma}^d_{ac} + \ell_{ab}\tilde{\Gamma}^d_{cb} = -\ell_{db} C^d_{ac} \tag{2.23}$$

$$\tilde{\Gamma}^a_{ba} = 0, \tilde{\Gamma}^a_{bc} = -\tilde{\Gamma}^a_{cb} \tag{2.24}$$

and of course $\tilde{Q}^a_{bc}(\tilde{\Gamma}) = 0$ where is a torsion of the connection $\tilde{\omega}^a_b$ We also introduce an inverse tensor of $g(\alpha)$

$$g(\alpha\beta)\tilde{g}^{(\alpha\gamma)} = \delta^\gamma_\beta \tag{2.25}$$

We introduce a second connection on P defined as

$$W^A{}_B = \omega^A{}_B - \frac{A}{3(\mathrm{n}+2)} \delta^A_B \overline{W} \tag{2.26}$$

$\overline{W}$ is a horizontal one form

$$\overline{W} = hor\overline{W} \tag{2.27}$$

$$\overline{W} = \overline{W}_\upsilon \theta^\upsilon = \frac{1}{2}(\overline{W}^\sigma_{\upsilon\sigma} - \overline{W}^\sigma_{\sigma\upsilon}) \tag{2.28}$$

In this way we define on P all analogues of four- dimension al quantities from NGT [6,11]. It means, $(n+4)$ dimension al analogues from Moffat theory of gravitation, i.e. two connection s and a nonsymmetric metric γ_{AB}. Those quantities are right- invarian t with respect to an action of a group G on P. One can calculate a scalar curvature of a connection $W^A{}_B$ getting the following result [1-3].

$$R(W) = \overline{R}(\overline{W}) - \frac{\lambda^2}{4}(2\ell_{cd} H^{c} H^{d} - \ell_{cd} L^c \quad H \quad) + \tilde{R}(\tilde{\Gamma}) \tag{2.29}$$

Where

$$R(W) = \gamma^{AB}(\mathrm{R}^{\mathrm{C}}{}_{\mathrm{ABC}}(\mathrm{W}) + \frac{1}{2}\mathrm{R}^{\mathrm{C}}{}_{CAB}(\mathrm{W})) \tag{2.30}$$

is a Moffat–Ricci curvature scalar for the connection $W^A{}_B$, $\overline{R}(\overline{W})$ is a Moffat–Ricci curvature scalar for the connection $\overline{W}^\alpha_\beta$, and $\tilde{R}(\tilde{\Gamma})$ is a Moffat–Ricci curvature scalar for the connection $\tilde{\omega}^a_b$

$$H^\alpha = g^{[\mu\nu]} H^a_{\mu\nu} \tag{2.31}$$

$$L^{\alpha\mu\nu} = g^{\alpha\mu} g^{\beta\mu} L^a_{\alpha\beta} \tag{2.32}$$

Usually in ordinary (symmetric) Kaluza–Klein Theory one has $\lambda = 2\frac{\sqrt{G_N}}{c^2}$ where GN is a Newtonian gravitational constant and c is the speed of light. In our system of units this is the same as in Non symmetric Kaluza–Klein Theory in an electromagnetic case [3,4] In the non- Abelian Kaluza –Klein Theory which unifies GR and Yang–Mills field theory we have a Yang–Mills lagrangian and a cosmological term. Here we have

$$\mathcal{L}_{\mathrm{YM}} = \frac{1}{8\pi}\ell_{cd}(2H^{\mathrm{c}} H^d - L^{\mathrm{c}\mu\nu} H^d_{\mu\nu}) \tag{2.33}$$

and $\tilde{R}(\tilde{\Gamma})$ plays a role of a cosmolog ical term.In order to incorporate a spontaneous symmetr y breaking and Higgs' mechanism in our geometrical unification of gravitation and Yang–Mills' fields we consider a fiber bundle $\underline{P}$ over a base manifold $E \times G/G_0$, where E is a space-time, $G_0 \subset G$, G_0, G are semisimple Lie groups. Thus we are going to combine a Kaluza–Klein theory with a dimension al reduction procedure.

Let $\underline{P}$ be a principal fiber bundle over $V=E \times M$ with a structural group H and with a projection π, where

$M= G/G_0$ is a homogeneous space, G is a semisimple Lie group and G_o its semisimple Lie subgroup. Let us suppose that (V, γ) is a manifold with a nonsymmetric metric tensor

$$\gamma_{AB} = \gamma_{(AB)} + \gamma_{[AB]} \tag{2.34}$$

The signature of the tensor γ is $(+,---,\underbrace{-\cdots-}_{n_1},\underbrace{--\cdots-}_{n})$ Let us introduce a natural Phenomenon

$$\theta^{\bar{A}} = (\pi^*(\theta^A), \theta^0 = \lambda\omega^\alpha) \tag{2.35}$$

It is convenient to introduce the following notation. Capital Latin indices with tilde $\tilde{A}, \tilde{B}, \tilde{C}$ run $1,2,3\ldots,m+4$, m=dim$H$+ dim$M$=n+ dim$M$=+n_1 , $n_{1=}$ dim$M$, n=dim$H$. Lower Greek indices $\alpha,\beta,\gamma,\delta=1,2,3,4$ and lower Latin indices a,b,c,d=n_1+5, n_2+5,…., n_1+6,…,m+4 Capital Latin indices A,B,C=1,2,…, n_1+4 . Lower Latin indices with tilde $\tilde{a},\tilde{b},\tilde{c}$ run 5,6…, n_1+4 The symbol over θ^A and other quantities indicates that these quantities are defined on V. We have of course $n_{1=}$ dim$G$-dim G_0=n_2-(n_2-n_1) where dim G=$n_2$dim G_0=n_2-n_1, m=n_1+n.

On the group H we define a bi- invarian t (symmetr ic) Killing–Cartan tensor

$$h(A,B) = h_{ab} A^a B^b \tag{2.36}$$

We suppose H is semisimple, it means det(h_{ab})≠0 . We define a skew- symmetr ic right- invarian t tensor on H

$k(A,B) = k_{bc} A^b B^c$ $k_{bc} = -k_{cb}$

Let us turn to the nonsymmetric metrization of $\underline{P}$.

