TOKYO J. MATH.
VOL. 16, No. 2, 1993

A Brownian Ball Interacting with Infinitely Many Brownian Particles in R^d

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(Communicated by Y. Maeda)

1. Introduction and main results.

In this paper we construct a system of a hard ball with radius $r(\in(0,\infty))$ interacting with infinitely many point particles in R^d $(d\geq 2)$. All particles and the ball are undergoing Brownian motions and when the distance between a particle and the center of the ball attains a given constant r, they repel each other instantly. Saisho and Tanaka [5] constructed a system of mutually reflecting finitely many hard balls by solving certain stochastic differential equation of Skorohod type. Following the idea of [5], Saisho [4] constructed a system of mutually repelling finitely many particles of m types: the number of particles of type k is n_k $(\sum_{k=1}^{m} n_k=n<\infty)$ and when the distance between two particles of different type attains a constant r, they repel each other instantly. In case each type consists of only one particle, the model of [4] is reduced to that of [5]. Our present model in this paper is formally regarded as the case of $m=2$, $n_1=1$ and $n_2=\infty$ in the model of [4].

Let $\mathfrak{M}_0$ be the set of all countable subsets η of $R^d\setminus U_r(0)$ satisfying $N_K(\eta)\equiv \sharp(\eta\cap K)<\infty$ for any compact subset K, where $U_r(x)=\{y\in R^d:|x-y|<r\}$. The configuration space of a hard ball with radius r and infinitely many point particles is defined by

$$X=\{x=(x_0,x_1,\cdots)\in(R^d)^\infty:\{x_i-x_0,\ i\in N\}\in\mathfrak{M}_0\}\ ,$$

where x_0 is the position of center of the hard ball and x_i is that of the i-th point particle. We put $W_0=C(w:[0,\infty)\to R^d,\ w(0)=0)$ and $W=W_0^\infty$. Given $x=(x_0,x_1,\cdots)\in X$ and $w=(w_0,w_1,\cdots)\in W$, we consider the following equation (1.1) under the conditions (1.2), (1.3) and (1.4):

Received August 21, 1992

* Research supported in part by Grant-in-Aid for Scientific Research (No. 03740101), Ministry of Education, Science and Culture.

$$(1.1)\qquad \begin{cases} \xi_0(t)=x_0+w_0(t)+\sum_{j=1}^{\infty}\int_0^t (\xi_0(s)-\xi_j(s))dL_j(s)\,, \\ \xi_i(t)=x_i+w_i(t)+\int_0^t (\xi_i(s)-\xi_0(s))dL_i(s)\,, \qquad i\in \boldsymbol{N}\,, \end{cases}$$

$$(1.2)\qquad \xi_i\in \boldsymbol{C}([0,\infty)\to \boldsymbol{R}^d)\,, \qquad i\in \boldsymbol{Z}_+\,,$$

$$(1.3)\qquad |\xi_i(t)-\xi_0(t)|\geq r\,, \qquad i\in \boldsymbol{N}\,, \quad t\in[0,\infty)\,,$$

(1.4) L_i, $i\in \boldsymbol{N}$ are continuous nondecreasing functions with $L_i(0)=0$ and

$$L_i(t)=\int_0^t \mathbb{I}_{\{r\}}(|\xi_i(s)-\xi_0(s)|)dL_i(s)\,,$$

where $\mathbb{I}_A$ stands for the indicator function of a set A.

Let P_W be a Wiener measure on W_0 and $P=P_W^{\otimes\infty}$. We denote by μ_λ a Poisson distribution on $\boldsymbol{R}^d\setminus U_r(0)$ with intensity measure λdx, that is, for any disjoint system $\{A_1, A_2, \cdots, A_m\}\subset\mathscr{B}(\boldsymbol{R}^d\setminus U_r(0))$ such that $|A_i|=\int_{A_i}dx<\infty$, $i=1, 2, \cdots, m$, and $\lambda>0$, N_{A_i}, $i=1, 2, \cdots, m$ are independent random variables with

$$\mu_\lambda(N_{A_i}=n)=\frac{(\lambda|A_i|)^n}{n!}\exp(-\lambda|A_i|)\,, \qquad i=1, 2, \cdots, m\,, \quad n\in \boldsymbol{N}\cup\{0\}\,.$$

Let Γ be the map from $\boldsymbol{X}$ to $\mathfrak{M}_0$ defined by

$$\Gamma(\boldsymbol{x})=\Gamma(x_0, x_1, \cdots)=\{x_i-x_0 : i\in \boldsymbol{N}\}\,.$$

Our main result of this paper is the following theorem.

