149. *A New Theory of Relativity under the Non-Locally Extended Lorentz Transformation Group*

By Tsurusaburo TAKASU
Tohoku University, Sendai

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The present author has established [16] an ameliorated theory of relativity under the group of *extended* Lorentz transformations:

(1) $$\varepsilon_l \bar{\xi}^l = a^l_m(\xi^p)\varepsilon_m \xi^m + \varepsilon_l a^l_0, \ (a^l_0 = \text{const.}, \ \varepsilon_l = (-1)^{\frac{1}{2}(1+\delta^4_l)}),$$

(2) $$\varepsilon_l \xi^l = \omega^l_\mu(x^\sigma)\varepsilon_\mu x^\mu + \varepsilon_l \omega^l_0, \ (\omega^l_0 = \text{const.}, \ \varepsilon_\mu = (-1)^{\frac{1}{2}(1+\delta^4_\mu)}),$$

$l, m, \cdots; \lambda, \mu, \cdots = 1, 2, 3, 4$; $x^1 = x$, $x^2 = y$, $x^3 = z$, $x^4 = ir = ict$; ((x, y, z): rectangular Cartesian coordinates, t = time); $(a^l_m(\xi^p))$ and $(\omega^l_\mu(x^\sigma))$: orthogonal matrices with determinant $\neq 0$; (x^σ), (ξ^l), and $(\bar{\xi}^l)$: II-*geodesic rectangular coordinates* ♯ [1···16]; δ's: Kronecker deltas which are 3-*dimensional extended equiform Laguerre transformations* *, the Einstein space ($R_{\mu\nu} = 0$) $[dS^2 = g_{\mu\nu}(x^\sigma)dx^\mu dx^\nu = (-1)^{1+\delta^4_l}\omega^l\omega^l > 0, \ g_{\mu\nu} = \omega^l_\mu \omega^l_\nu, \ \omega^l = \omega^l_\mu(x^\sigma)dx^\mu]$ being the map of the Minkowski space (x^σ) by the inverse transformation of the extended Lorentz transformation (2), so that *connection is not necessary* [28]. Thereby the physical interpretations of the geometrical objects were as follows:

(3) dS = action, $\omega^l_\mu(x^\sigma)$ = momentum-potential vector; principle of equivalence = invariancy of physical laws under the group * (physical change); "relativity" = referring to ♯; physical lines = II-geodesic curves (straight lines inclusive);

(4) *Hamilton's principle:* $\delta S = 0 \rightarrow$ equations of motion:

$$\frac{d^2\xi^l}{dS^2} = \frac{d}{dS}\frac{\omega^l}{dS} = \omega^l_\lambda\left\{\frac{d^2x^\lambda}{dS^2} + \Lambda^\lambda_{\mu\nu}\frac{dx^\mu}{dS}\frac{dx^\nu}{dS}\right\} = \omega^l_\lambda\left\{\frac{d^2x^\lambda}{dS^2} + \left\{{\lambda \atop \mu\nu}\right\}\frac{dx^\mu}{dS}\frac{dx^\nu}{dS}\right\} = 0,$$

where

(5) $$\Lambda^\lambda_{\mu\nu} = \Omega^\lambda_l \partial_\nu \omega^l_\mu \equiv -\omega^l_\mu \partial_\nu \Omega^\lambda_l, \ [\Omega^\lambda_l \omega^l_\mu = \delta^\lambda_\mu \Longleftrightarrow \Omega^\lambda_k \omega^h_\lambda = \delta^h_k],$$

the (4) representing II-*geodesics* (in the present author's sense) in 4-dimension, which are in 3-dimension "Kanalflächen" enveloped by oriented II-*geodesic spheres* (in the present author's sense) with the particle (x^1, x^2, x^3) as center and a II-*geodesic radius* $r = \int \frac{\omega^4}{dS} dS$. The theory was resumed ([16], p. 623) in the comparison of the present author's theory with the Einstein's, proving the immortal character (comparable with that of the Newton's law) of the former.

