17. *Probability-theoretic Investigations on Inheritance. IV*$_7$. *Mother-Child Combinations*[1)]

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6. Illustration by several blood types

The general discussions developed in the preceding sections have mainly concerned genotypes. However, in cases where recessive genes may be existent, we have only to bring together the combination-probabilities with respect to the phenotypes; cf. (1.25). The probabilities on mother-child combinations for *ABO* blood type have been tabulated in § 1. For a later purpose, we shall supplement here some corresponding results on mother-child or mother-children combinations for several human blood types; *MN*, *ABO*, A_1A_2BO, *Q* and $Qq_{\pm}$ types. The frequencies of genes *M*, *N*; *A*, *B*, *O*; A_1, A_2; *Q*, *q* and q_-, q_+ are denoted, as usual, by s, t; p, q, r; p_1, p_2; u, v and v_1, v_2 respectively[2),3)].

Mother \ Child	*M*	*N*	*MN*
M	s^3	0	s^2t
N	0	t^3	st^2
MN	s^2t	st^2	st

Mother \ Child	*Q*	*q*
Q	$u(1+uv)$	uv^2
q	uv^2	v^3

1) Continued from IV$_6$. Proc. Japan Acad. **27** (1951), 615-620.
2) Concerning the mode of inheritance of $Qq_{\pm}$ types, cf. VI. Proc. Japan Acad. **28** (1952), 54-58.
3) We shall write here merely π instead of Π in (1.24).

$v=v_1+v_2$

Mother \ Child	Q	q_-	q_+
Q	$u(1+uv)$	$uv_1(v+v_2)$	uv_2^2
q_-	$uv_1(v+v_2)$	$v_1(v^2+v_1v_2)$	$v_1v_2^2$
q_+	uv_2^2	$v_1v_2^2$	v_2^3

$p=p_1+p_2$

M. \ C.	O	A_1	A_2	B	A_1B	A_2B
O	r^3	p_1r^2	p_2r^2	qr^2	0	0
A_1	p_1r^2	$\begin{cases} p_1(p_1^2+3p_1(p_2+r) \\ \quad +(p_2+r)^2) \end{cases}$	$p_1p_2(p_2+2r)$	p_1qr	$p_1q(p+r)$	p_1p_2q
A_2	p_2r^2	$p_1p_2(p_2+2r)$	$p_2(p_2^2+3p_2r+r^2)$	p_2qr	0	$p_2q(p_2+r)$
B	qr^2	p_1qr	p_2qr	$q(q^2+3qr+r^2)$	$p_1q(q+r)$	$p_2q(q+r)$
A_1B	0	$p_1q(p+r)$	0	$p_1q(q+r)$	$p_1q(p_1+q)$	p_1p_2q
A_2B	0	p_1p_2q	$p_2q(p_2+r)$	$p_2q(q+r)$	p_1p_2q	$p_2q(p_2+q)$

M. \ 1st C. \ 2nd C.	M	N	MN
M	M	$\frac{1}{2}s^3(1+s)$	0	$\frac{1}{2}s^3t$
	N	0	0	0
	MN	$\frac{1}{2}s^3t$	0	$\frac{1}{2}s^2t(1+t)$
N	M	0	0	0
	N	0	$\frac{1}{2}t^3(1+t)$	$\frac{1}{2}st^3$
	MN	0	$\frac{1}{2}st^3$	$\frac{1}{2}st^2(1+s)$
MN	M	$\frac{1}{4}s^2t(1+s)$	$\frac{1}{4}s^2t^2$	$\frac{1}{2}s^2t$
	N	$\frac{1}{4}s^2t^2$	$\frac{1}{4}st^2(1+t)$	$\frac{1}{2}st^2$
	MN	$\frac{1}{2}s^2t$	$\frac{1}{2}st^2$	$\frac{1}{2}st$