THEOREM 1. *Let $\hat{\mu}$ be a probability measure on $\boldsymbol{X}$ such that $\Gamma\hat{\mu}=\mu_\lambda$ for some $\lambda>0$, where $\Gamma\hat{\mu}$ is the image measure of $\hat{\mu}$ under the map Γ. Then, for almost all $(\boldsymbol{x}, \boldsymbol{w})$ with respect to $\boldsymbol{P}=\hat{\mu}\otimes P$ there exists a unique solution $(\xi(t), L(t))$ of the equation* (1.1). *Furthermore, the distribution of $\Gamma\xi(t)$ is μ_λ for all $t\geq 0$.*

For the proof of Theorem 1, we first construct a system of a Brownian ball colliding with finitely many Brownian particles on some torus by using a Skorohod equation and the same procedure as that of [5] (Section 2). We also give an estimate concerning the motion of the Brownian ball in a way uniform with respect to the number of the particles (Lemma 2.6). The key idea of the proof of Lemma 2.6 is the decomposition of additive functionals of reversible processes which is originally obtained by Lyons and Zheng [2] for symmetric Markov processes associated with regular Dirichlet forms. We generalize this decomposition by employing the penalty method given in [6]. Secondly we show that under the assumption of Theorem 1 the summation in the equation (1.1) is in fact a finite sum for each $t\geq 0$ by Lemma 2.6. The proof of Theorem 1 is given in Section 3.

2. Skorohod equation for a torus.

For $M, n \in N$ and $r \in (0, 1/2)$, we define a domain D_M in $\boldsymbol{R}^{(n+1)d}$ by

$$D_M = \{x = (x_0, x_1, \cdots, x_n) \in \boldsymbol{R}^{(n+1)d} : \rho_M(x_i, x_0) > r,\ 1 \le i \le n\}\ ,$$

where $\rho_M(x_i, x_0) = \min\{|x_i - x_0 + Mz| : z \in \boldsymbol{Z}^d\}$. We also define the set $\mathcal{N}_x$ of inward normal unit vectors at $x \in \partial D_M$ by

$$\mathcal{N}_x = \bigcup_{l>0} \mathcal{N}_{x,l}\ ,$$

$$\mathcal{N}_{x,l} = \{\boldsymbol{n} \in \boldsymbol{R}^{(n+1)d} : |\boldsymbol{n}| = 1,\ U_l(x - l\boldsymbol{n}) \cap D_M = \varnothing\}\ .$$

First we prove the following lemma.

LEMMA 2.1. *For each $M, n \in N$, D_M satisfies Conditions* (A) *and* (B):
Condition (A). *There exists a constant $l_0 > 0$ such that*

$$\mathcal{N}_x = \mathcal{N}_{x,l_0} \neq \varnothing \qquad \text{for any } x \in \partial D_M\ .$$

Condition (B). *There exist constants $\delta > 0$ and $\beta \in [1, \infty)$ with the following property: for any $x \in \partial D_M$ there exists a unit vector $\boldsymbol{l}_x$ such that*

$$\langle \boldsymbol{l}_x, \boldsymbol{n} \rangle \ge 1/\beta \qquad \text{for any } \boldsymbol{n} \in \bigcup_{y \in U_\delta(x) \cap \partial D_M} \mathcal{N}_y\ ,$$

where $\langle \cdot, \cdot \rangle$ denotes the usual inner product in $\boldsymbol{R}^{(n+1)d}$.
In addition we have

$$\mathcal{N}_x = \left\{\boldsymbol{n} : |\boldsymbol{n}| = 1,\ \boldsymbol{n} = \sum_{i \in I} c^i \boldsymbol{n}^i,\ c^i \ge 0\right\}, \qquad x = (x_0, x_1, \cdots, x_n) \in \partial D_M\ ,$$