$$k(X,Y) = \gamma(X,Y) + \lambda^2 \ell_{\mathrm{bc}} \omega^a(X) \omega^b(Y) \tag{2.37}$$

where

$$\ell_{\mathrm{ab}} = h_{\mathrm{ab}} + \xi K_{\mathrm{ab}} \tag{2.38}$$

is a nonsymmetric right-invarian t tensor on H.One gets in a matrix form (in the natural frame (2.35))

$$\begin{pmatrix} \gamma_{AB} & 0 \\ 0 & \ell_{ab} \end{pmatrix} \tag{2.39}$$

$\det(\ell_{ab}) \neq 0$, $\xi = const$ and real, then

$$\ell_{ab}\ell^{ac} = \ell_{ba}\ell^{ca} = \delta^c_b \tag{2.40}$$

The signature of the tensor k is $(+,---,\underbrace{-\cdots-}_{n_1},\underbrace{-\cdots-}_{n})$. As usual, we have commutation relations for Lie algebra of H, $\mathfrak{h}$

$$[X_a, X_b] = C^c_{ab} X_c \tag{2.41}$$

This metrization of $\underline{P}$ is right- invarian t with respect to an action of H on P.Now we should nonsymmetric ally metrize $M=G/G_0$. M is a homogeneous space for G (with left action of group G). Let us suppose that the Lie algebra of G, g has the following reductive decomposition

$$\mathfrak{g} = \mathfrak{g}_0 \dotplus \mathfrak{m} \tag{2.42}$$

where $\mathfrak{g}_0$ is a Lie algebra of G_0 (a subalgebra of $\mathfrak{g}$) and m (the complement to the subalgebra $\mathfrak{g}_0$) is AdG_0 invarian t, + means a direct sum. Such a decomposition might be not unique, but we assume that one has been chosen. Sometimes one assumes a stronger condition for m, the so called symmetr y requirement

$$[\mathfrak{m}, \mathfrak{m}] \subset \mathfrak{g}_0 \tag{2.43}$$

Let us introduce the following notation for generators of $\mathfrak{g}$:

$$Y_i \in \mathfrak{g}, Y_{\tilde{i}} \in \mathfrak{g}, Y_{\hat{a}} \in \mathfrak{g}_0 \tag{2.44}$$

This is a decomposition of a basis of $\mathfrak{g}$ according to (2.42). We define a symmetr ic metric on M using a Killing–Cartan form on G in a classical way. We call this tensor h_0. Let us define a tensor field $h^0(x)$ on G/G_0, $x \in G/G_0$, using tensor field h on G. Moreover, if we suppose that h is a bi invarian t metric on G (a Killing–Cartan tensor) we have a simpler construction.The complement m is a tangent space to the point $\{\varepsilon G_0\}$ of M, ε is a unit element of. We restrict h to the space m only. Thus we have $h^0\{\varepsilon G_0\}$ at one point of M. Now we propagate $h^0\{\varepsilon f G_0\}$ using a left action of the group G $h^0(\{fG_0\}) = (L_f^{-1})^*(h^0(\{\varepsilon G_0\}))$. $h^0(\{\varepsilon G_0\})$ is of course AdG_0 invarian t tensor defined on m and $L_f^* h^0 = h^0$

We define on M a skew- symmetr ic 2-form k^0. Now we introduce a natural frame on M. Let $f^i{}_j$, be structure constant s of the Lie algebra $\mathfrak{g}$, i.e.

$$[Y_j, Y_k] = f^i_{jk} Y_i \tag{2.45}$$

Y_j are generators of the Lie algebra g. Let us take a local section s:V® → G/G_0 of a natural bundle $G \mapsto G/G_0$ where.$V \subset M = G/G_0$ The local section s can be considered as an introduction of a coordinate system on M.

Let ω_{MC} be a left- invarian t Maurer–Cartan form and let

$$\omega^\sigma{}_{\hat{I}\,C} = \sigma^* \omega_{\hat{I}\,C} \tag{2.46}$$

Using de composition (2.42) we have

$$\omega^\sigma \hat{I}\ C = \omega^\sigma{}_0 + \omega^\sigma{}_m = \theta^{\hat{i}} Y_i + \overline{\tau}^{\hat{a}} Y_{\hat{a}} \tag{2.47}$$

It is easy to see that $\overline{\theta}^{\tilde{a}}$ is the natural (left- invarian t) frame on M and we have

$$h^0 = \overset{0}{h}_{\tilde{a}\tilde{b}} \overline{\theta}^{\tilde{a}} \otimes \overline{\theta}^{\tilde{b}} \tag{2.48}$$

$$k^0 = \overset{0}{k}_{\tilde{a}\tilde{b}} \overline{\theta}^{\tilde{a}} \wedge \overline{\theta}^{\tilde{b}} \tag{2.49}$$

According to our notation $\tilde{a}, \tilde{b} = 5, 6, \ldots, n_1 + 4$.

Thus we have a nonsymmetric metric on M

$$\gamma_{\tilde{a}\tilde{b}} = r^2 (\overset{0}{h}_{\tilde{a}\tilde{b}} + \zeta \overset{0}{k}_{\tilde{a}\tilde{b}}) = r^2 g_{\tilde{a}\tilde{b}} \tag{2.50}$$

Thus we are able to write down the nonsymmetric metric on $V=E \times M=E \times G/G_0$

$$\gamma_{AB} = \begin{pmatrix} g_{\alpha\beta} & 0 \\ 0 & r^2 g_{\tilde{a}\tilde{b}} \end{pmatrix} \tag{2.51}$$

where

$$g_{\alpha\beta} = g(\alpha\beta) + g[\alpha\beta]$$
$$g_{\tilde{a}\tilde{b}} = h^0_{\tilde{a}\tilde{b}} + \zeta k^0 b^0_{\tilde{a}\tilde{b}}$$
$$k^0_{\tilde{a}\tilde{b}}, = -k^0_{\tilde{b}\tilde{a}}$$
$$h^0_{\tilde{a}\tilde{b}}, = -h^0_{\tilde{b}\tilde{a}}$$

$\alpha, \beta = 1, 2, 3, 4$, $\tilde{a}, \tilde{b} = 5, 6, \ldots, n_1 + 4 = \dim M + 4 = \dim G - \dim G_0 + 4$.

The frame $\overline{\theta}^{\tilde{a}}$ is unholonomic:

$$d\overline{\theta}^{\tilde{a}} = \frac{1}{2} k^{\tilde{a}}_{\tilde{b}\tilde{c}} \overline{\theta}^{\tilde{b}} \wedge \overline{\theta}^{\tilde{c}} \tag{2.52}$$

where $\kappa, b\tilde{c}$, are coefficient s of nonholonomicity and depend on the point of the manifold $M=G/G_0$ (they are not constant in general). They depend on the section s and on the constants $f^{\tilde{a}}_{\tilde{b}\tilde{c}}$,. We have here three groups H,G, G_0 H,G, G_0. Let us suppose that there exists a homomorphism μ between G_0 and H, $\mu(G_0)$

$$\mu : G_0 \to H \tag{2.53}$$

such that a centralizer of $\mu(G_0)$ in H, C^μ is isomorphic to G. C^μ, a centralizer of $\mu(G_0)$ in H, is a set of all element s of H which commute with element s of $\mu(G_0)$, which is a subgroup of H. This means that H has the following structure, $C^\mu=G$.

$$\mu(G_0) \otimes G \subset H \tag{2.54}$$

If μ is a iso morphi sm between G_0 and $\mu(G_0)$ one gets

$$G_0 \otimes G \subset H \tag{2.55}$$

Let us denote by μ' a tangent map to μ at a unit element. Thus μ' is a differential of μ acting on the Lie algebra element s. Let us suppose that the connection ω on the fiber bundle $\underline{P}$ is invarian t under group action of G on the manifold $V=E \times G/G_0$. According to Kobayashi [14-17] this means the following.

