251. On the Propositional Calculus with a Variable Functor, $C\delta pC\delta Np\delta q$

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In this note, we shall prove that $C\delta pC\delta Np\delta q$ implies

1) $CpCqp$,
2) $CCpCqrCCpqCpr$,
3) $CCNpNqCqp$.

As the substitution rules for δ are well-known, we omit these. (For example, see [1] Lukasiewicz, [2] Meredith or [3] Prior.) For the details on the propositional calculus with a variable functor δ, see [1], [2], [3].

Proof.

1 $C\delta pC\delta Np\delta q$.

1 $\delta/C'r$ *2,

2 $CCprCCNprCqr$.

1 $\delta/'$ *3,

3 $CpCNpq$.

1 $\delta/CpCNp'$, q/Np *$C3$ q/p—4,

4 $CCpCNpNpCpCNpNp$.

3 $p/CpCNpNp$, $q/NCpCNpNp$ *$C3$ q/Np—5,

5 $CNCpNpNpNCpNpNp$.

1 δ/C'', $p/CpCNpNp$, q/p *$C4$—$C5$—6,

6 Cpp.

1 δ/C'' *$C6$—7,

7 $CCNpNpCqq$.

1 $\delta/CC''Cqq$, p/Np, q/r *$C7$—$C7$ p/Np—8,

8 $CCrrCqq$.

3 p/Cpp *$C8$—9,

9 $CNCppq$.

1 $\delta/C'Cqq$, p/Cpp, q/r *$C8$ r/p—$C9$ q/Cqq—10,

10 $CrCqq$.

1 $\delta/CrC'r$, p/r *$C10$ q/r—$C3$ p/r, q/r—11,

11 $CrCqr$.

1 δ/Cr' *12,

12 $CCrpCCrNpCrq$.

12 p/Np, q/p, r/Np *$C6$ p/Np—13,

13 $CCNpNNpCNpp$.

1 $\delta/CCNpN'C'p$ *$C8$ r/Np, q/p—$C13$—14,

14 $CCNpNqCqp$.

11 q/CNpCpr, r/CpCNpr *C3 q/r—15,

15 CCNpCprCpCNpr.

1 δ/CC'CprCpC'r *C6 p/CpCpr—C15—16,

16 CCqCprCpCqr.

12 r/p *C6—17,

17 CCpNpCpq.

11 r/CCpNpCpr, q/CNpr *C17 q/r—18,

18 CCNprCCpNpCpr.

1 δ/CC'rCCp'Cpr *C11 r/Cpr, q/Cpp—C18—19,

19 CCqrCCpqCpr.

16 q/Cqr, p/Cpq, r/Cpr *C19—20,

20 CCpqCCqrCpr.

16 q/Cpq, r/q *C6 p/Cpq—21,

21 CpCCpqq.

21 p/CNCppq, q/NCpp *C9 p/s—22,

22 CCCNCppqNCppNCpp.

11 r/Cpp, q/CCCppqCpp *C6—23,

23 CCCCppqCppCpp.

1 δ/CCC'q'', p/Cpp, q/p *C23—C22—24,

24 CCCpqpp.

20 p/CCpCqrCqCpr, q/CCCpsCpCqrCCpsCqCpr,
r/CCCsCqrCCpsCpCqrCCsCqrCCpsCqCpr *C19
p/Cps, q/CpCqr, r/CqCpr—C19 p/CsCqr,
q/CCpsCpCqr, r/CCpsCqCpr—C16 p/q, q/p—C19
q/s,r/Cqr—25,

CCCCpCqrCqCprCCCpsCpCqrCCpsCqCprCCCCCpsCpCqr
CCpsCqCprCCCsCqrCCpsCpCqrCCsCqrCCpsCqCprCCCp
CqrCqCprCCCsCqrCCpsCpCqrCCsCqrCCpsCqCpr,

25 CCsCqrCCpsCqCpr.

25 p/s, q/Cpq, r/Cpr, s/Cqr *C19—26,

26 CCsCqrCCpqCsCpr.

25 p/s, q/Cqr, r/Cpr, s/Cpq *C20—27,

27 CCsCpqCCqrCsCpr.

20 p/CpCpq, q/CCCpqqCpq, r/Cpq *C20 q/Cpq,
r/q—C24 p/Cpq—28,

28 CCpCpqCpq.

27 p/Cpq, q/CpCpr, r/Cpr, s/CpCqr *C26
s/p—C28 q/r—29,

29 CCpCqrCCpqCpr.

Theses 11, 14, and 29 are axioms by J. Lukasiewicz. Therefore the proof is complete.

References

[1] J. Lukasiewicz: Aristotle's Syllogistic. 2nd Edition, Oxford (1963).
[2] C. A. Meredith: On an extended system of the propositional calculus. Proc. Royal Irish Acad., Dublin, **54** A3 (1951).
[3] A. N. Prior: Formal Logic. 2nd Edition, Oxford (1962).