where

$$I = \{i \in \{1, 2, \cdots, n\} : \rho_M(x_i, x_0) = r\}\ ,$$

$$\boldsymbol{n}^i = \Bigg(\frac{\tilde{x}_0 - \tilde{x}_i}{\sqrt{2}\, r}, 0, \cdots, 0, \underset{(i+1)\text{-th}}{\frac{\tilde{x}_i - \tilde{x}_0}{\sqrt{2}\, r}}, 0, \cdots, 0\Bigg)\ ,$$

and $\tilde{x} = (0, \tilde{x}_1, \cdots, \tilde{x}_n) \in \boldsymbol{R}^{(n+1)d}$ with $\tilde{x}_i \in K_M = [-M/2, M/2)^d$ and $\tilde{x}_i = x_i - x_0 \pmod{M\boldsymbol{Z}^d}$ for $i = 1, \cdots, n$.

PROOF. We introduce the domain D_0 defined by

$$D_0 = \{x = (x_0, x_1, \cdots, x_n) \in \boldsymbol{R}^{(n+1)d} : |x_i - x_0| > r,\ 1 \le i \le n\}\ .$$

It is easy to see that

(2.1) $$D_M = \bigcap_{z_0 \in \boldsymbol{Z}^d} (D_0 + (Mz_0, 0, \cdots, 0)),$$

(2.2) $$D_M = D_M + (Mz_0 + y_0, Mz_1 + y_0, \cdots, Mz_n + y_0),$$

$\forall z_0, \cdots, z_n \in \boldsymbol{Z}^d$, $\forall y_0 \in \boldsymbol{R}^d$, where $A + x = \{y + x : y \in A\}$. If $x \in \partial D_M$, (2.1) and (2.2) yield $\tilde{x} \in \partial D_M$ and

(2.3) $$D_M \cap U_a(x) = (D_M \cap U_a(\tilde{x})) + x - \tilde{x}, \qquad a > 0.$$

If we take $a \in (0, (M/2 - r)/\sqrt{2})$, for any $y \in U_a(\tilde{x})$ and $z_0 \in \boldsymbol{Z}^d \setminus \{0\}$, we have

$$\begin{aligned} |y_i - y_0 + Mz_0| &= |y_i - y_0 + Mz_0 - (\tilde{x}_i - \tilde{x}_0) + (\tilde{x}_i - \tilde{x}_0)| \\ &\geq |\tilde{x}_i - \tilde{x}_0 + Mz_0| - |y_i - \tilde{x}_i| - |y_0 - \tilde{x}_0| \\ &\geq M/2 - \sqrt{2}\,|y - \tilde{x}| > r. \end{aligned}$$

Thus, we have $\tilde{x} \in \partial D_0$ and

$$D_M \cap U_a(\tilde{x}) = D_0 \cap U_a(\tilde{x}).$$

Combining this with (2.3), we obtain

$$D_M \cap U_a(x) = (D_0 \cap U_a(\tilde{x})) + x - \tilde{x}.$$

Moreover, it has been proved in [4] that D_0 satisfies Conditions (A) and (B) for

$$\mathcal{N}_x(D_0) = \left\{ \boldsymbol{n} : |\boldsymbol{n}| = 1, \boldsymbol{n} = \sum_{i \in \boldsymbol{I}_0} c^i \boldsymbol{m}^i, c^i \geq 0 \right\}, \qquad x = (x_0, x_1, \cdots, x_n) \in \partial D_0,$$

where

$$\boldsymbol{I}_0 = \{ i \in \{1, 2, \cdots, n\} : |x_i - x_0| = r \},$$

$$\boldsymbol{m}^i = \Bigg(\frac{x_0 - x_i}{\sqrt{2}\, r}, 0, \cdots, 0, \underset{(i+1)\text{-th}}{\frac{x_i - x_0}{\sqrt{2}\, r}}, 0, \cdots, 0 \Bigg).$$

Thus, we obtain Lemma 2.1. □

For given $w \in W_0^{n+1} = \boldsymbol{C}(w : [0, \infty) \to \boldsymbol{R}^{(n+1)d}, w(0) = 0)$ and $x \in \overline{D_M}$, *Skorohod equation* for D_M with reflecting boundary condition is written in the form

(2.4) $$\zeta(t) = x + w(t) + \varphi(t), \qquad t \geq 0,$$

where a solution (ζ, φ) should be found under the following two conditons (2.5) and (2.6) (we also call ζ a solution of (2.4)).

