129. *There are no Transitive Anosov Diffeomorphisms on Negatively Curved Manifolds**)

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Let $f: M \to M$ be an Anosov diffeomorphism. (See [4] for the definition.) We call f *transitive* if $\Omega(f)=M$. The aim of this note is to prove the following

Theorem.**) *Let M be a closed manifold which admits a Riemannian metric of negative sectional curvature. Then there are no transitive Anosov diffeomorphisms on M.*

Proof. It suffices to prove the theorem in the case that $\dim M \geqq 3$. Assume the contrary, i.e., there exists a transitive Anosov diffeomorphism $f: M \to M$. If necessary, taking a finite covering of M, we may assume that the stable and the unstable foliations of f are orientable. We may also assume that the dimension u of the unstable foliation is greater than one. Then, Ruelle-Sullivan [3] says that there are a $\lambda > 1$ and an $\alpha \neq 0$ in $H_u(M; \boldsymbol{R})$ such that $f_*(\alpha) = \pm \lambda \alpha$. Taking the value of Gromov's pseudo-norm Γ (for the definition, see [1] or [2]), we get $\lambda \Gamma(\alpha) = \Gamma(f_* \alpha) \leqq \Gamma(\alpha)$ and thus $\Gamma(\alpha) = 0$. This contradicts a result obtained by Gromov [1] and Inoue-Yano [2] and thus the proof completes.

References

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