Let e be a local section of $\underline{P}$, $e: V \subset U \to P$ and $A = e^*\omega$. Then for every $g \in G$ there exists a gauge transformation ρ_g such that

$$f^*(g) A = Ad_{\rho^{-1}g} + \rho^{-1}{}_g dg_g \tag{2.56}$$

f^*means a pull-back of the action f of the group G on the manifold V. According to Hlavaty [13-25] we are able to write a general form for such an ω. Following [17] we have

$$\omega = \tilde{\omega}_E + \mu' \circ \omega^\sigma_0 + \Phi \circ \omega^\sigma_m \tag{2.57}$$

(An action of a group G on $V=E \times G/G_0$ means left multiplication on a homogeneous space $M=G/G_0$.) where $\omega^\sigma{}_0 + \omega^\sigma{}_m = \omega^\sigma{}_M$, are components of the pull-back of the Maurer–Cartan form from the de composition (2.47) $\tilde{\omega}_E$ is a connection defined on a fiber bundle Q over a space-time E with structural group C^μ and a projection π_E. Moreover, $C^\mu=G$ and $\tilde{\omega}_E$ is a 1-form with values in the Lie algebra $\mathfrak{g}$.

Citation: Kalinowski MW (2016) A Hierarchy of Symmetry Breaking in the Nonsymmetric Kaluza–Klein (Jordan–Thiry) Theory. J Phys Math 7: 152. doi:10.4172/2090-0902.1000152

This connection describes an ordinary Yang–Mills' field gauge group $C^{\mu}=G$ on the space-time E. Φ is a function on E with values in the space $\tilde{S}$ of linear maps

$$\Phi : \mathfrak{m} \rightarrow \mathfrak{h} \tag{2.58}$$

satisfying Φ

$$\Phi[X_0, X] = \Phi[\mu' X_0, \Phi(X)] \tag{2.59}$$

Thus

$$\begin{aligned} \tilde{\omega}_E &= \tilde{\omega}^i_E Y_i, Y_i \in \mathfrak{g}, \\ \omega^{\sigma}_0 &= \theta^{\hat{i}} Y_{\hat{i}}, Y_{\hat{i}} \in \mathfrak{g}_0, \\ \omega^{\sigma}_m &= \overline{\theta}^{\tilde{a}} Y_{\tilde{a}}, Y_{\tilde{a}} \in \mathfrak{m}. \end{aligned} \tag{2.60}$$

Let us write condition (2.57) in the base of left-invariant t form $\theta^{\hat{i}}, \overline{\theta}^{\tilde{a}}$, which span respectively dual spaces to g_0 and m [24,25]. It is easy to see that

$$\Phi \circ \omega^{\sigma}_m = \Phi^a_{\tilde{a}}(x) \overline{\theta}^{\tilde{a}} X_a, X_a \in \mathfrak{h} \tag{2.61}$$

and

$$\mu' = \mu^a_{\hat{i}} \theta^{\hat{i}} X_a \tag{2.62}$$

From (2.59) one gets

$$\Phi^c_{\tilde{b}}(x) f^{\tilde{b}}_{\hat{i}\tilde{a}} = \mu^a_{\hat{i}} \Phi^b_{\tilde{a}}(x) C^c_{ab} \tag{2.63}$$

where $f^{\tilde{b}}_{\hat{i}\tilde{a}}$ are structure constant s of the Lie algebra g and $C^c{}_a$, are structure constant s of the Lie algebra $\mathfrak{h}$ Equation (2.63) is a constraint on the scalar field $\Phi^a_{\tilde{a}}(x)$. For a curvature of ω one gets

$$\Omega = \frac{1}{2} H^C AB^{\theta^A} \wedge \theta^B X_C = \frac{1}{2} \tilde{H}^i_{\mu\nu} \theta^{\mu} \wedge \theta^{\nu} \alpha^c_i X_c + \overset{gauge}{\nabla}_{\mu} \Phi^c_{\tilde{a}} \theta^{\mu} \wedge \theta^{\tilde{a}} X_c + \frac{1}{2} C^c_{ab} \Phi^a_{\tilde{a}} \Phi^b_{\tilde{b}} \theta^{\tilde{a}} \wedge \theta^{\tilde{b}} X_c - \frac{1}{2} \Phi^c_{\tilde{d}} f^{\tilde{d}}_{\tilde{a}\tilde{b}} \theta^{\tilde{a}} \wedge \theta^{\tilde{b}} X_c \tag{2.64}$$

Thus we have

$$H^C_{\mu\nu} = \alpha^c_i \tilde{H}^i_{\mu\nu} \tag{2.65}$$

$$H^C_{\mu\tilde{a}} = \overset{gauge}{\nabla}_{\mu} \Phi^c_{\tilde{a}} = -H^C_{\tilde{a}\mu} \tag{2.66}$$

$$H^C_{\tilde{a}\tilde{b}} = C^c_{ab} \Phi^a_{\tilde{a}} \Phi^b_{\tilde{b}} - \mu^c_{\hat{i}} f^{\hat{i}}_{\tilde{a}\tilde{b}} - \Phi^c_{\tilde{d}} f^{\tilde{d}}_{\tilde{a}\tilde{b}} \tag{2.67}$$

where $\overset{gauge}{\nabla}_{\mu}$ means gauge derivative with respect to the connection $\tilde{\omega}_E$ defined on a bundle q over a space-time E with a structur al group G

$$Y_{\hat{i}} = \alpha^c_{\hat{i}} X_c \tag{2.68}$$

$\tilde{H}^i_{\mu\nu}$ is the curvature of the connection $\tilde{\omega}_E$ in the base $\{Y_i\}$, generators of the Lie algebra of the Lie group G g, α^c_i is the matrix which connects $\{Y_i\}$ with $\{X_c\}$. Now we would like to remind that indices a,b,c refer to the Lie algebra h, $\tilde{a},\tilde{b},\tilde{c}$ to the space m (tangent space to M), $\hat{i},\hat{j},\hat{k}$ to the Lie algebra g_0 and i,j,k to the Lie algebra of the group G, g. The matrix α^c_i establishes a direct relation between generators of the Lie algebra of the subgroup of the group H iso morphi c to the group G.

Let us come back to a construction of the Nonsymmetric Kaluza–Klein Theory on a manifold P. We should define connection s. First of all, we should define a connection compatible with a nonsymmetric tensor γ_{AB}, Equation (2.51)

$$\overline{\omega}^A B = \overline{\Gamma}^A BC\theta^C \tag{2.69}$$

$$\overline{D}\gamma AB = \gamma AD \overline{Q}^D BC(\overline{\Gamma})\theta^C \tag{2.70}$$

$$\overline{Q}^D BD(\overline{\Gamma}) = 0$$

where $\overline{D}$ is the exterior covariant derivative with respect to $\overline{\omega}^A B$ and $\overline{Q}^D BC(\overline{\Gamma})$ its torsion. Using (2.51) one easily finds that the connection (2.69) has the following shape

$$\overline{\omega}^A B = \begin{pmatrix} \pi^*_E(\omega^{-\alpha}\beta) & 0 \\ 0 & \hat{\overline{\omega}}^{\tilde{a}}_{\tilde{b}} \end{pmatrix} \tag{2.71}$$

where $\omega^{-\alpha}\beta = \overline{\Gamma}^{\alpha}\beta\gamma\overline{\theta}^{\gamma}$ is a connection on the space-time E and on the manifold $M= G/G_0$ with the following properties .