(2.5) $$\zeta \in \boldsymbol{C}([0, \infty) \to \overline{D_M}).$$

(2.6) φ is an $\boldsymbol{R}^{(n+1)d}$-valued continuous function with bounded variation on each finite time interval satisfying $\varphi(0)=0$ and

$$\varphi(t)=\int_0^t \boldsymbol{n}(s)d\|\varphi\|_s ,$$

$$\|\varphi\|_t=\int_0^t \mathbb{1}_{\partial D_M}(\zeta(s))d\|\varphi\|_s ,$$

where

$$\boldsymbol{n}(s)\in\mathcal{N}_{\zeta(s)} \qquad \text{if } \zeta(s)\in\partial D_M ,$$

$$\|\varphi\|_t=\text{the total variation of } \varphi \text{ on } [0, t] .$$

The existence and uniqueness of solutions of Skorohod equations were studied by many authors (Lions-Sznitman [1], Saisho [3], Tanaka [8]). By Lemma 2.1 we can apply Theorem 4.1 in [3] and obtain the following result.

PROPOSITON 2.2. *For each $n\in\boldsymbol{N}$ and $M\in\boldsymbol{N}$ the Skorohod equation* (2.4) *for D_M has a unique solution.*

REMARK 2.1. We denote by $\zeta(t, x, w)$ the unique solution of the Skorohod equation (2.4) for D_M, $x\in D_M$, $w\in W_0^{n+1}$. From (2.2), for any $y_0\in\boldsymbol{R}^d$ we see that $x'\equiv(x_0+y_0, x_1+y_0, \cdots, x_n+y_0)\in D_M$ and

$$\zeta_i(t, x', w)=\zeta_i(t, x, w)+y_0 , \qquad i=1, 2, \cdots, n .$$

By the same procedure as in Section 2 of [6], we can construct a continuous function $V(x)$ on $\boldsymbol{R}^{(n+1)d}$ with the following properties (2.7), (2.8) and (2.9):

$$V(x)=V(x+Mz) \qquad \text{for any } x\in\boldsymbol{R}^{(n+1)d} ,\quad z\in\boldsymbol{Z}^{(n+1)d} , \tag{2.7}$$

$$V(x)=\inf_{y\in D_M}|x-y|^2 \qquad \text{if } \inf_{y\in D_M}|x-y|\le l_0 , \tag{2.8}$$

$$\nabla V \text{ is bounded and Lipschitz continuous} . \tag{2.9}$$

Given $w\in\boldsymbol{W}_0^{n+1}$, $x\in\boldsymbol{R}^{(n+1)d}$ and $m\in\boldsymbol{N}$, we denote by $\zeta^m(t)$ the solution of

$$\zeta^m(t)=x+w(t)-\frac{m}{2}\int_0^t \nabla V(\zeta^m(s))ds . \tag{2.10}$$

The following result is Theorem 2 in [6].

PROPOSITION 2.3 ([6]). *Let $T>0$ and x^m, $m\in\boldsymbol{N}$ be a sequence of $\overline{D_m}$ which converges to $x\in\overline{D_M}$. Then the process $\zeta^m(t, x^m, w)$ converges to the solution $\zeta(t, x, w)$ of the equation* (2.4) *uniformly in $t\in[0, T]$ as $m\to\infty$.*

Now let π_M be the natural projection from $\boldsymbol{R}^d$ to $T_M=\boldsymbol{R}^d/M\boldsymbol{Z}^d\cong K_M$ and define $\pi_M:\boldsymbol{R}^{(n+1)d}\to T_M^{n+1}$ by

$$\pi_M(x_0, x_1, \cdots, x_n) = (\pi_M x_0, \pi_M x_1, \cdots, \pi_M x_n) .$$

Put $\widetilde{D_M} = \pi_M \overline{D_M}$ and $\tilde{\zeta}(t, x, w) = \pi_M \zeta(t, x, w)$. We denote by Q the uniform distribution on $\widetilde{D_M}$. The following proposition is the immediate consequence of Theorem 1 in [6].

