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Correction to: A Characterization of the Poisson Kernel Associated with $SU(1, n)$

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In Corollary 6 (iii), which appears in page 45 of the paper above, the numerator in the right hand side of the second equation should be equal to 4 instead of 2. As a consequence the numerators $2(2n^2-9n+1)$ and $4n(6n^2+5n-5)$ in (21) must be changed into $-2(n^2+3n+2)$ and $8n(3n^2+n-1)$ respectively, and then the equation below (24) into $A=\bar{A}$. Therefore, the argument in p. 51 that deduces (8d) collapses. We replace it as follows.

Let F be a real valued, C^2 function on G/K satisfying $F(0)=1$ and (2a), (2b), (2c) in Lemma 1. We here put $[F](g)=\int_M f(mg)dm$ $(g\in G)$ and $R=F-[F]$. Then $[F]$ satisfies $[F](0)=1$, (2a) and (2c), and R satisfies $R(0)=0$, (2a) and $(\partial R/\partial\zeta_i)(0)=0$ $(1\le i\le n)$. Especially, if we denote by $[F]=\sum_{N=0}^{\infty}[F]_N$ (resp. $R=\sum_{N=0}^{\infty}R_N$) a homogeneous expansion of $[F]$ (resp. R) with respect to $\zeta_1, \bar{\zeta}_1, \zeta_2, \bar{\zeta}_2, \cdots, \zeta_n, \bar{\zeta}_n$, we see that

(1a) $$[F]_0=1\,, \qquad [F]_1=n(\zeta_1+\bar{\zeta}_1)\,,$$

(1b) $$R_0=R_1=0\,.$$

Since $P(\zeta)=P(\zeta, e_1)$ and $[F]$ are M-invariant eigenfunctions of D, it follows from Proposition 7 that they have expansions of the forms:

(2a) $$P(\zeta)=\sum_{p,q\ge 0}P_{pq}(r)\phi_{pq}(\dot{\zeta})=\sum_{p,q\ge 0}Q^0_{pq}(r)\zeta_1^p\bar{\zeta}_1^q\,,$$

(2b) $$[F](\zeta)=\sum_{p,q\ge 0}C_{pq}P_{pq}(r)\phi_{pq}(\dot{\zeta})=\sum_{p,q\ge 0}Q_{pq}(r)\zeta_1^p\bar{\zeta}_1^q\,,$$

where $r^2=|\zeta|^2$, $\dot{\zeta}=\zeta/r$, $C_{pq}\in \boldsymbol{C}$ and ϕ_{pq} is a spherical harmonic on K/M (see [1], p. 144). Since $\phi_{00}(\zeta)=1$, $\phi_{10}(\zeta)=\zeta_1$ and $\phi_{01}(\zeta)=\bar{\zeta}_1$, it follows from (1a) that

(3a) $$Q_{00}=Q^0_{00}=(1-r^2)^n\,,$$

(3b) $$Q_{10}=Q_{01}=Q^0_{10}=Q^0_{01}=(1-r^2)^n n\,.$$

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Moreover, comparing with the coefficient of $1=\zeta_1^0\bar{\zeta}_1^0$ in $D[F]=0$, we see from (3a) that

(4) $$Q_{11}=Q_{11}^0=(1-r^2)^n n^2 .$$

Therefore, noting the relations among coordinates in p. 41, we can deduce that $[F]$ is of the form:

(5) $$[F]=1+n(\xi+\bar{\xi})+\alpha\xi^2+\bar{\alpha}\bar{\xi}^2+n^2|\xi|^2-nr^2+\cdots.$$

We next substitute $F=[F]+R$ for (2b) in Lemma 1:

(6) $$8n^2([F]^2+2[F]R+R^2)=|\nabla|^2([F])+2\nabla([F], R)+|\nabla|^2(R),$$

where $\nabla(f, g)=\Delta(fg)-\Delta(f)g-f(\Delta g)$ and $|\nabla|^2(f)=\nabla(f^2)$. Since $[\nabla([F], R)]=[\Delta([F]R)]=\Delta([F][R])=0$, the average of (6) over M is given by

(7) $$8n^2([F]^2+[R^2])=|\nabla|^2([F])+[|\nabla|^2(R)] .$$

Then, comparing with the homogeneous polynomials of degree 2 in (7), we see from (1b) that

(8) $$8\left[\sum_{i=1}^{n}\left|\frac{\partial R_2}{\partial\zeta_i}\right|^2\right]=\text{the homogeneous polynomial of degree 2}$$
$$\text{in } 8n^2[F]^2-|\nabla|^2([F]) .$$

We here let $\zeta=\zeta_0=(0, \zeta_2, \cdots, \zeta_n)$. Then (3) implies that

(9) $$\left[\sum_{i=1}^{n}\left|\frac{\partial R_2}{\partial\zeta_i}\right|^2(\zeta_0)\right]=0 .$$

This means that $\partial R_2/\partial\zeta_i=\partial R_2/\partial\bar{\zeta}_i=0$ $(2\le i\le n)$, so R_2 is a function of ζ_1 and $\bar{\zeta}_1$. Since $[R]=0$, we can deduce that

(10) $$R_2=0 .$$

Then, it follows from (1b) and (10) that $F_i=[F]_i$ $(i=0, 1, 2)$ and thus, F is of the same form as (5). Therefore, noting the relations among coordinates in p. 41, we see that $G=e^{-2n\tau}F$ is of the form:

(11) $$G=1+a\xi^2+\bar{a}\bar{\xi}^2+\cdots .$$

Therefore, in $H_2(\xi, z)$ (see p. 49) $B=D_i=0$ $(2\le i\le n)$. Then it follows from (15) and (16) that $B=\mathrm{Re}(A)=0$, so we recover (8d).

The idea used in this correction can be generalized to the case of $Sp(n, 1)$ (see [2]).

References

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[2] T. Kawazoe, A characterization of the Poisson kernel on the classical rank one symmetric spaces, Tokyo J. Math. **15** (1992), 365–379.

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