191. A Characterization of the NB-System

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(Comm. by Kinjirô Kunugi, m.j.a., Oct. 12, 1966)

In my paper [2], we introduced a new system of propositional calculus which is called the *NB-system*. This system is given by the following axioms:

1    $CpCqp$,
2    $CCpCqrCCpqCpr$,
3    $CCpNqCqNp$.

In the positive implicational calculus satisfying the conditions 1 and 2, if we introduce a propositional constant 0, and we put $Np = Cp0$, then we obtain the *NB*-system (see [3]). In this note, we shall give a new axiom system of the *NB*-system.

In his paper [4], Professor B. Sobociński gave an axiom system of three valued logic. His system is given by

4    $CCpqCCqrCpr$,
5    $CpCCpqq$,
6    $CCpCpqCpq$,
7    $CpCqCNqp$,
8    $CCNpNqCqp$.

From the theses 4, 5, he proved a commutative law:

9    $CCpCqrCqCpr$.

As well known, by the theses 9, 4, we have

10    $CCqrCCpqCpr$.

As already shown in [1], from the theses 4, 5, 6, 9, and 10, we have

11    $CCpCqrCCpqCpr$.

Therefore the thesis 11 is obtained from the theses 4, 5, and 6. Hence, if we introduce a propositional constant 0, and we define $Np = Cp0$, then the thesis 9 implies the following

12    $CCpNqCqNp$.

Then we have the following characterization of the *NB*-system.

Theorem. *The NB-system is characterized by the axiom system:*

$$CpCqp, \quad CCpqCCqrCpr, \quad CpCCpqq, \quad CCpCpqCpq, \quad Np = Cp0.$$

References

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