PROPOSITION 2.4. *Under $Q \otimes P_W^{\otimes(n+1)}$ the process $\tilde{\zeta}(t, x, w)$ is a $\widetilde{D_M}$-valued reversible diffusion process.*

As a corollary of Proposition 2.4 we obtain the following result.

COROLLARY 2.5. *Under $Q \otimes P_W^{\otimes(n+1)}$ the process*

$$\tilde{\eta}(t, x, w) = (\tilde{\zeta}_1(t, x, w) - \tilde{\zeta}_0(t, x, w), \cdots, \tilde{\zeta}_n(t, x, w) - \tilde{\zeta}_0(t, x, w))$$

is a stationary process with stationary measure Q_0, where Q_{x_0}, $x_0 \in T_M$ is the n-fold product distribution of the uniform distribution on $T_M \setminus U_r(x_0)$.

The following lemma is a key part of the proof of Theorem 1.

LEMMA 2.6. *Let $T > 0$. Then there exists a positive constant C, which depends only on d and T, such that*

$$\int_{\widetilde{D_M} \times W_0^{n+1}} \exp(\sup_{t \in [0,T]} |\tilde{\zeta}_0(t, x, w) - \tilde{\zeta}_0(0, x, w)|) Q(dx) P_W^{\otimes(n+1)}(dw) \leq C .$$

PROOF. Let $\zeta^m(t)$ be the solution of (2.10). Put $\widetilde{\zeta^m}(t) = \pi_M \zeta^m(t)$ and introduce a probability measure Q^m on T_M^{n+1} defined by

$$Q^m(dx) = \frac{1}{Z_m} \exp(-mV(x)) dx , \qquad Z_m = \int_{T_M^{n+1}} \exp(-mV(x)) dx .$$

It is known that the process $\widetilde{\zeta^m}(t)$ is a reversible Markov process under $Q^m \otimes P_W^{\otimes(n+1)}$ (see for instance Lemma 7.1 in [6]). If we define an additive functional F_t by

$$F_t(\widetilde{\zeta^m}(\cdot)) = \widetilde{\zeta^m}(t) - \widetilde{\zeta^m}(0) + \frac{m}{2} \int_0^t \nabla V(\widetilde{\zeta^m}(s)) ds ,$$

then

$$\begin{aligned} F_t(\widetilde{\zeta^m}(T - \cdot)) &= \widetilde{\zeta^m}(T - t) - \widetilde{\zeta^m}(T) + \frac{m}{2} \int_0^t \nabla V(\widetilde{\zeta^m}(T - s)) ds \\ &= \widetilde{\zeta^m}(T - t) - \widetilde{\zeta^m}(T) + \frac{m}{2} \int_{T-t}^T \nabla V(\widetilde{\zeta^m}(s)) ds . \end{aligned}$$

Put $\widehat{w^m}(t) = \widehat{w^m}(t, x, w) = F_t(\widetilde{\zeta^m}(T - \cdot))$. Under $Q^m \otimes P_W^{\otimes(n+1)}$, by the reversibility of the process $\widetilde{\zeta^m}(t)$, $\widehat{w^m}(t)$ has the same distribution as that of $F_t(\widetilde{\zeta^m}(\cdot)) = \pi_M w(t)$ and so is a Brownian motion on T_M^{n+1}. Using Proposition 2.3, for any sequence x^m, $m \in N$, of $\overline{D_M}$ which converges to $x \in \overline{D_M}$ we have

$$\widehat{w^m}(t, x^m, w) \to \hat{w}(t, x, w), \qquad \text{uniformly in } t \in [0, T] \text{ as } m \to \infty,$$

where $\hat{w}(t, x, w) = \hat{w}(t) = \tilde{\zeta}(T-t) - \tilde{\zeta}(T) + \pi_M \varphi(T-t) - \pi_M \varphi(T)$. Then we see that $\hat{w}(t)$ is a Brownian motion on T_M^{n+1} under $Q \otimes P_W^{\otimes(n+1)}$. Since

$$\tilde{\zeta}(t) - \tilde{\zeta}(0) = \frac{1}{2} \pi_M w(t) + \frac{1}{2} (\hat{w}(T-t) - \hat{w}(T)),$$

we obtain Lemma 2.6 from Doob's inequality. □

REMARK 2.2. From Remark 2.1 we see that the distribution of the process $(\tilde{\eta}(t, x, w), \delta_{x_0} \otimes Q_{x_0} \otimes P_W^{\otimes(n+1)})$ does not depend on $x_0 \in T_M$ and coincides with that of the process $(\tilde{\eta}(t, x, w), Q \otimes P_W^{\otimes(n+1)})$.

