65. *Nonexistence Theorem of Expansive Flows on Certain 3-manifolds*

By Takashi INABA*) and Shigenori MATSUMOTO**)

(Communicated by Kunihiko KODAIRA, M. J. A., Sept. 12, 1989)

§1. **Statement of the result.** This is a research announcement. Details including a full proof will appear elsewhere.

Expansive homeomorphisms have been studied for some time by various authors including Mañé [6] and Hiraide [4]. In particular Hiraide [4] showed that every expansive homeomorphism on a closed surface is topologically conjugate to a pseudo-Anosov diffeomorphism (including Anosov). Specifically this implies that there are no expansive homeomorphisms on S^2.

The present report deals with a nonsingular expansive flow. Recall that a nonsingular continuous flow φ on a metric space X is said to be *expansive* if it satisfies the following property: for every $\varepsilon>0$, there exists $\delta>0$ such that if for $x, y \in X$ and for an increasing homeomorphism $h: \boldsymbol{R} \to \boldsymbol{R}$ with $h(0)=0$ one has $d(\varphi_t(y), \varphi_{h(t)}(x))<\delta$ for all $t \in \boldsymbol{R}$, then $y=\varphi_t(x)$ for some $|t|<\varepsilon$.

Fundamental properties of expansive flows are studied in [1] and [5]. Especially some equivalent definitions are found in [1].

In this report, we are concerned exclusively with a nonsingular expansive flow on a closed oriented 3-manifold.

Our main result is:

Theorem 1. *There are no nonsingular expansive flows on a closed oriented non-aspherical* 3-*manifold.*

Recall that reducible 3-manifolds as well as 3-manifolds with finite fundamental groups are non-aspherical. Thus in particular S^3 does not admit nonsingular expansive flows.

§2. **Outline of the proof.** Let φ be a nonsingular expansive flow on a closed 3-manifold M^3. For $x \in M^3$ and for $\varepsilon>0$, define the ε-*stable set* $W^s_\varepsilon(x)$ to be the set of points $y \in M^3$ such that $d(\varphi_t(x), \varphi_{h(t)}(y))<\varepsilon$ for every $t>0$ and for some increasing homeomorphism $h: \boldsymbol{R} \to \boldsymbol{R}$ with $h(0)=0$.

Oka [8] applied methods originated by Hiraide [4] to investigate the topological nature of the intersection of $W^s_\varepsilon(x)$ with a local cross section ([5]). To state his result in a form suited for our purpose, we need the following definition.

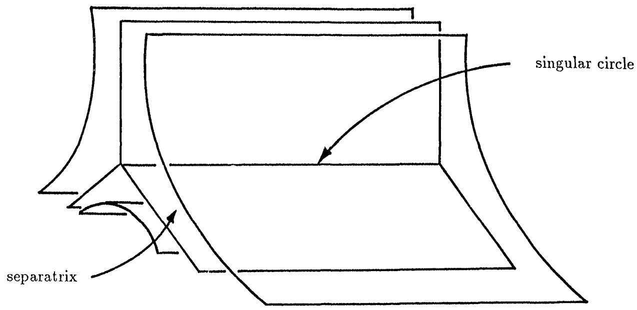
A codimension one singular C^0 foliation $\mathcal{F}$ on a manifold M is called a

---

*) College of Arts and Sciences, Chiba University.

**) Department of Mathematics, College of Science and Technology, Nihon University.

foliation with circle prong singularities if it admits no singularities other than those depicted in the following picture.



Singularities are to consist of a finite union of embedded circles. A leaf which ends at a singular circle is called a *separatrix*. When intersected with a transverse disk, a singular circle is assumed to induce a singular point with at least 3 separatrices on the disk. Because of a possible twisting along a singular circle, however, the number of its separatrices may be smaller in global. $\mathcal{F}$ yields a decomposition of M by *extended leaves*, given by viewing a singular circle and its separatrices belonging to the same extended leaf.

The following theorem is essentially due to Oka [8].

Theorem 2. *Let φ be a nonsingular flow on a closed oriented 3-manifold M^3 and let $\varepsilon>0$. Let $\approx$ be the equivalence relation generated by the relation $\sim$: $x \sim y$ if and only if $x, y \in W^s_\varepsilon(z)$ for some $z \in M^3$. For sufficiently small $\varepsilon>0$, $\approx$ gives a decomposition of M^3 by extended leaves of a certain foliation $\mathcal{F}$ with circle prong singularities. Furthermore $\mathcal{F}$ satisfies:*

1) *Every separatrix is homeomorphic to an open cylinder and exactly one of the two ends terminates at a singular circle.****)

2) *$\mathcal{F}$ has no compact leaf.*

Now the main step of the proof of Theorem 1 is the following generalization of Novikov's compact leaf theorem.

Theorem 3. *A closed oriented non-aspherical 3-manifold does not admit a foliation with circle prong singularities which satisfies* 1) *and* 2) *of Theorem* 2.

Although there appears to exist little difficulty in the generalization, one must take into account the fact the $\mathcal{F}$ may not be transversely oriented. Thus arguments based upon the Poincaré-Bendixson theorem must be carefully avoided. Other than this, however, the proof goes along the same line of the well-known proof of Novikov's theorem ([2], [7]). Notice that it is known to hold for C^0 foliations thanks to a C^0 general position argument ([3], [9]).

***) That is, in a usual terminology, there are no *connections* between singular circles.

References

[1] R. Bowen and P. Walters: Expansive one-parameter flows. J. Diff. Eq., **12**, 180–193 (1972).
[2] A. Haefliger: Travaux de Novikov sur les feuilletages. Seminare Bourbaki, **339** (1967/68).
[3] G. Hector and U. Hirsch: Introduction to the geometry of foliations. part B. Vieweg, Braunschweig (1983).
[4] K. Hiraide: Expansive homeomorphisms on compact surfaces (Tsukuba Univ. preprint).
[5] H. B. Keynes and M. Sears: Real-expansive flows and topological dimension. Ergod. Th. and Dynam. Sys., **1**, 179–195 (1981).
[6] R. Mañé: Expansive diffeomorphisms. Lect. Notes in Math., vol. 468, Springer-Verlag, pp. 162–174 (1975).
[7] S. P. Novikov: Topology of foliations. Trudy Moskov. Mat. Obshch., **14**, 248–278 (1965).
[8] M. Oka: Expansive flows on 3-manifolds have Markov families (in preparation).
[9] V. V. Solodov: Components of topological foliations. Math. Sb., **119**, 340–354 (1982).