124. A Fact, Which is Unfavorable to the Theory of General Relativity of A. Einstein

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As for the theory of special relativity of A. Einstein, except for the *author's three-dimensional Laguerre-geometrical interpretation,*[1] *which is at the same time a concrete physical interpretation,* there remains no question. As for the theory of general relativity of A. Einstein and his generalized gravitation theory of 1953,[2] their situations are quite different. In this note *a fact extremely unfavorable to the former* will be pointed out and then it will be shown that *the latter implies a self-contradiction,* being thus lead to the *actual theory* as the author's three-dimensional non-holonomic Laguerre fibre bundle geometry[5] realized in the ordinary three-dimensional Cartesian space teleparallelismically torsioned by the nascency of an (in general non-holonomic) action field caused by the charge of a particle.

1. *Preliminaries.* When a particle without charge lies in the three-dimensional Cartesian space, it may be represented by a geometrical point (x^i, $i=1,2,3$: Cartesian). But so soon as it gets charged, it emits some energy with components $\omega^l/dt=\omega^l_\mu(x^\lambda)dx^\mu/dt$, say, in unit of time, so that the ω^l are the components of the action, $l, \lambda, \mu=1,2,3,4$. Let ω^l be an orthogonal system thereby. Then the metric

$$dS^2=\omega^l\omega^l=g_{\mu\nu}dx^\mu dx^\nu,\ (\lambda, \mu, \nu, \cdots=1,2,3,4),\ |\omega^l_\mu|\neq 0$$

arises, where the dS is the resultant action and the ω^l are of invariant forms, so that hereafter the x^λ may be considered to be curvelinear coordinates. Thereby the summation convention is: $A^lB^l\equiv A^4B^4-A^iB^i$, $(i=1,2,3)$. Evidently the ω^l_μ are the covariant components of the momentum, the fourth ω^l_4 being the statical potential, when the x^4 is the time t. For the ω^l_μ arisen, we obtain the contravariant components Ω^λ_l of the momentum by the conditions:

$$\omega^l_\mu\Omega^\lambda_l=\delta^\lambda_\mu,\quad \Omega^\lambda_m\omega^l_\lambda=\delta^l_m.$$

Utilizing the Dirac matrices $\gamma_1, \gamma_2, \gamma_3$ and $\gamma_5(\gamma_h\gamma_k+\gamma_k\gamma_h=2\delta_{hk};\ h, k=1,2,3,5)$ with $\gamma_4=i\gamma_5$, we put

$$dS=\gamma_l\omega^l,\quad (dS^2=-dSdS=\omega^l\omega^l),\quad (|dS|=dS),$$

whence we obtain the following relations:

$$g_{\mu\nu}=g_{\underline{\mu\nu}}+g_{\underset{\vee}{\mu\nu}},\quad g_{\underline{\mu\nu}}=g_{\underline{\nu\mu}},\quad g_{\underset{\vee}{\mu\nu}}=-g_{\underset{\vee}{\nu\mu}},\quad g_{\underline{\mu\nu}}=\omega^l_\mu\omega^l_\nu,$$

$$g_{\underset{\vee}{\mu\nu}}=\gamma_4\gamma_1(\omega^4_\mu\omega^1_\nu-\omega^1_\mu\omega^4_\nu)+\cdots+\gamma_2\gamma_3(\omega^2_\mu\omega^3_\nu-\omega^3_\mu\omega^2_\nu)+\cdots,$$

$$g^{\underline{\mu\nu}}=\Omega^\mu_l\Omega^\nu_l,\quad \omega^l_\mu=g_{\underline{\mu\nu}}\Omega^\nu_l,\quad \Omega^\lambda_l=g^{\underline{\lambda\mu}}\omega^l_\mu.$$

The teleparallelism parameter is

$$(1) \qquad \Lambda^{\lambda}_{\mu\nu}=\Omega^{\lambda}_{l}\frac{\partial\omega^{l}_{\mu}}{\partial x^{\nu}}\equiv-\omega^{l}_{\mu}\frac{\partial\Omega^{\lambda}_{l}}{\partial x^{\nu}}$$

and we can easily prove the identity:

$$(2) \qquad \left\{ {\lambda \atop \mu\nu} \right\}\equiv\Lambda^{\lambda}_{\mu\nu}-g^{\lambda\sigma}\Delta_{\sigma\mu\nu},$$

where

$$\Delta_{\lambda\mu\nu}\equiv\begin{vmatrix} \omega^{l}_{\lambda} & \omega^{l}_{\mu} & \omega^{l}_{\nu} \\ \dfrac{\partial}{\partial x^{\lambda}} & \dfrac{\partial}{\partial x^{\mu}} & \dfrac{\partial}{\partial x^{\nu}} \\ \omega^{l}_{\lambda} & \omega^{l}_{\mu} & \omega^{l}_{\nu} \end{vmatrix}.$$

We have $\left\{ {\lambda \atop \mu\nu} \right\}=\Lambda^{\lambda}_{\mu\nu}$, when and only when $\Delta_{\lambda\mu\nu}=0$, i.e. when and only when the Pfaffians ω^{l} are all total (holonomic).

2. *The momentum vector Ω^{λ}_{l} is parallel along the path of the teleparallelism $\Lambda^{\lambda}_{\mu\nu}$.*

Proof. The identity $\Lambda^{\lambda}_{\mu\nu}\equiv-\omega^{l}_{\mu}\dfrac{\partial\Omega^{\lambda}_{l}}{\partial x^{\nu}}$ and the condition

$$\frac{\partial\Omega^{\lambda}_{l}}{\partial x^{\nu}}+\Lambda^{\lambda}_{\mu\nu}\Omega^{\mu}_{l}=0$$

are reversible. The latter means that

$$\frac{d\Omega^{\lambda}_{l}}{dS}+\Lambda^{\lambda}_{\mu\nu}\frac{dx^{\nu}}{dS}\Omega^{\mu}_{l}=0,$$

i.e. that the momentum vector Ω^{λ}_{l} is parallel along the path

$$(3) \qquad \frac{d^{2}x^{\lambda}}{dS^{2}}+\Lambda^{\lambda}_{\mu\nu}\frac{dx^{\mu}}{dS}\frac{dx^{\nu}}{dS}=0$$

of the teleparallelism.

