83. On Axiom Systems of Propositional Calculi. XX

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In their note ([1], [2]), Y. Arai and K. Iséki discuss on some theses of equivalential calculus introduced by S. Leśniewski (see, [3]).

The equivalential calculus satisfies the following two fundamental axioms:

$E1$ $EEEprEEqpErq$,

$E2$ $EEpEqrEEpqr$,

where E is the truth functor in the calculus (see, [4]).

In his paper [2], Prof. K. Iséki has given a new axiom set and has proved that the equivalential calculus is characterized by it, using some metatheorems. His results are read as below:

Lemma 1. *The equivalential calculus is characterized by*

(1) Epp,

(2) $EEpqEqp$,

(3) $EEpqEEqrEpr$.

Lemma 2. *The above axiom set is equivalent to the single axiom* $EEpqEEprErq$ (see, [2]).

In this paper, we shall also give a new axiom set of the equivalential calculus and prove that its set characterizes the equivalential calculus.

We use the two rules of inference, i.e., substitution and detachment: α and $E\alpha\beta$ imply β.

First we shall prove the following

Theorem 1. *The following axiom set, i.e.,*

1 $EEpEqrEEsqEsEpr$,

2 $EEpqEqp$,

implies the axiom set, i.e.,

(1) Epp,

(2) $EEpqEqp$,

(3) $EEpqEEqrEpr$.

For the proof we shall use prooflines by J. Lukasiewicz.

Proof. From the axioms 1 and 2, i.e.,

1 $EEpEqrEEsqEsEpr$,

2 $EEpqEqp$,

we deduce the following theses:

1 p/Erq $*C2$—3,

3 $EEsqEsEErqr$.

2 *p/EpEqr*, *q/EEsqEsEpr* **C*1—4,

4 *EEEsqEsEprEpEqr*.

4 *s/Epr*, *q/Epr* **C*2 *p/Epr*, *q/Epr*—5,

5 *EpEEprr*.

2 *p/Esq*, *q/EsEErqr* **C*3—6,

6 *EEsEErqrEsq*.

6 *s/p*, *r/p*, *q/p* **C*5 *r/p*—7,

7 *Epp*.

1 *p/EpEqr*, *q/Esq*, *r/EsEpr*, *s/t* **C*1—8,

8 *EEtEsqEtEEpEqrEsEpr*.

8 *t/Esq* **C*7 *p/Esq*—9,

9 *EEsqEEpEqrEsEpr*.

9 *s/q* **C*7 *p/q*—10,

10 *EEpEqrEqEpr*.

10 *p/EpEqr*, *r/Epr* **C*10—11,

11 *EqEEpEqrEpr*.

1 *p/q*, *q/EpEqr*, *r/Epr* **C*11—12,

12 *EEsEpEqrEsEqEpr*.

1 *p/Eqr* **C*7 *p/Eqr*—13,

13 *EEsqEsEEqrr*.

12 *s/Epq*, *q/Eqr* **C*13 *s/p*—14,

14 *EEpqEEqrEpr*.

Next we shall give the proof of the following theorem.

Theorem 2. *The single axiom of the equivalential calculus, i.e.,*

1 *EEpqEEprErq*

implies the following axiom set, i.e., EEpEqrEEsqEsEpr, EEpqEqp.

For the proof we use some results from the axiom *EEpqEEprErq* (for the details and prooflines, see [5]).

Lemma 3. *The axiom EEpqEEprErq implies*

2 *EEprErp*,

3 *EEpqEErpErq*,

4 *EEqErsErEqs*.

4 *q/Epq*, *q/Erp*, *s/Erq* *C3—5,

5 *EErpEEpqErq*.

3 *p/q*, *q/Epr*, *r/s* *6

6 *EEqEprEEsqEsEpr*.

5 *r/EpEqr*, *p/EqEpr*, *q/EEsqEsEpr* **C*4 *q/p*, *r/q*, *s/r*—*C*6—7,

7 *EEpEqrEEsqEsEpr*.

If we put *B* for *CCqrCCpqCpr*, *C* for *CCpCqrCqCpr*, *I* for *Cpp*, then *the system BCI* is equivalent to *I* together with

CCpCqrCCsqCsCpr (see [2]). Hence, if *I* and *CCpCqrCCsqCsCpr* are independent, then it is clearly seen that *Epp* and *EEpEqrEEsqEsEpr* are independent. Further *CCpqCqp* is not a thesis in *the system BCI.* Hence *EEpqEqp* is independent from *Epp* and *EEpEqrEEsqEsEpr*.

References

[1] Y. Arai: On axiom systems of propositional calculi. XVII. Proc. Japan Acad., **42**, 351–354 (1966).

[2] K. Iséki: On axiom systems of propositional calculi. XV. Proc. Japan Acad., **42**, 217–220 (1966).

[3] S. Leśniewski: Grundzüge eines neuen Systems der Grundlagen der Mathematik. Fund. Math., **14**, 1–81 (1929).

[4] A. N. Prior: Formal Logic. Oxford (1962).

[5] S. Tanaka: On axiom systems of propositional calculi. XVIII. Proc. Japan Acad., **42**, 000–000 (1966).