$$\overline{D}_{\alpha\beta} = g\alpha\delta\overline{Q}^{\delta}\beta\gamma(\overline{\Gamma})\overline{\theta}^{\gamma} = 0 \tag{2.72}$$

$$\overline{Q}^{\alpha}\beta\alpha(\overline{\Gamma}) = 0$$

$$\hat{\overline{D}} g_{\tilde{a}\tilde{b}} = g_{\tilde{a}\tilde{d}} \hat{\overline{Q}}^{\tilde{d}}_{\tilde{b}\tilde{c}}(\hat{\overline{\Gamma}}) \tag{2.73}$$

$$\overline{Q}^{\tilde{d}}_{\tilde{b}\tilde{d}}(\hat{\overline{\Gamma}}) = 0 \tag{2.74}$$

($\overline{D}$ is an exterior covariant derivative with respect to a connection $\omega^{-\alpha}\beta.\overline{Q}^{\alpha}_{\beta\gamma}$ is a tensor of torsion of a connection $\hat{\overline{D}}$ is an exterior covariant derivative of a connection $\hat{\overline{\omega}}^{\tilde{a}}_{\tilde{b}}$ and $\hat{\overline{Q}}^{\tilde{a}}_{\tilde{b}\tilde{c}}$ its torsion.On a space-time E we also define the second affine connection such that

$$\overline{W}^{\alpha}\beta = \overline{\omega}^{\alpha}{}_{\beta} - \frac{2}{3}\delta^{\alpha}\beta\overline{W} \tag{2.75}$$

$$\overline{W} = \overline{W}\gamma\overline{\theta}^{\gamma} = \frac{1}{2}(\overline{W}^{\sigma}_{\gamma\sigma} - \overline{W}^{\sigma}_{\gamma\sigma})$$

We proceed a nonsymmetric metrization of a principal fiber bundle $\underline{P}$ according to (2.51). Thus we define a right-invariant t connection with respect to an action of the group H compatible with a tensor $\kappa_{\tilde{A}\tilde{B}}$

$$\begin{aligned} D_{K\tilde{A}\tilde{B}} &= K_{\tilde{A}\tilde{D}} Q^{\tilde{D}}_{\tilde{B}\tilde{C}}(\Gamma)\theta^{\tilde{C}} \\ Q^{\tilde{D}}_{\tilde{B}\tilde{D}}(\Gamma) &= 0 \end{aligned} \tag{2.76}$$

where $\omega^{\tilde{A}}{}_{\tilde{B}} = \Gamma^{\tilde{A}}{}_{\tilde{B}}, \tilde{\theta}^{\tilde{C}}$ D is an exterior covariant derivative with respect to the connection $\omega^{\tilde{A}}{}_{\tilde{B}}$ and $Q^{\tilde{A}}{}_{\tilde{B}}$, its torsion. After some calculations one finds

$$\omega^{\tilde{A}}{}_{\tilde{B}} = \left(\frac{\pi^*(\overline{\omega}^A B) - \ell_{db}\gamma^{MA} L^d MB\theta^b L^a BC\theta^C}{\ell_{db}\gamma^{AB}(2H^d_{CB} - L^d_{CB})\theta^C \tilde{\omega}^a_b} \right) \tag{2.77}$$

Where

$$L^d{}_{MB} = -L^d{}_{BM} \tag{2.78}$$

$$\ell_{dc}\gamma MB\gamma^{CM} L^d_{CA} + \ell_{cd}\gamma AM\gamma^{MC} L^d_{BC} = 2l_{cd}\gamma AM\gamma^{MC} H^d_{BC} \tag{2.79}$$

$L^d{}_C$, is Ad-type tensor with respect to H (Ad- covariant on $\underline{P}$)

$$\tilde{\omega}^a_b = \tilde{\Gamma}^a_b \theta^C \tag{2.80}$$

$$\ell_{db}\tilde{\Gamma}^d_{ac} + \ell_{ab}\tilde{\Gamma}^d_{cb} = -\ell_{db} C^d_{ac} \tag{2.81}$$

$$\tilde{\Gamma}^d_{ac} = -\tilde{\Gamma}^d_{ca}, \tilde{\Gamma}^a_{ad} = 0 \tag{2.82}$$

We define on P a second connection

$$W^{\tilde{A}}{}_{\tilde{B}} = W^{\tilde{A}}{}_{\tilde{B}} - \frac{A}{3(m+2)}\delta^{\tilde{A}}{}_{\tilde{B}}\overline{W} \tag{2.83}$$

Thus we have on P all *(m+4)* dimension al analogues of geometrical quantities from NGT, i.e. $W^{\tilde{A}}{}_{\tilde{B}}$, $\omega^{\tilde{A}}{}_{\tilde{B}}$ *and* $\kappa_{\tilde{A}\tilde{B}}$.

Let us calculate a Moffat–Ricci curvature scalar for the connection $W^{\tilde{A}}{}_{\tilde{B}}$

Citation: Kalinowski MW (2016) A Hierarchy of Symmetry Breaking in the Nonsymmetric Kaluza–Klein (Jordan–Thiry) Theory. J Phys Math 7: 152. doi:10.4172/2090-0902.1000152

Hierarchy of a Symmetry Breaking

Let us incorporate in our scheme a hierarchy of a symmetry breaking. In order to do this let us consider a case of the manifold

$$M = M_0 \times M_1 \times \ldots \times M_{k-1} \tag{3.1}$$

where

$$\dim M_i = \bar{n}_i, \quad i = 0,1,2,\ldots,k-1 \tag{3.2}$$

$$\dim M = \sum_{i=0}^{k-1} \bar{n}_i, \tag{3.3}$$

$$M_i = G_{i+1} / G_i. \tag{3.4}$$

Every manifold M_i is a manifold of vacuum states if the symmetr y is breaking from G_{i+1} to G_i, $G_k = G$.

Thus

$$G_0 \subset G_1 \subset G_2 \subset \ldots \subset G_k = G. \tag{3.5}$$

We will consider the situation when

$$M \simeq G / G_0. \tag{3.6}$$

This is a constraint in the theory. From the chain (3.5) one gets

$$\mathfrak{g}_0 \subset \mathfrak{g}_1 \subset \ldots \subset \mathfrak{g}_k = \mathfrak{g} \tag{3.7}$$

and

$$\mathfrak{g}_{i+1} = \mathfrak{g}_i \dot{+} \mathfrak{m}_i, \quad i = 0,1,\ldots,k-1. \tag{3.8}$$

The relation (3.6) means that there is a diffeomorphism g onto G/G_0 such that

$$g : \prod_{i=0}^{k-1} (G_{i+1} / G_i) \to G / G_0. \tag{3.9}$$

This diffeomorphism is a deformation of a product (3.1) in G/G_0. The theory has been constructed for the case considered before with G_0 and G. The multiplet of Higgs' fields Φ breaks the symmetr y from G to G_0 (equivalently from G to G_0 in the false vacuum case). $\mathfrak{g}_i$ mean Lie algebras for groups G_i and $\mathfrak{m}_i$ a complement in a decomposition (3.8) . On every manifold M_i we introduce a radius r_i (a "size" of a manifold) in such a way that $r_i > r_{i+1}$. On the manifold G/G_0 we define the radius r as before. The diffeomorphism g induces a contragradient transformation for a Higgs field Φ in such a way that

$$g^* \Phi = (\Phi_0, \Phi_1, \ldots, \Phi_{k-1}). \tag{3.10}$$

The fields Φ_i, i=0,…,k-1.