In this note, *the said theory will be extended further* by extending the extended Lorentz transformations to "non-locally" ([17–20]) *extended Lorentz transformations. The general procedure consists in considering*

(6) $g_{\mu\nu}(x, \dot{x}, \cdots, \overset{(m)}{x}),\ \omega^l_\mu(x, \dot{x}, \cdots, \overset{(m)}{x}),\ \Omega^\lambda_l(x, \dot{x}, \cdots, \overset{(m)}{x}),\ a^l_m(\xi, \dot{\xi}, \cdots, \overset{(m)}{\xi}),$
where $\dot{x}^\sigma = dx^\sigma/dS$, etc. and *the process is quite parallel and analogous to that of* [16], so that we may omit most calculations here and we shall understand by $g_{\mu\nu}$, ω^l_μ, Ω^λ_l, a^l_m, etc. those in (6). The results will further be viewed through the Finsler-Craig-Synge-Kawaguchi spaces (cf. Art. 4) as well as through the non-local field theory of H. Yukawa (cf. Art. 5).

1. Non-locall II-geodesic curves. Since

$$\omega^l = \omega^l_\mu(x, \dot{x}, \cdots, \overset{(m)}{x})\, dx^\mu \tag{1.1}$$

is written in an invariant form, *the* (x^σ) *in the Minkowski space may also be local curvilinear coordinates.*

$$\Lambda^\lambda_{\mu\nu} = \Omega^\lambda_l \partial_\nu \omega^l_\mu \equiv -\omega^l_\mu \partial_\nu \Omega^\lambda_l;\ \Omega^\lambda_l \omega^l_\mu = \delta^\lambda_\mu \Longleftrightarrow \Omega^\lambda_k \omega^h_\lambda = \delta^h_k. \tag{1.2}$$

Non-local II-*geodesic curves:*

$$\frac{d^2\xi^l}{dS^2} = \frac{d}{dS}\frac{\omega^l}{dS} \equiv \omega^l_\lambda\left\{\frac{d^2x^\lambda}{dS^2} + \Lambda^\lambda_{\mu\nu}(x, \dot{x}, \cdots, \overset{(m)}{x})\frac{dx^\mu}{dS}\frac{dx^\nu}{dS}\right\}$$
$$= \omega^l_\lambda\left\{\frac{d^2x^\lambda}{dS^2} + \left\{ {\lambda \atop \mu\nu} \right\}(x, \dot{x}, \cdots, \overset{(m)}{x})\frac{dx^\mu}{dS}\frac{dx^\nu}{dS}\right\} = 0. \tag{1.3}$$

$$d\Omega^\lambda_l + \Lambda^\lambda_{\mu\nu}\Omega^\mu_l dx^\nu = 0,\ d\omega^l_\mu - \Lambda^\lambda_{\mu\nu}\omega^l_\lambda dx^\nu = 0. \tag{1.4}$$

Differential and finite equations of non-local II-geodesic curves:

$$d\xi^l = \omega^l = \alpha^l dS,\ (\alpha^l = \text{const.}),\ \xi^l = \alpha^l S + \gamma^l = \int \frac{\omega^l}{dS} dS,\ (\gamma^l = \text{const.}). \tag{1.5}$$

$$d\xi^l/dS = a^m \Omega^l_m(\xi, \dot{\xi}, \cdots, \overset{(m)}{\xi}) = \alpha^l, \tag{1.6}$$

$$dx^\lambda/dS = a^m \Omega^\lambda_m(x, \dot{x}, \cdots, \overset{(m)}{x}) = \alpha^\lambda,\ (\alpha^\lambda = \text{const.}). \tag{1.6$'$}$$

The coordinates $(x^\sigma) = (x, y, z, r = ct)$, (ξ^l) and $(\bar{\xi}^l)$ will be called *the non-local* II-*geodesic rectangular coordinates.*

The non-local II-*geodesic curves* (1.5) *behave, as for meet and join as well as for the extremal* $\delta S = 0$ *like straight lines.* A special kind of (1.5) is $(x^\sigma) = (x, y, z, r = ct) = (a^\sigma S + c^\sigma)$, $(a^\sigma, c^\sigma$: const.), which represent a straight line in 4-dimension and *a circular cone enveloped by an oriented sphere with center* (x, y, z) *and radius* $r = ct$.