3. Proof of Theorem 1.

In this section we give a proof of Theorem 1. Without loss of generality we can assume $r \in (0, 1/2)$. First we introduce a system of a Brownian ball colliding with finitely many Brownian particles. Let M, $n \in \boldsymbol{N}$. Given $w = (w_0, w_1, \cdots, w_n) \in W_0^{n+1}$ and $x = (x_0, x_1, \cdots, x_n) \in \boldsymbol{R}^{(n+1)d}$ with $\rho_M(x_i, x_0) \geq r$, $1 \leq i \leq n$, we consider the following equation (3.1) under the conditions (3.2), (3.3) and (3.4):

$$(3.1) \qquad \begin{cases} \xi_0^M(t) = x_0 + w_0(t) + \sum_{j=1}^{n} \sum_{z \in \boldsymbol{Z}^d} \int_0^t (\xi_0^M(s) - \xi_j^M(s) + Mz) dL_{j,z}^M(s), \\ \xi_i^M(t) = x_i + w_i(t) + \sum_{z \in \boldsymbol{Z}^d} \int_0^t (\xi_i^M(s) - \xi_0^M(s) + Mz) dL_{i,z}^M(s), \qquad 1 \leq i \leq n, \end{cases}$$

$$(3.2) \qquad \xi_i^M \in \boldsymbol{C}([0, \infty) \to \boldsymbol{R}^d), \qquad 0 \leq i \leq n,$$

$$(3.3) \qquad \rho_M(\xi_i^M(t), \xi_0^M(t)) \geq r, \qquad 1 \leq i \leq n, \quad t \in [0, \infty),$$

(3.4) $L_{i,z}^M$, $z \in \boldsymbol{Z}^d$, $1 \leq i \leq n$, are continuous nondecreasing functions with $L_{i,z}^M(0) = 0$ and

$$L_{i,z}^M(t) = \int_0^t \mathbb{I}_{\{r\}}(|\xi_i^M(s) - \xi_0^M(s) + Mz|) dL_{i,z}^M(s).$$

The following Proposition is obtained by the equivalence of the Skorohod equation (2.4) for D_M and the equation (3.1), which can be shown by the same procedure as that of the proof of Theorem 4.1 in [5].

PROPOSITION 3.1. *For each $n \in \boldsymbol{N}$ and $M \in \boldsymbol{N}$ the equation* (3.1) *has a unique solution.*

We denote the unique solution of the equation (3.1) for x and w by $\xi^M(t, x, w) = (\xi_i^M(t, x, w), 0 \leq i \leq n)$, $L^M(t, x, w) = (L_{i,z}^M(t, x, w), 1 \leq i \leq n, z \in \boldsymbol{Z}^d)$.

For $x=(x_0, x_1, \cdots)\in X$ and $w=(w_0, w_1, \cdots)\in W$ we put

$$\{i_0, i_1, \cdots, i_n\}=\{i\in N: x_i-x_0\in K_M\}, \qquad 0=i_0<i_1<\cdots<i_n.$$

We also put $x_M=(x_{i_0}, x_{i_1}, \cdots, x_{i_n})$, $w_M=(w_{i_0}, w_{i_1}, \cdots, w_{i_n})$ and

$$\xi_i^M(t, x, w)=\begin{cases}\xi_k^M(t, x_M, w_M), & \text{if } i=i_k \text{ for some } k\in\{0, 1, \cdots, n\},\\ x_i+w_i(t), & \text{otherwise},\end{cases}$$

$$L_i^M(t, x, w)=\begin{cases}\sum_{z\in Z^d} L_{k,z}^M(t, x_M, w_M), & \text{if } i=i_k \text{ for some } k\in\{1, 2, \cdots, n\},\\ 0, & \text{otherwise}.\end{cases}$$