In this way we get the following decomposition for a kinetic part of the field Φ and for a potential of this field:

$$\mathcal{L}_{kin}(\overset{\text{gauge}}{\nabla} \Phi) = \sum_{i=0}^{k-1} \mathcal{L}^i_{\text{kin}}(\overset{\text{gauge}}{\nabla} \Phi_i)0 \tag{3.11}$$

$$V(\Phi) = \sum_{i=0}^{k-1} V^i(\Phi_i)1 \tag{3.12}$$

where

$$V_i = \int_{M_i} \sqrt{|\tilde{g}_i|}\, d^{\bar{n}_i} x \tag{3.13}$$

$$\tilde{g} = \det(g_{i b_i a_i}) \tag{3.14}$$

$$g_{i\tilde{b}_i \tilde{a}_i} \tag{3.15}$$

Equation (3.5) is a non symmetr ic tensor on a manifold M_i.

$$V^i(\Phi_i) = \frac{l_{ab}}{V_i} \int_{M_i} \sqrt{|\tilde{g}_i|}\, d^{\bar{n}_i} x [2 g^{[\tilde{m}_i \tilde{n}_i]} (C^a_{cd} \Phi^c_{i\tilde{m}_i} \Phi^d_{i\tilde{n}_i} - \mu^a_{\hat{i}_i} f^{\hat{i}_i}_{\tilde{m}_i \tilde{n}_i} - \Phi^a_{i\tilde{e}_i} f^{\tilde{e}_i}_{\tilde{m}_i \tilde{n}_i}) \times g^{[\tilde{a}_i \tilde{b}_i]}$$
$$(C^b_{ef} \Phi^e_{i\tilde{a}_i} \Phi^f_{i\tilde{b}_i} - \mu^b_{\hat{j}_i} f^{\hat{j}_i}_{\tilde{a}_i \tilde{b}_i} - \Phi^b_{i\tilde{a}_i} f^{\tilde{d}_i}_{\tilde{a}_i \tilde{b}_i}) - g_i^{\tilde{a}_i \tilde{m}_i} g_i^{\tilde{b}_i \tilde{n}_i} L^a_{\tilde{a}_i \tilde{b}_i} (C^b_{cd} \Phi^c_{i\tilde{m}_i} \Phi^d_{\tilde{n}_i} - \mu^b_{\hat{i}_i} f^{\hat{i}_i}_{\tilde{m}_i \tilde{n}_i} - \Phi^b_{\tilde{e}_i} f^{\tilde{e}_i}_{\tilde{m}_i \tilde{n}_i})],5 \tag{3.16}$$

$f^{\hat{j}_i}_{\tilde{a}_i \tilde{b}_i}$ are structure constant s of the Lie algebra $\mathfrak{g}_i$. The scheme of the symmetr y breaking acts as follows from the group G_{i+1} to G_i (G_t) (if the symmetr y has been broken up to G_{i+1}). The potential $V^i(\Phi_i)$ has a minimum (global or local) for Φ^k_{icrt}, $k = 0,1$. The value of the remaining part of the sum (3.12) for fields Φ_j, $j < i$, is small for the scale of energy is much lower ($r_j > r_i$, $j < i$). Thus the minimum of $V^i(\Phi_i)$ is an approximate minimum of the remaining part of the sum (3.12)). In this way we have a descending chain of truncations of the Higgs potential. This gives in principle a pattern of a symmetr y breaking. However, this is only an approximate symmetr y breaking. The real symmetr y breaking is from G to G_0 (or to G_0 in a false vacuum case). The important point here is the diffeomorphism g.

$$g^* \Phi^b = (\Phi^b_0, \Phi^b_1, \ldots, \Phi^b_{k-1}) \tag{3.17}$$

$$\Phi^b_i = \Phi^b_{i\tilde{a}_i} \tilde{\theta}^{\tilde{a}_i}, \quad i = 0,\ldots,k-1 \tag{3.18}$$

The shape of g is a true indicator of a reality of the symmetr y breaking pattern. If

$$g = \mathrm{Id} + \delta g \tag{3.19}$$

where δg is in some sense small and Id is an identity, the sums (3.11)-(3.12) are close to the analogous formulae from the expanation of Kalinowski [5,10]. The smallness of g is a criterion of a practical application of the symmetr y breaking pattern (3.5) . It seems that there are a lot of possibilities for the condition (3.9). Moreover, a smallness of δg plus some natural conditions for groups G_i can narrow looking for grand unified models. Let us notice that the decomposition of M results in decomposition of cosmolog ical terms

$$\underline{\tilde{P}} = \sum_{i=0}^{k-1} \underline{\tilde{P}}_i 9 \tag{3.20}$$

where

$$\underline{\tilde{P}}_i = \frac{1}{r_i^2 V_i} \int_{M_i} \sqrt{|\tilde{g}_i|}\, \hat{\overline{R}}_i(\hat{\overline{\Gamma}}_i)\, d^{n_i} x 0 \tag{3.21}$$

where $\hat{\overline{\Gamma}}_i$ is a non symmetr ic connection on M_i compatible with the non symmetr ic tensor $g_{i\tilde{a}_i \tilde{b}_i}$ and $\hat{\overline{R}}_i(\hat{\overline{\Gamma}}_i)$ its curvature scalar. The truncation procedure can be proceeded in several ways. Finally let us notice that the energy scale of broken gauge bosons is fixed by a radius r_i at any stage of the symmetr y breaking in our scheme.

Let us consider Equation (3.10) in more details. One gets

$$A^{\tilde{a}}_{i\tilde{a}_i}(y) \Phi^b_{\tilde{a}}(y) = \Phi^b_{\tilde{a}_i}(y_i), \quad y \in M, y_i \in M_i \tag{3.22}$$

where

$$g^*(y) = (A_0 \mid A_1 \mid A_2 \mid \ldots \mid A_{k-1}) \tag{3.23}$$

$$A_i = (A^{\tilde{a}}_{i\tilde{a}_i})_{\tilde{a}=1,2,\ldots,n_1, \tilde{a}_i=1,2,\ldots,\bar{n}_i}, \; i = 0,1,2,\ldots,k \tag{3.24}$$

is a matrix of Higgs' fields transformation.

According to our assumptions one gets also:

$$\left(\frac{r_i}{r}\right)^2 g_{i\tilde{a}_i \tilde{b}_i}(y_i) = A^{\tilde{a}}_{\tilde{a}_i}(y) A^{\tilde{b}}_{\tilde{b}_i}(y) g_{\tilde{a}\tilde{b}}(y) \tag{3.25}$$

For g is an invertible map we have $\det g^*(y) \neq 0$.