The conditions for that the non-local II-geodesic curves $d^2\xi^l/dS^2 = 0$ may be transformed into the non-local II-geodesic curves $d^2\bar{\xi}^l/dS^2 = 0$ are (cf. [1–16]):

$$da^l_m(\xi, \dot{\xi}, \cdots, \overset{(m)}{\xi}) d\xi^m = 0, \quad da^l_m(\xi, \dot{\xi}, \cdots, \overset{(m)}{\xi}) \xi^m = 0. \tag{1.7}$$

2. Non-locally extended Lorentz transformation group. By virtue of (1.7), the differential equations

$$\varepsilon_l d\bar{\xi}^l = a^l_m(\xi, \dot{\xi}, \cdots, \overset{(m)}{\xi}) \varepsilon_m d\xi^m, \tag{2.1}$$

$$\varepsilon_l \dot{\bar{\xi}}^l = a^l_m(\xi, \dot{\xi}, \cdots, \overset{(m)}{\xi}) \varepsilon_m \dot{\xi}^m, \tag{2.1$'$}$$

which arise from (1.5), may be integrated, resulting to the *non-locally extended Lorentz transformation formulas:*

$$\varepsilon_l \bar{\xi}^l = a^l_m(\xi, \dot{\xi}, \cdots, \overset{(m)}{\xi}) \varepsilon_m \xi^m + \varepsilon_l a^l_0,\ (a^l_0 = \text{const.}), \tag{2.2}$$

of which a special ones are

(2.3) $\varepsilon_l\xi^l=\omega^l_\mu(x,\dot{x},\cdots,\overset{(m)}{x})\varepsilon_\mu x^\mu+\varepsilon_l\omega^l_0,\ (\omega^l_0=\text{const.}),$

(2.3)′ $\varepsilon_l\dot{\xi}^l=\omega^l_\mu(x,\dot{x},\cdots,\overset{(m)}{x})\varepsilon_\mu\dot{x}^\mu.$

(2.2) and (2.3) are generalizations of (1) resp. (2).

That the totality of the non-locally extended Lorentz transformations (2.2) *forms a group,* which we will call *the non-locally extended equiform Laguerre group in* 3*-dimension* and *the non-locally extended Lorentz transformation group in* 4*-dimension* may be proved quite as in [28], p. 7.

Since our new physical space undergo this transformation group, *the geometry under this group belongs to the "Erlanger Programm" of Felix Klein* (1872), *so that connection is not necessary* [28].

By virtue of (1.6) and (1.6)′, the formulas (1.6), (1.6)′, (1.7), (2.1), (2.1)′, (2.2), (2.3) and (2.3)′ may be rewritten as follows:

(1.6) $d\xi^l/dS=\alpha^m\Omega^l_m(\xi,\alpha^p,0,\cdots,0)=\alpha^l,$

(1.6)′ $dx^\lambda/dS=\alpha^m\Omega^\lambda_m(x,\alpha^\sigma,0,\cdots,0)=\alpha^\lambda,$

(1.7) $da^l_m(\xi,\alpha^p,0,\cdots,0)\,d\xi^m=0,\quad da^l_m(\xi,\alpha^p,0,\cdots,0)\,\xi^m=0,$

(2.1) $\varepsilon_l\bar{\omega}^l=\varepsilon_l d\bar{\xi}^l=a^l_m(\xi,\alpha^p,0,\cdots,0)\,\varepsilon_m d\xi^m=a^l_m(\xi,\alpha^p,0,\cdots,0)\,\varepsilon_m\omega^m,$

(2.1)′ $\varepsilon_l\dot{\bar{\xi}}^l=\varepsilon_l\bar{\alpha}^l=a^l_m(\xi,\alpha^p,0,\cdots,0)\,\varepsilon_m\alpha^m=a^l_m(\xi,\alpha^p,0,\ldots,0)\varepsilon_m\dot{\xi}^m,$

(2.2) $\varepsilon_l\bar{\xi}^l=a^l_m(\xi,\alpha^p,0,\cdots,0)\,\varepsilon_m\xi^m+\varepsilon_l a^l_0,$

(2.3) $\varepsilon_l\xi^l=\omega^l_\mu(x,\alpha^\sigma,0,\ldots,0)\,\varepsilon_\mu x^\mu+\varepsilon_l\omega^l_0,$

(2.3)′ $\varepsilon_l\dot{\xi}^l=\varepsilon_l\alpha^l=\omega^l_\mu(x,\alpha^\sigma,0,\cdots,0)\,\varepsilon_\mu\alpha^\mu=\omega^l_\mu(x,\alpha^\sigma,0,\cdots,0)\,\varepsilon_\mu\dot{x}^\mu,$

whereby 4 *transformation parameters* ($\alpha^1,\alpha^2,\alpha^3,\alpha^4$) *arise anew.* (2.1)′ *and* (2.3)′ *show us that* (α^m) *and* (α^μ) *undergo respective non-locally extended orthogonal transformations.*

3. Physical interpretations of the geometrical objects. *The interpretation*

(3.1) $(\omega^l_\mu(x,\dot{x},\cdot\cdot,\overset{(m)}{x}))=(\omega^l_\mu(x,\alpha^\sigma,0,\cdot\cdot,0))=$ *momentum-potential vector* *is seen to be quite natural owing to* (1.6)′ *and* (1.2).