For $T>0$ and $M\in N$ we introduce measurable subsets $\Lambda_1(M)$, $\Lambda_2(M)$, $\Lambda_3(M)$ and $\Lambda(M)$ of $X\times W$ by

$$\Lambda_1(M)=\{(x, w): \exists i\in N, \exists t\in[0, T] \text{ s.t. } x_i-x_0\in K_M, x_i-x_0+w_i(t)\notin K_{3M/2}\},$$

$$\Lambda_2(M)=\{(x, w): \exists i\in N, \exists t\in[0, T] \text{ s.t. } x_i-x_0\notin K_M, x_i-x_0+w_i(t)\in K_{M/2}\},$$

$$\Lambda_3(M)=\{(x, w): \sup_{t\in[0, T]} |\xi_0^M(t, x, w)-x_0|>M/8\},$$

$$\Lambda(M)=\Lambda_1(M)\cup\Lambda_2(M)\cup\Lambda_3(M).$$

REMARK 3.1. For any $(x, w)\in\Lambda(M)^c$ and any $t\in[0, T]$ we have

$$\min_{z\in Z^d\setminus\{0\}} |\xi_0^M(t, x, w)-\xi_i^M(t, x, w)+Mz|>r, \qquad i\in\{i_1, \cdots, i_n\},$$

$$|\xi_0^M(t, x, w)-\xi_i^M(t, x, w)|>r, \qquad i\notin\{i_1, \cdots, i_n\},$$

and so $(\xi^M(t, x, w), L^M(t, x, w))$, $t\in[0, T]$ is the unique solution of (1.1).

LEMMA 3.2.

$$\sum_{M=1}^{\infty} P(\Lambda(M))<\infty.$$

PROOF. First put $\Delta(M)=\{(x, w): \sharp(\Gamma x\cap K_M)>\exp(M)\}$. Since $\Gamma\hat{\mu}=\mu_\lambda$, by Chebychev's inequality we have

$$P(\Delta(M))\leq\lambda M^d\exp(-M).$$

Using Doob's inequality for the submartingale $(\exp(8|w(t)|), P_W)$, we obtain

$$\begin{aligned}P(\Lambda_1(M))&\leq P(\Delta(M))+P(\Lambda_1(M)\setminus\Delta(M))\\ &\leq\lambda M^d\exp(-M)+\exp(M)P_W(\sup_{t\in[0, T]}|w(t)|>M/4)\\ &\leq\exp(-M)\{\lambda M^d+c(T)\},\end{aligned}$$

where $c(T)=\int_{W_0}\exp(8|w(T)|)P_W(dw)$. Thus,

$$\sum_{M=1}^{\infty} \boldsymbol{P}(\Lambda_1(M))<\infty . \tag{3.5}$$

We introduce the measurable sets $\Lambda_{2,k}(M)$, $k\in\boldsymbol{N}$ defined by

$$\Lambda_{2,k}(M)=\{(\boldsymbol{x}, \boldsymbol{w}) : \exists i\in\boldsymbol{N}, \exists t\in[0, T] \text{ s.t. } x_i-x_0\in K_{M+k+1}\setminus K_{M+k}\,,\ x_i-x_0+w_i(t)\in K_{M/2}\} .$$

Then we have

$$\begin{aligned}\boldsymbol{P}(\Lambda_2(M))&\leq\sum_{k=0}^{\infty}\boldsymbol{P}(\Lambda_{2,k}(M))\\&\leq\sum_{k=0}^{\infty}\left\{\boldsymbol{P}(\Delta(M+k))+\exp(M+k)P_W\left(\sup_{t\in[0,T]}|w(t)|>\frac{1}{4}(M+2k)\right)\right\}\\&\leq\sum_{k=0}^{\infty}\exp(-(M+k))\{\lambda(M+k)^d+\exp(-2k)c(T)\}\,,\end{aligned}$$

which implies

$$\sum_{M=1}^{\infty}\boldsymbol{P}(\Lambda_2(M))<\infty . \tag{3.6}$$

We denote by $\mu_{\lambda,M}$ a Poisson distribution on $K_M\setminus U_r(0)$ with intensity measure λdx. Noting that the distribution of $\Gamma(\boldsymbol{x}_M)$ under $\hat{\mu}$ is $\mu_{\lambda,M}$, by the equivalence of the equations (2.4) and (3.1) we obtain