We have also

Citation: Kalinowski MW (2016) A Hierarchy of Symmetry Breaking in the Nonsymmetric Kaluza–Klein (Jordan–Thiry) Theory. J Phys Math 7: 152. doi:10.4172/2090-0902.1000152

$$n_1 = \sum_{i=0}^{k-1} \bar{n}_i \quad (3.26)$$

and

$$\Phi_{\tilde{a}}^{b}(y) = \sum_{i=0}^{k-1} \widetilde{A}_{i\tilde{a}}^{\tilde{a}_i}(y)\Phi_{\tilde{a}_i}^{b}(y_i) \quad (3.27)$$

or

$$\begin{pmatrix} \tilde{A}_0 \\ \tilde{A}_1 \\ \dots \\ \tilde{A}_{k-1} \end{pmatrix} \quad (3.28)$$

$$\widetilde{A}_i = (\widetilde{A}_{i\tilde{a}}^{\tilde{a}_i})_{\tilde{a}_i=1,2,\dots,\bar{n}_i,\ \tilde{a}=1,2,\dots,n_1} \quad (3.29)$$

such that

$$g(y_0,\dots,y_{k-1}) = y \quad (3.30)$$

$$(y_0,y_1,\dots,y_{k-1}) = g^{-1}(y) \quad (3.31)$$

For an inverse tensor $g^{\tilde{a}\tilde{b}}$ one easily gets

$$(\frac{r_i^2}{r^2})g^{\tilde{a}\tilde{b}} = \sum_{i=0}^{k-1} \widetilde{A}_{i\tilde{a}_i}^{\tilde{a}} g_i^{\tilde{a}_i\tilde{b}_i} A_{i\tilde{b}_i}^{\tilde{b}} \quad (3.32)$$

We have

$$r^{2n_1}\det(g_{\tilde{a}\tilde{b}}) = \prod_{i=0}^{k-1} r_i^{2\bar{n}_i}\det(g_{i\tilde{a}_i\tilde{b}_i}). \quad (3.33)$$

In this way we have for the measure

$$d\mu(y) = \prod_{i=0}^{k-1} d\mu_i(y_i) \quad (3.34)$$

where

$$d\mu(y) = \sqrt{\det g}\, r^{n_1} d^{n_1}y \quad (3.35)$$

$$d\mu_i(y_i) = \sqrt{\det g_i}\, r_i^{\bar{n}_i} d^{\bar{n}_i}y_i. \quad (3.36)$$

In the case of $\mathcal{L}_{int}(\Phi,\widetilde{A})$ one gets

$$\mathcal{L}_{int}(\Phi,\widetilde{A}) = \sum_{i=0}^{k-1} \mathcal{L}_{int}(\Phi_i,\widetilde{A}) \quad (3.37)$$

where

$$\mathcal{L}_{int}(\Phi_i,\widetilde{A}) = h_{ab}\mu_i^a \widetilde{H}^i \underline{g}_i^{[\tilde{a}_i\tilde{b}_i]}(C_{cd}^b \Phi_{i\tilde{a}_i}^c \Phi_{i\tilde{b}_i}^d - \mu_i^b f_{\tilde{a}_i\tilde{b}_i}^{\hat{i}} - \Phi_{\tilde{d}_i}^b f_{\tilde{a}_i\tilde{b}_i}^{\tilde{d}_i}) \quad (3.38)$$

where

$$\underline{g}_i^{[\tilde{a}_i\tilde{b}_i]} = \frac{1}{V_{i\,M_i}} \sqrt{|\widetilde{g}_i|}\, d^{\bar{n}_i} x\, g_i^{[\tilde{a}_i\tilde{b}_i]}, \quad i=0,1,2,\dots,k-1 \quad (3.39)$$

Moreover, to be in line in the full theory we should consider a chain of groups H_i, $i=0,1,2,\dots,k-1$, in such a way that

$$H_0 \subset H_1 \subset H_2 \subset \dots \subset H_{k-1} = H \quad (3.40)$$

For every group H_i we have the following assumptions

$$G_i \subset H_i \quad (3.41)$$

and G_{i+1} is a centralizer of G_i in H_i. Thus we should have

$$G_i \otimes G_{i+1} \subset H_i, \quad i=0,1,2,\dots,k-1.0 \quad (3.42)$$

We know from elementary particles physics theory that

$$G_0 = U_{el} \otimes SU(3)_{color},$$

$$G_1 = SU(2)_L \otimes U(1)_Y \otimes SU(3)_{color}$$

and that $G2$ is a group which plays the role of H in the case of a symmetr y breaking from $SU(2)_L \otimes U_Y(1)$ to $U_{el}(1)$. However, in this case because of a factor $U(1)$, $M=S^2$. Thus $M_0=S^2$ and $G2 \subset H_0$.

It seems that in a reality we have to do with two more stages of a symmetr y breaking. Thus $k=3$. We have

$$M \simeq S^2 \times M_1 \times M_2 \quad (3.43)$$

$$M = G/(U(1)\otimes SU(3)) \quad (3.44)$$

$$M_1 = G_1/(SU(2)\times U(1)\times SU(3))$$

$$U(1)\otimes SU(3) \subset SU(2)\otimes U(1)\otimes SU(3) \subset G_2 \otimes SU(3) \subset G_3 = G3 \quad (3.45)$$

and

$$G_1 \subset H_1 \subset H \quad (3.46)$$

$$U(1)\otimes SU(3)\otimes G \subset H \quad (3.47)$$

$$(U(1)\otimes SU(2)\otimes SU(3))\otimes G_2 \subset H_1 \quad (3.48)$$

and

$$G_2 \otimes G \subset H_2 = H \quad (3.49)$$

$$M_2 = G/G_1 \quad (3.50)$$

We can take for G, $SU(5)$, $SU(10)$, $E6$ or $SU(6)$. Thus there are a lot of choices for G_2, H_1 and H.We can suppose for a trial that

$$G2 \otimes SU(3) \subset H_0 \quad (3.51)$$

We have also some additional constraints

$$rank(G) \ge 4 \quad (3.52)$$

Thus

$$rank(H_0) \ge 4 \quad (3.53)$$

We can try with $F4 = H_0$.

In the case of H

$$rank(H) \ge rank(G) + 3 \ge 7 \quad (3.54)$$

Thus we can try with E_7, E_8

$$rank(H_1) \ge rank(G_2) + 4$$

$$rank(H) \ge rank(G_2) + rank(G) \ge rank(G_2) + 4 \ge rank(G) + 4 \ge 8 \quad (3.55)$$

In this way we have

$$rank(H) \ge 8.5f \quad (3.56)$$

Thus we can try with

$$H = E8.5g \quad (3.57)$$

But in this case

rank (G_2)= *rank* (G)=4

This seems to be nonrealistic. For instance, if $G= SO(10)$, $E6$,

rank(SO(10))=5rankE6=6

In this case we get

rank (H)=9 rank (H)=10

And H could be $SO(10)$, $SO(18)$, $SO(20)$.In this approach we try to consider additional dimensions connecting to the manifold M more seriously, i.e. as physical dimensions, additional space-like dimensions. We remind to the reader that gauge-dimensions connecting to the group H have different meaning. They are dimensions connected to local gauge symmetr ies (or global) and they cannot be directly observed. Simply saying we cannot travel along them. In the case of a manifold M this possibility still exists. However, the manifold M is diffeomorphically equivalent to the product of some manifolds M_i, $i=0,1,2,\ldots,k-1$, with some characteristic sizes r_i The radii r_i represent energy scales of symmetr y breaking. The lowest energy scale is a scale of weak interactions (Weinberg–Glashow–Salam model) $r_0 \simeq 10^{-16}$ cm. In this case this is a radius of a sphere S^2 The possibility of this "travel" will be considered in the concept explanied by Kalinowski [26]. In this case a metric on a manifold M can be dependent on a point $x\in E$ (parametrically).It is interesting to ask on a stability of a symmetr y breaking pattern with respect to quantum fluctuations. This difficult problem strongly depends on the details of the model. Especially on the Higgs sector of the practical model. In order to preserve this stability on every stage of the symmetr y breaking we should consider remaining Higgs' fields (after symmetry breaking) with zero mass. According to S. Weinberg, they can stabilize the symmetry breaking in the range of energy

$$\frac{1}{r_i}(\frac{\hbar}{c}) < E < \frac{1}{r_{i+1}}(\frac{\hbar}{c}), \quad i=0,1,2,\ldots,k-1,6 \tag{3.58}$$

i.e. for a symmetry breaking from G_{i+1} to G_i.