(3.2) $dS=$ *action.*

The expressibility

(3.3) $dS^2=g_{\mu\nu}(x,\dot{x},0,\cdots,\overset{(m)}{x})\,dx^\mu dx^\nu=(-1)^{1+\delta^4_l}\omega^l\omega^l$

is proved to be valid but for undergoing non-locally extended orthogonal transformations (2.1).

4. Considerations from the view points of Finsler-Craig-Synge-Kawaguchi spaces. The metric space, in which an arc length S along a curve $x^\lambda=x^\lambda(t)$, ($t=$curve parameter), is given by an integral

(4.1) $S=\int_{t_0}^{t_1}F(x,x',\cdots,x^{(m)})dt,\ (x'=dx/dt,\ \text{etc.}),$

satisfying the so-called Zermero's conditions:

(4.2) $$\begin{cases} \Delta F=\sum_{\alpha=1}^{m}\alpha x^{(\alpha)\lambda}\dfrac{\partial F}{\partial x^{(\alpha)\lambda}}=F, \quad (x^{(\alpha)}=d^\alpha x/dt^\alpha), \\ \Delta_k F=\sum_{\alpha=K}^{m}\binom{\alpha}{K}x^{(\alpha-K+1)\lambda}\dfrac{\partial F}{\partial x^{(\alpha)\lambda}}=0, \ (K=2, 3, \cdots, m)\leftarrow(K=2, 3), \end{cases}$$

where the last relation has been shown 1938 by A. Kawaguchi and H. Hombu, is called Kawaguchi space of order m, whose special cases are:

Finsler space: $m=1$. | Craig-Synge space: $m=2$.

Since dS in the present author's sense is the action, it is natural in case $n=4$ *to interpret* $F(x, x', \cdots, x^{(m)})$, ($t$=time) *as the energy. The Kawaguchi space is reducible to the Finsler space having n transformation parameters* $(\alpha^\lambda)=(\alpha^1, \alpha^2, \cdots, \alpha^n)$ *in addition by specializing the coordinates* (x^λ) *to* II-*geodesic rectangular coordinates in the base manifold* [28], *so that* $dx^\lambda/dS=\alpha^\lambda$. Now for the Finsler space corresponding to

(4.3) $$dS^2=F^2(x, \dot{x})^2(dt)^2,$$

where F is of degree one in $\dot{x}=dx/dS$, we have

(4.4) $$dS^2=g_{\mu\nu}(x, \dot{x})dx^\mu dx^\nu,$$

(4.5) $$g_{\mu\nu}(x, \dot{x})=\frac{1}{2}\frac{\partial^2F^2(x, \dot{x})}{\partial\dot{x}^\mu\partial\dot{x}^\nu} \quad [\text{cf. } (6)].$$

We can render (4.4) into the form

(4.6) $$dS^2=\omega^l\omega^l, \ (\omega^l=\omega^l_\mu(x, \dot{x})dx^\mu)$$

but for undergoing non-locally extended orthogonal transformations.

Another procedure is to adopt the metric tensor ([25], p. 724, $^*g_{ij}$):

(4.6) $$g_{\mu\nu}(x, \dot{x}, \cdots, \overset{(m)}{x})=mF^{2m}F_{(m)\mu}F_{(m)\nu}+\overset{m}{\mathfrak{E}}_\mu\overset{m}{\mathfrak{E}}_\nu+{}^*\overset{\perp}{\mathfrak{E}}{}^*_\mu\overset{\perp}{\mathfrak{E}}_\nu,$$

