

## Erratum to “On a Degenerate Quasilinear Elliptic Equation with Mixed Boundary Conditions”

(Tokyo Journal of Mathematics, Vol. 10 (1987), pp. 437–470)

Kazuya HAYASIDA

*Kanazawa University*

The paper [1] was in collaboration with Prof. Yasuhiko Kawai but he died at 1998. I alone write this erratum.

The proof of Lemma 5.3 is incorrect. So the equality (6.1) does not hold. We have assumed that  $\partial\Omega$  and  $S$  are of class  $C^\omega$  in Theorems 2 and 3. But we remove this assumption and replace it with the following weaker one:  $\partial\Omega$  and  $S$  are of class  $C^2$ . This assumption has been set in our recent work [2], where the similar result was obtained for the equation of linear elastostatics with mixed boundary condition, concerning the regularity property of solutions.

Under the above new assumption we revise the beginning of Section 6 in [1] as follows:

Let  $x'(x)$  be the original (new) coordinate, respectively, which are connected with the mapping  $x = \Psi(x')$  in a neighborhood of the origin. More precisely  $\Psi$  and  $\Psi^{-1}$  are of class  $C^2$ , satisfying  $\Psi(O) = O$ ,  $\Psi(U \cap S) \subset \{x_{n-1} = x_n = 0\}$ ,  $\Psi(U \cap \Omega) \subset \{x_n > 0\}$ ,  $\Psi(U \cap \partial_1\Omega) \subset \{x_{n-1} > 0, x_n = 0\}$  and  $\Psi(U \cap \partial_2\Omega) \subset \{x_{n-1} < 0, x_n = 0\}$ .

The other notations are the same as in [1]. Under the above condition the equation (0.3) becomes

$$(|Eu|^{p-2}Eu, Ev \cdot J)_\Sigma + (|u|^\alpha u, vJ)_\Sigma = (f, vJ)_\Sigma, \quad v \in V(\Sigma')$$

in place of (6.2), where  $Eu = (\frac{\partial x_k}{\partial x'_1} \partial_{x_k} u, \dots, \frac{\partial x_k}{\partial x'_n} \partial_{x_k} u)$  and  $J = |D(x'_1, \dots, x'_n)/D(x_1, \dots, x_n)|$  ( $> 0$ ).

Accordingly (6.3) should be revised as follows:

$$(|Eu|^{p-2}Eu, EP_h(\zeta^2 w) \cdot J) + (|u|^\alpha u, P_h(\zeta^2 w) \cdot J) = (f, P_h(\zeta^2 w) \cdot J).$$

Replacing  $\nabla u$  in Section 6 with  $Eu$  newly, we proceed in parallel with the original proof. Then in place of (6.12) we have

$$(6.12') \quad \int_\Sigma \zeta^2 (|S_h Eu|^{p-2} + |Eu|^{p-2}) |P_h Eu|^2 dx \leq CA.$$

Since the mapping  $\Psi$  is non-singular, it holds that for some positive constant  $c_0$

$$|Eu| \geq c_0 |\nabla u| \quad \text{and} \quad |S_h Eu| \geq c_0 |S_h \nabla u|.$$

On the other hand we see that

$$\begin{aligned} P_h Eu &= \left( S_h \left( \frac{\partial x_k}{\partial x'_1} \right) P_h(\partial_{x_k}), \dots, S_h \left( \frac{\partial x_k}{\partial x'_n} \right) P_h(\partial_{x_k}) \right) \\ &\quad + \left( P_h \left( \frac{\partial x_k}{\partial x'_1} \right) \partial_{x_k} u, \dots, P_h \left( \frac{\partial x_k}{\partial x'_n} \right) \partial_{x_k} u \right). \end{aligned}$$

Thus

$$|P_h(\nabla u)| \leq C(|P_h Eu| + |\nabla u|).$$

From the above we obtain

$$\begin{aligned} \int_{\Sigma} \zeta^2 (|S_h \nabla u|^{p-2} + |\nabla u|^{p-2}) |P_h \nabla u|^2 dx &\leq C \left[ \int_{\Sigma} \zeta^2 (|S_h Eu|^{p-2} + |Eu|^{p-2}) |P_h Eu|^2 dx \right. \\ &\quad \left. + \int_{\Sigma} \zeta^2 (|S_h Eu|^{p-2} + |Eu|^{p-2}) |\nabla u|^2 dx \right]. \end{aligned}$$

We apply (6.12') to the first integral on the right-hand side. By Cauchy-Hölder's inequality the second integral is estimated from above by

$$\int_{\Sigma} \zeta^2 (|S_h Eu|^p + |Eu|^p + |\nabla u|^p) dx.$$

This is equivalent to  $\int_{\Sigma} \zeta^2 |\nabla u|^p dx$ . On the other hand from (2.6)

$$\int_{\Sigma} \zeta^2 |\nabla u|^p dx \leq C(\|f\|_{p^*})^{p^*}.$$

Therefore Theorems 2 and 3 hold under the new assumption.

### References

- [ 1 ] K. HAYASIDA and Y. KAWAI, On a degenerate quasilinear elliptic equation with mixed boundary conditions, Tokyo J. Math. **10** (1987), 437-470.
- [ 2 ] K. HAYASIDA and K. WADA, On the regularity property for solutions of the equation of linear elastostatics with discontinuous boundary condition, Japan J. Indust. Appl. Math. **16** (1999), 377-399.

*Present Address:*

FUKUI UNIVERSITY OF TECHNOLOGY,  
GAKUEN, FUKUI, 910-8505 JAPAN.