Cor. The momentum vector is tangential to the path of the teleparallelism in three-dimension.

Later the formula (5) yields: $\dfrac{dx^{\lambda}}{dS}=a^{l}\Omega^{\lambda}_{l}$.

This physical fact characterizes the teleparallelism $\Lambda^{\lambda}_{\mu\nu}$ and makes the key for the author's new theory of relativity as a non-holonomic Laguerre fibre bundle geometry realized in the three-dimensional Cartesian space teleparallelismically torsioned by the action field caused by the charge of the particle.

3. *A fact, which is extremely unfavorable to the theory of general relativity of A. Einstein*

The momentum vector Ω^{λ}_{l} is not parallel along the geodesic curves:

$$\frac{d^{2}x^{\lambda}}{dS^{2}}+\left\{ {\lambda \atop \mu\nu} \right\}\frac{dx^{\mu}}{dS}\frac{dx^{\nu}}{dS}=0$$

based on the $g_{\mu\nu}(\delta S=0)$ in the space-time as the path of the free particle.

Proof. This fact follows immediately from (2), the Fiesecke's condition $\left\{ {\lambda \atop \mu\nu} \right\} = \Lambda^{\lambda}_{\mu\nu} + 2\delta^{\lambda}_{\mu}\psi_{\nu}$ for that $\left\{ {\lambda \atop \mu\nu} \right\}$ and $\Lambda^{\lambda}_{\mu\nu}$ may give one and the same parallelism and the fact stated in Art. 2.

Cor. The momentum vector is not tangential to the conjectured path of Einstein of the free particle.

N.B. This fact seems to be *fatal* to the theory of general relativity of A. Einstein.

4. *The generalized gravitation theory of A. Einstein of 1953 implies a self-contradiction.*

Proof. When the ω^l_μ are purely gravitational, the $g_{\underline{\mu\nu}} = \omega^l_\lambda \omega^l_\nu$ and the

$$g_{\underset{\vee}{\mu\nu}} = \gamma_4\gamma_1(\omega^4_\mu\omega^1_\nu - \omega^1_\mu\omega^4_\nu) + \cdots + \gamma_2\gamma_3(\omega^2_\mu\omega^3_\nu - \omega^3_\mu\omega^2_\nu) + \cdots$$

must also be purely gravitational contrary to the Einstein's conjecture, that the $g_{\underset{\vee}{\mu\nu}}$ are electromagnetic.

5. *Conclusion*

(*1*) *The path of a free charged particle must be the path of the teleparallelism corresponding to the momentum ω^l_μ arisen by the charge.* (*2*) *The radii of the energy front generalized sphere must also be the paths of the same teleparallelism.* (*3*) *The receptacle of the physical phenomena is the ordinary three-dimensional Cartesian space; only that it gets teleparallelismical torsion by the charge of the particle.* (*4*) *The paths of the teleparallelism behave as for meet and join like straight lines* as their equations

$$(4) \qquad \frac{d}{dS}\frac{\omega^l}{dS} \equiv \omega^l_\sigma \left(\frac{d^2x^\sigma}{dS^2} + \Lambda^\sigma_{\mu\nu}\frac{dx^\mu}{dS}\frac{dx^\nu}{dS} \right) = 0$$

shows:

$$(5) \qquad \xi^l \equiv \int_0^S \frac{\omega^l}{dS} dS = a^l S, \quad (a^l a^l = 1).$$

It is the solution of the principle of least action (*as well as of least action function*)*: $\delta S = 0$ for the variable parameters* (*ξ^l and*) *ω^l/dS,* (cyclic case!). (*5*) *The equations of motion* (*3*) *represent generalized homocentric systems.*

Thus we are lead to the

CONCLUSION. *The A. Einstein's theory of general relativity must be replaced by the author's as a three-dimensional non-holonomic Laguerre fibre bundle geometry*[1)3)-5)] *of the second kind* (*20-dimensional in abstract sense*) *realized in the ordinary three-dimensional Cartesian space, the energy front generalized spheres with the paths of the teleparallelism under consideration as generalized radii being taken as spatial elements. It proves extreme naturality in describing physical phenomena. It is equally supported by the three well-known observational data, which have supported the Einstein's theory, because the related well-known equation*

$$\frac{d^2u}{d\varphi^2}+u=\frac{m}{h^2}+3mu^2 \tag{6}$$

for the planetary motion is also found among the paths of the teleparallelism under consideration, namely that which arises from the quadratic form

$$dS^2=\gamma(\rho)dt^2-(1/\bar{\gamma}(\rho))d\rho^2-\rho^2 d\theta^2-\rho^2\sin^2\theta d\varphi^2, \tag{7}$$

where

$$-\bar{\gamma}(\rho)=h^2\left(1-2mu-\frac{2m}{u}\right)+\frac{C}{u^2}, \quad (C=\text{const.}), \tag{8}$$

$$\gamma(\rho)=\text{any continuous function of } \rho. \tag{9}$$

The author's theory unifies any kinds of action fields, which are unified linearly superposedly in ω^l. *When the mass is a function, the corresponding geometry is the author's non-holonomic parabolic Lie fibre bundle geometry.*

References

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