It seems that in order to create a realistic grand unified model based on non symmetr ic Kaluza–Klein (Jordan–Thiry) theory it is necessary to nivel cosmolog ical terms. This could be achieved in some models due to choosing constant s ξ and ζ and μ . After this we can control the value of those terms, which are considered as a selfinteraction potential of a scalar field Υ. The scalar field Υ can play in this context a role of a quintessence .

Let us notice that using the equation

$$\Phi^c_{\tilde{b}}(x) f^{\tilde{b}}_{i\tilde{a}} = \mu^a_i \Phi^b_{\tilde{a}}(x) C^c_{ab}, w \tag{3.59}$$

and (3.27) one gets

$$\sum_{i=0}^{k-1} \tilde{A}^{\tilde{a}_i}_{i\tilde{b}} \Phi^c_{\tilde{a}_i} f^{\tilde{b}}_{i\tilde{a}} = C^c_{ab} \mu^a_i \sum_{i=0}^{k-1} \tilde{A}^{\tilde{a}_i}_{i\tilde{a}} \Phi^b_{\tilde{a}_i} \tag{3.60}$$

In this way we get constraints for Higgs' fields, Φ_0, Φ_1, Φ_{k-1}

$$\Phi_i = (\Phi^b_{\tilde{a}_i}), \quad i=0,1,\ldots,k-1.$$

Solving these constraints we obtain some of Higgs' fields as functions of in dependent components [26]. This could result in some cross terms in the potential (3.12) between Φ's with different i. For example a term

$$V(\Phi'_i, \Phi'_j),$$

where Φ' means in dependent fields. This can cause some problems in a stability of symmetry breaking pattern against radiative corrections. This can be easily seen from Equation (3.59) solved by in dependent Φ' ,

$$\Phi = B\Phi' \tag{3.61}$$

$$\Phi^c_{\tilde{b}} = B^{\bar{c}\tilde{\bar{b}}}_{\tilde{b}\bar{c}} \Phi'^{\bar{c}}_{\tilde{\bar{b}}} \tag{3.62}$$

Where B is a linear operator transforming in dependent Φ' into Φ.

We can suppose for a trial a condition similar to (3.59) for every $i=0,\ldots,k-1$,

$$\Phi^{c_i}_{\tilde{b}_i} f^{\tilde{b}_i}_{i\hat{i}\tilde{a}_i} = \mu^{a_i}_{\hat{i}i} \Phi^{b_i}_{\tilde{a}_i} C^{c_i}_{a_i b_i} \tag{3.63}$$

where $C^{c_i}_{a_i b_i}$ are structure constant s for the Lie algebra h_i of the group H_i. $f^{\tilde{b}_i}_{i\hat{i}\tilde{a}_i}$ are structure constant s of the Lie algebra $\mathfrak{g}_{i+1}$, $\hat{i}_i$ are indices belonging to Lie algebra g_i and $\tilde{a}_i$ to the complement m_i.

In this way

$$\Phi^c_{\tilde{b}_i} = \Phi^{c_i}_{\tilde{b}_i} \delta^c_{c_i} \tag{3.64}$$

In this case we should have a consistency between (3.63) and (3.60) which impose constraints on $C, f\, \mu$ and $C^i\, f^{\,i}, \mu^i$ where $C^i\, f^{\,I}, \mu^i$ refer to H_i, G_{i+1}. Solving (3.63) via introducing in dependent fields Φ'_i one gets

$$\Phi^{c_i}_{i\tilde{b}_i} = B^{c_i\tilde{\bar{b}}_i}_{i\bar{c}_i\tilde{b}_i} \Phi'^{\bar{c}_i}_{\tilde{\bar{b}}_i} \tag{3.65}$$

Combining (3.62) , (3.64) , (3.65) one gets

$$B^{\bar{c}\tilde{\bar{b}}}_{\tilde{b}\bar{c}} \Phi'^{\bar{c}}_{\tilde{\bar{b}}} = \sum_{i=0}^{k-1} \tilde{A}^{\tilde{a}_i}_{i\tilde{b}} \delta^c_{c_i} B^{c_i\tilde{\bar{b}}_i}_{i\bar{c}_i\tilde{b}_i} \Phi'^{\bar{c}_i}_{\tilde{\bar{b}}_i} \tag{3.66}$$

Equation (3.66) gives a relation between in dependent Higgs' fields Φ' and Φ'_i . Simultaneously it is a consistency condition between Equation (3.59) and Equation (3.63). However, the condition (3.63) seems to be too strong and probably it is necessary to solve a weaker condition (3.60) which goes to the mentioned terms $V(\Phi'_i, \Phi'_j)$. The conditions (3.63) plus a consistency (3.66) avoid those terms in the Higgs potential. This problem demands more investigation. ϕ (g)= $\{gG_1\}$

It seems that the condition (3.9) could be too strong. In order to find a more general condition we consider a simple example of (3.5). Let $G_{0=}$ $\{e\}$ and $K=2$ In this case we have

$$\{e\} \subset G_1 \subset G_2 = G \tag{3.67}$$

$$M_0 = G_1, \quad M_1 = G/G_1 \tag{3.68}$$

$$g : G_1 \times G/G_1 \to G \tag{3.69}$$

In this way $G_1\times G/G_1$ is diffeomorphically equivalent to G. Moreover, we can consider a fibre bundle with base space G/G_1 and a structural group G_1 with a bundle manifold G. This construction is known in the theory of induced group representation done by Trautman [27]. The projection $\phi : G^{\circledast} G/G_1$ is defined by ϕ (g)= $\{gG_1\}$. The natural extension of (3.69) is to consider a fibre bundle (G, G/G_1, G_1, ϕ). In this way we have in a place of (3.69) a local condition

$$g_U : G_1 \times U \underset{\text{in}}{\to} G \tag{3.70}$$

where $U\subset G/G_1$ is an open set. Thus in a place of (3.9) we consider a local diffeomorphism

$$g_U : M_0 \times M_1 \times \ldots \times M_{k-1} \underset{\text{in}}{\to} G/G_0 \tag{3.71}$$

where

$$U = U_0 \times U_1 \times \ldots \times U_{k-1},$$

$U_i \subset M_i$, $i=0,1,2,\ldots,k-1$, are open sets. Moreover we should define projectors φ_i , $i=0,1,2,\ldots,k-1$,

$$\varphi_i : G/G_0 \to G_{i+1}/G_i \tag{3.72}$$

i.e.

$$\varphi_i(\{gG_0\}) = \{g_{i+1}G_i\} \tag{3.73}$$

$gG,\ g\notin G_{i+1},\ G_0\subset G_i\subset G_{i+1}\subset G$

in a unique way. This could give us a fibration of G/G_0 in $\prod_{i=0}^{k-1}(G_{i+1}/G_i)$.

