21. The Groups $J_G(*)$ for Compact Abelian Topological Groups G*)

By Katsuo KAWAKUBO
Universität Bonn and Osaka University

(Communicated by Kunihiko KODAIRA, M. J. A., March 13, 1978)

§1. **Introduction.** In [4], we defined $J_G(X)$ for a compact group G and for a compact G-space X. When X is a point, we denote it by $J_G(*)$. Similar groups $JO(G)$ were defined and studied by Atiyah and Tall [2], Snaith [7], and Lee and Wasserman [5]. Our definition is more rigid than those of $JO(G)$ in [2], [5], [7] and is given from the geometrical point of view as follows.

Two orthogonal representation spaces V, W of a compact topological group G are said to be J-equivalent if there exist an orthogonal representation space U and a G-homotopy equivalence $f: S(V\oplus U) \to S(W\oplus U)$ where $S(V\oplus U)$ and $S(W\oplus U)$ denote the unit spheres in $V\oplus U$ and $W\oplus U$ respectively. Then the group $J_G(*)$ is defined as the quotient of the orthogonal representation ring $RO(G)$ by the the subgroup

$$T_G(*)=\{V-W \mid V \text{ is } J\text{-equivalent to } W\}.$$

The natural epimorphism $RO(G)\to J_G(*)$ is also denoted by J_G.

The purpose of the present paper is to announce the group structure of $J_G(*)$ for G an arbitrary compact abelian topological group (Theorem 1).

In a forthcoming paper, we shall study $J_G(*)$ for G an arbitrary p-group.

The full exposition and proofs will also appear later.

§2. **The groups $J'_{Z_n}(*)$.** Let n be an integer greater than one and $n=2^k\cdot p_1^{r(1)}\cdots p_t^{r(t)}$ be the prime decomposition of n. Denote by Z_n the cyclic group Z/nZ of order n. Then we define a group $J'_{Z_n}(*)$ as follows.

Case 1. $k\geqq 2$. We set

$$J'_{Z_n}(*)=Z\oplus Z_{2^{k-2}}\oplus\bigoplus_{i=1}^{t} Z_{(p_i^{r(i)}-p_i^{r(i)-1})}.$$

Case 2. $k=0$ or 1. we set

$$J'_{Z_n}(*)=Z\oplus\left\{\bigoplus_{i=1}^{t} Z_{(p_i^{r(i)}-p_i^{r(i)-1})}\right\}/Z_2$$

where the inclusion of Z_2 into

*) Research partially supported by S. F. B. grant Bonn and Fūjukai grant.

$$\bigoplus_{i=1}^{t} Z_{(p_i^{r(i)}-p_i^{r(i)-1})} \quad \text{is given by} \quad 1 \mapsto \bigoplus_{i=1}^{t} \frac{p_i^{r(i)}-p_i^{r(i)-1}}{2}.$$

§ 3. The groups $J_G(*)$. Let G be a compact abelian topological group and F_0 (resp. F_1) be the family of all closed subgroups H ($\neq G$) of G such that G/H is isomorphic to the circle group S^1 (resp. a finite cyclic group). Then we have

Theorem 1. *We have the following isomorphism*

$$J_G(*) \cong Z \oplus Z(F_0) \oplus \bigoplus_{H \in F_1} J'_{G/H}(*)$$

where $Z(F_0)$ denotes the free abelian group generated by F_0 and $J'_{G/H}(*)$ is the group given in § 2.

Corollary 2. *Let V, W be orthogonal G-representation spaces. Then $S(V)$ is G-homotopy equivalent to $S(W)$ if and only if V is J-equivalent to W.*

§ 4. Normal representations of the fixed point sets of G-homotopy equivalent manifolds. Let G be a compact Lie group and M_1, M_2 be closed smooth G-manifolds. Denote by F_i^μ each component of the fixed point set of M_i ($i=1,2$). Suppose that we are given a G-homotopy equivalence $f: M_1 \to M_2$. Then the set $\{F_1^\mu\}$ of connected components is in one to one correspondence with the set $\{F_2^\mu\}$ such that $f(F_1^\mu)=F_2^\mu$. Denote by V_i^μ the normal representation of F_i^μ in M_i. Then we proved in [4] that V_1^μ is J-equivalent to V_2^μ. Therefore by combining this theorem and Corollary 2, we have

Theorem 3. *When G is a compact abelian Lie group, $S(V_1^\mu)$ and $S(V_2^\mu)$ are G-homotopy equivalent.*

§ 5. Equivariant Adams conjecture. In this section, we consider as a special case of § 3 abelian p-group actions and express $J_G(*)$ in terms of the equivariant Adams operations. Let p be an odd prime positive integer and r be a positive integer. Let G be an abelian p-group of order p^r. Denote by α a primitive root mod p^r. Let Ψ^s: $RO(G) \to RO(G)$ be the s-th Adams operation [2], [5].

Definition. Denote by $WO(G)$ the subgroup of $RO(G)$ generated by the elements

$$\{\Psi^{\alpha^i}(x) - i\Psi^{\alpha}(x) + (i-1)x\}$$

where $x \in RO(G)$, $i=2,3,\cdots,p^r-p^{r-1}$.

Then as a special case of Theorem 1, we have

Theorem 4. $J_G(*) \cong RO(G)/WO(G)$.

Remark. Theorems 1 and 4 show that $J_G(*)$ involves many torsion groups in general and the equivariant Adams conjecture does not hold in general in the form similar to the non-equivariant case [1], [3], [6]. These properties contrast with those of $JO(G)$ [2], [5], [7].

References

[1] Adams, J. F.: On the groups $J(X)$ I. Topology, **2**, 181–195 (1963).
[2] Atiyah, M. F., and Tall, D. O.: Group representations, λ-rings and the J-homomorphisms. Topology, **8**, 253–297 (1969).
[3] Becker, J. C., and Gottlieb, D. H.: The transfer map and fiber bundles. Topology, **14**, 1–12 (1975).
[4] Kawakubo, K.: Compact Lie group actions and fiber homotopy type (to appear).
[5] Lee, C. N., and Wasserman, A. G.: On the groups $JO(G)$. Memoirs of A. M. S., **159** (1975).
[6] Quillen, D.: The Adams conjecture. Topology, **10**, 67–80 (1971).
[7] Snaith, V. P.: J-equivalence of group representations. Proc. Camb. Phil. Soc., **70**, 9–14 (1971).