etc. for the (6), where

(4.7) $$F=(x, \dot{x}, \cdots, \overset{(m)}{x}),$$

$$\left[\begin{array}{l} {}^*\overset{K}{E}_\mu=\sum_{\sigma=K}^{m}(-1)^\alpha\dbinom{\alpha}{K}\dfrac{D^{\alpha-K}}{dt^{\alpha-K}}F_{(\alpha)\mu}, \quad \dfrac{D}{dt}f=\sum_{\alpha=0}^{m-1}f_{(\alpha)\mu}\overset{(\alpha+1)}{x^\mu}-f_{(m)\mu}H^\mu, \\ \overset{(m+1)}{x^\mu}=-H^\mu(x^\mu, \dot{x}^\mu, \cdots, \overset{(m)}{x^\mu}), \\ \text{the } A^\lambda_\mu \text{ being defined by the recurring formulae} \\ A^0_1=1, \ A^\lambda_\mu=\dfrac{dA^\lambda_{\mu-1}}{dt}+A^{\lambda-1}_\mu F, \quad (\lambda, \mu=0, 1, \cdots, m), \\ A^1_\nu=F^{(\nu-1)}, \ A^\nu_0=0, \ A^0_\omega=0, \quad (\nu=1, 2, \cdots, m; \ \omega=2, 3, \cdots, m). \end{array}\right]$$

(4.8) $${}^*\overset{K}{\mathfrak{E}}_\mu=F^{-1}\sum_{\sigma=K}^{m}{}^*\overset{\sigma}{E}_\mu A^K_{\sigma-K+1}, \quad (K=0, 1, \cdots, m),$$

(4.9) $$\overset{K}{\mathfrak{E}}_\mu=F^{-1}\sum_{\alpha=K}^{m}{}^*\overset{\sigma}{E}A^K_{\sigma-K+1}, \quad (K=0, 1, \cdots, m).$$

$$\left[\overset{K}{E}_\mu=\sum_{\alpha=K}^{m}(-1)^\alpha\binom{\alpha}{\mathrm{K}}(F_{(\alpha)K})^{(\alpha-K)}, \quad (\mathrm{K}=0, 1, \cdots, m).\right]$$

5. A generalization of H. Yukawa's non-local field theory. H. Yukawa ([17], [18]) has established a non-local (i.e. bilocal!) field theory by putting

$$(5.1)\qquad X^{\lambda}=\frac{1}{2}(\underset{1}{x^{\lambda}}+\underset{2}{x^{\lambda}}),\quad r^{\lambda}=\underset{2}{x^{\lambda}}-\underset{1}{x^{\lambda}}$$

in the space-time $(x^{\lambda})=(x, y, z, ict)$. If we put

$$(5.2)\qquad \underset{2}{x^{\lambda}}-\underset{1}{x^{\lambda}}=2\rho c^{\lambda},\quad (c^{\lambda}c^{\lambda}=1),$$

then (c^{λ}) are the direction cosines, so that $((X^{\lambda}), \rho, (c^{\lambda}))$ affords us a line-element space, which is a special Finsler space (cf. [20]).

Applying this principle to our space (ξ^{l}), we obtain a *non-locally extended line-element space* $((\xi^{l}), \rho, (\alpha^{l}))$, where

$$(5.3)\qquad \underset{2}{\xi^{l}}-\underset{1}{\xi^{l}}=2\rho\alpha^{l},\ (\alpha^{l}\alpha^{l}=1).\quad \Big|\quad d\xi^{l}=\alpha^{l}dS.$$

In this way, we can *generalize the H. Yukawa's non-local field theory.* Thus *the theory of relativity under the non-locally extended Lorentz transformation group of the present author is a generalization of the H. Yukawa's non-local field theory.*

6. Comparison of the theory of relativity of A. Einstein and the present theory of T. Takasu. This comparison may be done quite as in [16] being led to the conclusion:

The classical physics, the theory of special relativity, in which we shall have

$$(\omega^{l}_{\mu}(x, \dot{x}, \cdots, \overset{(m)}{x}))=(\omega^{l}_{\mu}(x, \alpha, 0, \cdots, 0))=\begin{pmatrix} m_0c^2 & 0 & 0 & 0 \\ 0 & m_0c & 0 & 0 \\ 0 & 0 & m_0c & 0 \\ 0 & 0 & 0 & m_0c \end{pmatrix},$$

the gravitation theory, the electromagnetic theory, the universally accepted part of quantum theory, the T. Takasu's theory [16], *the K. Kondo's theories* ([26], [27]) *and the H. Yukawa's non-local field theory* ([17–19]) *are in the*

A. Einstein's theory,	T. Takasu's theory,
which is a mere conjecture and an approximation theory, not unified.	*which is a decisive exact theory with immortal character as the Newton's law, unified.*

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