$$\begin{aligned}\boldsymbol{P}(\Lambda_3(M))&=\boldsymbol{P}(\sup_{t\in[0,T]}|\xi_0^M(t, \boldsymbol{x}_M, \boldsymbol{w}_M)-x_0|>M/8)\\&=\int_{\boldsymbol{R}^d}\widehat{\mu_0}(dx_0)\exp(-\lambda|K_M\setminus U_r(0)|)\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\\&\quad\cdot\int_{(K_M\setminus U_r(0)+x_0)^n}dx_1\cdots dx_n P_W^{\otimes(n+1)}(\sup_{t\in[0,T]}|\zeta_0(t, x, w)-x_0|>M/8)\,,\end{aligned} \tag{3.7}$$

where $\widehat{\mu_0}(dy)=\hat{\mu}(x_0\in dy)$. Using Remark 2.1 and Lemma 2.6, we have

$$\begin{aligned}&\int_{(K_M\setminus U_r(0)+x_0)^n}dx_1\cdots dx_n P_W^{\otimes(n+1)}(\sup_{t\in[0,T]}|\zeta_0(t, x, w)-x_0|>M/8)\\&=\int_{(T_M\setminus U_r(0))^n}dx_1\cdots dx_n P_W^{\otimes(n+1)}(\sup_{t\in[0,T]}|\tilde{\zeta}_0(t, x, w)-x_0|>M/8)\\&=|T_M\setminus U_r(0)|^n Q\otimes P_W^{\otimes(n+1)}(\sup_{t\in[0,T]}|\tilde{\zeta}_0(t)-\tilde{\zeta}_0(0)|>M/8)\end{aligned} \tag{3.8}$$

$$\leq |T_M \setminus U_r(0)|^n C \exp(-M/8) .$$

Combining (3.7) and (3.8), we obtain

$$P(\Lambda_3(M)) \leq C \exp(-M/8)$$

and

$$\sum_{M=1}^{\infty} P(\Lambda_3(M)) < \infty . \tag{3.9}$$

Thus, Lemma 3.2 is derived from (3.5), (3.6) and (3.9). □

PROOF OF THEOREM 1. Using Borel Cantelli's lemma and Lemma 3.2, we obtain

$$P\left(\bigcup_{m \geq 1} \bigcap_{M \geq m} \Lambda(M)^c\right) = 1 .$$

Thus, for almost all (x, w) with respect to P there exists $M_0 \in N$ such that $(x, w) \in \Lambda(M)^c$, $M \geq M_0$, and so

$$(\xi(t, x, w), L(t, x, w)) = \lim_{M \to \infty} (\xi^M(t, x, w), L^M(t, x, w))$$

$$= (\xi^{M_0}(t, x, w), L^{M_0}(t, x, w)) , \qquad t \in [0, T] .$$

Thus, we obtain the first assertion of Theorem 1 from Remark 3.1.

Since the distribution of Γx_M is $\mu_{\lambda,M}$ under $\hat{\mu}$, by Corollary 2.5 and Remark 2.2 we see that under P the process

$$\widetilde{\eta^M}(t, x, w) = \{\pi_M \xi_i^M(t, x, w) - \pi_M \xi_0^M(t, x, w) , \ i \in \{i_1, \cdots, i_n\}\}$$

is a stationary process whose stationary measure is a Poisson distribution on $T_M \setminus U_r(0)$ with intensity measure λdx. Since

$$\Gamma \xi(t, x, w) \cap K_{M/8-r} = \widetilde{\eta^M}(t, x, w) \cap K_{M/8-r}$$

for $(x, w) \in \Lambda(M)^c$, $t \in [0, T]$, we see that the distribution of $\widetilde{\eta^M}(t, x, w)$ vaguely converges to $\Gamma\hat{\mu}$ as $M \to \infty$. Therefore, the distribution of $\Gamma\xi(t)$ is μ_λ. □

ACKNOWLEDGEMENT. The authors would like to express their thanks to Professor H. Tanaka for helping them with valuable suggestion and constant encouragement.

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