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Correction to: "Existence and Regularity Results for Harmonic Maps with Potential"

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In the proof of Theorem 2.3 of the above mentioned paper, on p. 200, the author gave the following estimate.

$$\begin{aligned}\int_\Omega e(u)d\mu &\le \int_\Omega G(u)d\mu + E_G(f)\\ &\cdots\cdots\\ &\le E_G(f) + b_0\mathrm{vol}(\Omega) + b_1\int_\Omega \{\varepsilon|u|^{2^*} + \varepsilon^{-\frac{\gamma}{2^*-\gamma}}\}d\mu\\ &\le c_3(E_G(f), \Omega, g, \varepsilon, \gamma, b_0, b_1) + \varepsilon c_4(\Omega, g, h, b_1)\int_\Omega e(u)d\mu\,.\end{aligned}$$

However, the last inequality is not correct. In the last term c_4 depends on $\|u\|_{L^\infty}$ also. Therefore the remaining part of the proof is not valid. We must treat the term $\int |u|^{2^*}d\mu$ more carefully. Moreover, for the case that $m = 2$, since $2^* = +\infty$, some small changes are necessary. From the 14th line of page 200, the proof should be changed as follows.

Now, let us estimate the right hand side of (2.18). We proceed as if we are assuming that $m = 3$ or 4, however, by replacing 2^* with a sufficiently large number, the proof will be valid also for $m = 2$.

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Since we are assuming (2.4) and that $\|u\|_{L^\infty} \le R$, the minimality of u implies that

$$\begin{aligned}\int_\Omega e(u)d\mu &\le \int_\Omega G(u)d\mu + E_G(f)\\ &\le E_G(f) + b_0\mathrm{vol}(\Omega) + b_1\int_\Omega |u|^\gamma d\mu\\ &\le E_G(f) + b_0\mathrm{vol}(\Omega) + b_1\int_\Omega \{\varepsilon|u|^{2^*} + \varepsilon^{-\frac{\gamma}{2^*-\gamma}}\}d\mu\\ &\le c_3(E_G(f),\Omega,b_0) + b_1\mathrm{vol}(\Omega)\varepsilon^{-\frac{\gamma}{2^*-\gamma}} + \varepsilon c_4(\Omega,g,h,b_1)R^{2^*-2}\int_\Omega e(u)d\mu\,.\end{aligned}$$

Here, we used Young's inequality and the Poincaré inequality. By choosing $\varepsilon = 1/2c_4R^{2^*-2}$, we get the following a-priori estimate:

$$\int_\Omega |Du|^2dx \le c_5(g,h,\gamma,b_0,b_1,\Omega,E_G(f))(1+R^{\frac{2^*-2}{2^*-\gamma}\gamma}). \tag{2.19}$$

Using the Poincaré inequality and the assumption that $m \le 4$, from (2.19) we get

$$\begin{aligned}\|u\|_{L^4} &\le c_6\|u\|_{L^{2^*}} \le c_6R^{\frac{2^*-2}{2^*}}\|u\|_{L^2}^{\frac{2}{2^*}} \le c_6'R^{\frac{2^*-2}{2^*}}\|Du\|_{L^2}^{\frac{2}{2^*}}\\ &\le c_6'c_5^{\frac{1}{2^*}}R^{\frac{2^*-2}{2^*}}(1+R^{\frac{2^*-2}{2^*-\gamma}\gamma})^{\frac{1}{2^*}} \le c_6'c_5^{\frac{1}{2^*}}(1+R^{\frac{2^*-2}{2^*-\gamma}})\,.\end{aligned} \tag{2.20}$$

where c_6 is a positive constant depending only on m and Ω. It is nothing to see that c_5 satisfies

$$\lim_{b_0,b_1,E_G(f)\to 0} c_5 = 0\,. \tag{2.21}$$

On the other hand, using the condition (2.5), we see that

$$\left\| |u|\frac{\partial G}{\partial s}\right\|_{L^q} \le c_7(b_2,b_3,q,\Omega)\,\|u\|_{L^{2^*}} \quad \text{for} \quad q = \min\{2^*, 2^*/\gamma\} > m/2\,.$$

Thus, if $m \le 4$ and (2.5) holds, we obtain from (2.18)

$$\sup_\Omega |u|^2 \le c_2\{(c_6'+c_7)c_5^{\frac{1}{2^*}}(1+R^{\frac{2^*-2}{2^*-\gamma}}) + \|f\|_{L^{2q}}\} + \sup_\Omega |f|^2\,. \tag{2.22}$$

Now, from (2.21) and (2.22), we can see that if $b_0, b_1, b_2, b_3, E_G(f)$ and $\|f\|_{L^\infty(\Omega)}$ are sufficiently small we have (2.12).

When we can take $R_0 = +\infty$, for any given b_0, b_1, b_2, b_3 and f we can choose R sufficiently large so that R^2 is greater than the right hand side of (2.22). It is possible since we get

$$\frac{2^*-2}{2^*-\gamma} < 2 \tag{2.23}$$

from the assumption $\gamma < 4/(m-2)$ in (2.5).

For the case $m = 2$, for any γ we can proceed as in the above proof by replacing 2^* by a sufficiently large constant for which (2.23) holds. □

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