M. \ 1st C. \ 2nd C.		O	A	B	AB
O	O	$\frac{1}{2}r^3(1+r)$	$\frac{1}{2}pr^3$	$\frac{1}{2}qr^3$	0
	A	$\frac{1}{2}pr^3$	$\frac{1}{2}pr^2(1+p)$	$\frac{1}{2}pqr^2$	0
	B	$\frac{1}{2}qr^3$	$\frac{1}{2}pqr^2$	$\frac{1}{2}qr^2(1+q)$	0
	AB	0	0	0	0
A	O	$\frac{1}{4}pr^2(1+r)$	$\frac{1}{4}pr^2(1+2p+r)$	$\frac{1}{4}pqr^2$	$\frac{1}{4}pqr^2$
	A	$\frac{1}{4}pr^2(1+2p+r)$	$\frac{1}{4}p((2p^2+5pr+r^2)\times(1+p+r)+pr(1+p))$	$\frac{1}{4}pqr(2p+r)$	$\frac{1}{4}pq(2p^2+4pr+r^2)$
	B	$\frac{1}{4}pqr^2$	$\frac{1}{4}pqr(2p+r)$	$\frac{1}{4}pqr(1+q)$	$\frac{1}{4}pqr(1+q)$
	AB	$\frac{1}{4}pqr^2$	$\frac{1}{4}pq(2p^2+4pr+r^2)$	$\frac{1}{4}pqr(1+q)$	$\frac{1}{4}pq(2p+r)(1+q)$
B	O	$\frac{1}{4}qr^2(1+r)$	$\frac{1}{4}pqr^2$	$\frac{1}{4}qr^2(1+2q+r)$	$\frac{1}{4}pqr^2$
	A	$\frac{1}{4}pqr^2$	$\frac{1}{4}pqr(1+p)$	$\frac{1}{4}pqr(2q+r)$	$\frac{1}{4}pqr(1+p)$
	B	$\frac{1}{4}qr^2(1+2q+r)$	$\frac{1}{4}pqr(2q+r)$	$\frac{1}{4}q((2q^2+5qr+r^2)\times(1+q+r)+qr(1+q))$	$\frac{1}{4}pq(2q^2+4qr+r^2)$
	AB	$\frac{1}{4}pqr^2$	$\frac{1}{4}pqr(1+p)$	$\frac{1}{4}pq(2q^2+4qr+r^2)$	$\frac{1}{4}pq(2q+r)(1+p)$
AB	O	0	0	0	0
	A	0	$\frac{1}{4}pq(p+r)(1+p+r)$	$\frac{1}{4}pq(2r+pq)$	$\frac{1}{4}pq(2p+qr)$
	B	0	$\frac{1}{4}pq(2r+pq)$	$\frac{1}{4}pq(q+r)(1+q+r)$	$\frac{1}{4}pq(2q+pr)$
	AB	0	$\frac{1}{4}pq(2p+qr)$	$\frac{1}{4}pq(2q+pr)$	$\frac{1}{4}pq(p+q)(1+p+q)$

M. \ 2nd C. / 1st C.		Q	q
Q	Q	$\frac{1}{4}u(2+2u+5uv+u^2v)$	$\frac{1}{4}uv^2(2+u)$
	q	$\frac{1}{4}uv^2(2+u)$	$\frac{1}{4}uv^2(1+v)$
q	Q	$\frac{1}{2}uv^2(1+u)$	$\frac{1}{2}uv^3$
	q	$\frac{1}{2}uv^3$	$\frac{1}{2}v^3(1+v)$

$v=v_1+v_2$

M. \ 2nd C. / 1st C.		Q	q_-	q_+
Q	Q	$\frac{1}{4}u(2+2u+5uv+u^2v)$	$\frac{1}{4}uv_1(v+v_2)(2+u)$	$\frac{1}{4}uv_2^2(2+u)$
	q_-	$\frac{1}{4}uv_1(v+v_2)(2+u)$	$\frac{1}{4}uv_1(v(1+v)+v_2(1+v_1))$	$\frac{1}{4}uv_1v_2^2$
	q_+	$\frac{1}{4}uv_2^2(2+u)$	$\frac{1}{4}uv_1v_2^2$	$\frac{1}{4}uv_2^2(1+v_2)$
q_-	Q	$\frac{1}{2}uv_1(v+v_2)(1+u)$	$\frac{1}{2}uv_1(v^2+v_1v_2)$	$\frac{1}{2}uv_1v_2^2$
	q_-	$\frac{1}{2}uv_1(v^2+v_1v_2)$	$\frac{1}{4}v_1((v^2+vv_1+2v_1v_2)\times(1+v)+v_1v_2(1+v_1))$	$\frac{1}{4}v_1v_2^2(1+v+v_1)$
	q_+	$\frac{1}{2}uv_1v_2^2$	$\frac{1}{4}v_1v_2^2(1+v+v_1)$	$\frac{1}{4}v_1v_2^2(1+v_2)$
q_+	Q	$\frac{1}{2}uv_2^2(1+u)$	$\frac{1}{2}uv_1v_2^2$	$\frac{1}{2}uv_2^3$
	q_-	$\frac{1}{2}uv_1v_2^2$	$\frac{1}{2}v_1v_2^2(1+v_1)$	$\frac{1}{2}v_1v_2^3$
	q_+	$\frac{1}{2}uv_2^3$	$\frac{1}{2}v_1v_2^3$	$\frac{1}{2}v_2^3(1+v_2)$