For $g\notin G_{i+1}$ we simply define

$$\varphi_i(\{gG_0\})=\{gG_i\} \tag{3.74}$$

If $g\in G,\ g\notin G_{i+1}$, we define

$$\varphi_i(\{gG_0\})=\{G_i\} \tag{3.75}$$

Thus in general

$$\varphi_i(\{gG_0\})=\{p(g)G_i\} \tag{3.76}$$

where

$$p(g)=\begin{cases} g, & g\in G_{i+1} \\ e, & g\notin G_{i+1}. \end{cases} \tag{3.77}$$

Thus in a place of (3.9) we have to do with a structure

$$\{G/G_0,\prod_{i=0}^{k-1}(G_{i+1}/G_i),\varphi_0,\varphi_1,\dots,\varphi_{k-1}\} \tag{3.78}$$

such that

$$g_U\circ\varphi_{|U}=\mathrm{id} \tag{3.79}$$

where

$$\varphi_{|U}=\prod_{i=0}^{k-1}\varphi_{i|U_i} \tag{3.80}$$

This generalizes (3.9) to the local conditions (3.71). Now we can repeat all the considerations concerning a decomposition of Higgs' fields using local diffeomorphisms g_U (g^*_U) in the place of g (g^*). Let us also notice that in the chain of groups it would be interesting to consider as G_2

$$G_2=SU(2)_L\otimes SU(2)_R\otimes SU(4) \tag{3.81}$$

suggested by Salam and Pati, where $SU(4)$ unifies $SU(2)_{color}\otimes U(1)_Y$. This will be helpful in our future consideration concerning extension to super symmetr ic groups, i.e. $U(2,2)$ which unifies $SU(2)_L\otimes SU(2)_R$ to the super Lie group $U(2,2)$ considered by Mohapatra. Such models on the phenomenological level incorporate fermions with a possible extension to the super symmetric $SO(10)$ model. They give a natural framework for lepton flavour mixing going to the neutrino oscillations incorporating see-saw mechanism for mass generations of neutrinos. In such approaches the see-saw mechanism is coming from the grand unified models. Our approach after incorporating manifolds with anticommuting parameters, super Lie groups, super Lie algebras and in general supermanifolds (superfibrebundles) can be able to obtain this. However, it is necessary to develop a formalism (in the language of supermanifolds, superfibrebundles, super Lie groups, super Lie algebras) for non symmetr ic connections, non symmetr ic Kaluza–Klein (Jordan–Thiry) theory. In particular we should construct an analogue of Einstein–Kaufmann connection for supermanifold, a non symmetr ic Kaluza–Klein (Jordan–Thiry) theory for superfibrebundle with super Lie group. In this way we should define first of all a non symmetr ic tensor on a super Lie group and afterwards a non symmetr ic metrization of a superfibrebundle. Let us notice that on every stage of symmetr y breaking, i.e. from G_{i+1} to G_i, we have to do with group G_t (similar to the group G_0). Thus we can have to do with a true and a false vacuum cases which may complicate a pattern of a symmetr y breaking.

References

1. Kalinowski MW (1990) Non-symmetric Fields Theory and its Applications. World Scientific.
2. Kalinowski MW (1991) Can we get confinement from extra dimensions? Physics of Elementary Interactions. World Scientific, Singapore 294-304.
3. Kalinowski MW (1991) Non symmetric Kaluza-Klein (Jordan-Thiry) Theory in a general non-Abelian case. Int J Theor Phys 30: 281.
4. Kalinowski MW (1992) Nonsymmetric Kaluza-Klein and Jordan-Thiry Theory in the electromagnetic case.Int Journal of Theor Phys 31: 611.
5. Kalinowski MW (2015) Scalar fields in the Non-symmetric Kaluza-Klein (Jordan-Thiry) Theory.
6. Moffat JW (1982) Generalized theory of gravitation and its physical consequences.in: Proceeding of the VII International School of Gravitation and Cosmology.
7. Kalinowski MW (2013) Preliminary applications of the non-symmetric Kaluza-Klein (Jordan-Thiry) theory to Pioneer 10 and 11 spacecraft anomalous acceleration. CEAS Space Journal 5: 19.
8. Kalinowski MW (2015) pioneer 10 and 11 spacecraft anomalous acceleration in the light of the nonsymmetric kaluza-klein (jordan-thiry) theory.
9. Kalinowski MW (2014) On some developments in the Nonsymmetric Kaluza-Klein Theory. The European Physical Journal C 74: 2742.
10. Kalinowski MW (2014) The Nonsymmetric Kaluza-Klein Theory and Modern Physics. A novel approach.
11. Einstein A (1951) The Meaning of Relativity: Including the Relativistic Theory of the Non-Symmetric field. Princeton university press.
12. Einstein A (1950) On the generalized theory of gravitation. Sci Amer 182: 13.
13. Hlavaty V (1957) Geometry of Einstein Unified Field Theory.
14. Kobayashi S, Nomizu K (1969) Foundations of Differential Geometry. John Wiley and Sons.
15. Wang H (1958) On invariant connection on principal fibre bundle. Nagoya Math J 13: 1-19.
16. Harnad J, Shnider S, Tafel J (1980) Group actions on principal bundles and invariance conditions for gauge fields. J Math Phys 21: 2719.
17. Harnad J, Shnider S, Tafel J (1980) Group actions on principal bundles and dimensional reduction. Lett in Math Phys 4: 107.
18. Manton NS (1979) A new six-dimensional approach to the Weinberg-Salam model. Nucl Phys B 158: 141.
19. Forgacs P (1980) Manton NS Space-time symmetries in gauge theories. Comm Math Phys 72: 15.
20. Chapline G, Manton NS (1981) The geometrical significance of certain Higgs' potentials: An approach to grand unifications. Nucl Phys B 184: 391.
21. Meclenburg W (1992) Geometrical unification of gauge and Higgs' fields. Journal of Physics G: Nuclear Physics 6: 1-9.
22. Meclenburg W (1981) Towards a unified theory for gauge and Higgs' fields Phys Rev D 24: 3194.
23. Witten E (1977) Some exact multipseudo-particle solutions of classical Yang-Mil ls theory. Phys Rev Lett 38: 121.
24. Helgason S (1987) Differential Geometry, Lie Groups and Symmetric Spaces. Academic Press, New York.
25. Cheeger J, Ebin DG (1975) Comparison Theorem in Riemannian. Geometry.
26. Kalinowski MW (2003) A Warp Factor in the Non symmetric Kaluza-Klein (Jordan-Thiry) Theory.
27. Trautman A (1970) Fibre bundles associated with space-time. Rep of Math Phys 1: 29

Citation: Kalinowski MW (2016) A Hierarchy of Symmetry Breaking in the Nonsymmetric Kaluza–Klein (Jordan–Thiry) Theory. J Phys Math 7: 152. doi:10.4172/2090-0902.1000152