28. *Axiom Systems of Aristotle Traditional Logic*

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In this paper, we shall consider axiom systems of Aristotle traditional logic. Some axiom systems are obtained by J. Lukasiewicz ([3], [4]), I. Bocheński [1] and N. Kretzmann [2]. We shall give a method to find axiom systems. The method given below is useful for the purpose, and all axiom systems are found by our method.

As well known (see A. N. Prior [5]), in the Aristotle traditional logic, there are four types of categorical propositions, a and b being terms:

1) Every a is b.
2) At least one a is b.
3) At least one a is not b.
4) No a is b.

These propositions are denoted by Aab, Iab, Oab, Eab respectively. Let N be the negation functor, then $Eab=NIab$, $Oab=NAab$. By X, Y, Z, we denote the term functors A, I, O, E, then there are two different kinds of moods: *immediate inference* and *syllogism*. Moreover, the mood of immediate inference is divided into two types: $CXabYab$, $CXabYba$, where C is the implication functor. These are denoted by XY_1, XY_2 respectively. On the syllogism, we have four types of moods:

I) $CKXabYcaZcb$,

II) $CKXabYcbZca$,

III) $CKXabYacZcb$,

IV) $CKXabYbcZca$,

where K is the conjunction functor. These moods are denoted by XYZ_i $(i=1, 2, 3, 4)$ respectively.

In these symbols, the Lukasiewicz axiom system is written in the form of

L1) Aaa,

L2) Iaa,

L3) AAA_1,

L4) AII_3.

Assuming some theses of propositional calculus, we have II_1, II_2, EE_1, EE_2, AI_1, AI_2, EO_1, EO_2 (see J. Lukasiewicz [3]).

From theses of the classical propositional calculus, we have the following deduction rules:

$$(1)\quad CK\alpha\beta\gamma \begin{array}{l} \nearrow\ 1)\ CK\beta\alpha\gamma, \\ \rightarrow\ 2)\ CK\alpha N\gamma N\beta, \\ \searrow\ 3)\ CKN\gamma\beta N\alpha. \end{array}$$

$$(2)\quad CK\alpha\beta\gamma, C\delta\alpha \longrightarrow CK\delta\beta\gamma.$$

$$(3)\quad CK\alpha\beta\gamma, C\delta\beta \longrightarrow CK\alpha\delta\gamma.$$

$$(4)\quad CK\alpha\beta\gamma, C\gamma\delta \longrightarrow CK\alpha\beta\delta, CK\beta\alpha\delta.$$

From the deduction rules (1), 2) and 3), we have the followings:

$$XYZ_1 \begin{array}{l} \nearrow XNZNY_2 \\ \searrow NZYNX_3 \end{array}$$

$$XYZ_2 \begin{array}{l} \nearrow XNZNY_1 \\ \searrow NZYNX_3 \end{array}$$

$$XYZ_3 \begin{array}{l} \nearrow XNZNY_2 \\ \searrow NZYNX_1 \end{array}$$

$$XYZ_4 \begin{array}{l} \nearrow XNZNY_1 \\ \searrow NZYNX_1 \end{array}$$

Hence L3) implies $ANANA_2$, $NAANA_3$. These means AOO_2, OAO_3. Similarly, L4) implies AEE_2, EIO_1. For example, AOO_2 and EO_2 imply AEO_4 by applying the deduction rule (3). This means the order change of terms contained in the second functor O, and a replacement $O \rightarrow E$.

As the rules of order changes, we have the following results, where $\tilde{X}$ means the order change of terms contained in X.

$$XYZ_1 \begin{array}{l} \nearrow \tilde{X}YZ_2 \\ \searrow X\tilde{Y}Z_3 \end{array}$$

$$XYZ_2 \begin{array}{l} \nearrow \tilde{X}YZ_1 \\ \searrow X\tilde{Y}Z_4 \end{array}$$

$$XYZ_3 \begin{array}{l} \nearrow \tilde{X}YZ_4 \\ \searrow X\tilde{Y}Z_1 \end{array}$$

$$XYZ_4 \begin{array}{l} \nearrow \tilde{X}YZ_1 \\ \searrow X\tilde{Y}Z_3 \end{array}$$

Therefore, by AOO_2 and EO_2, we have $A\tilde{O}O_4 \rightarrow AEO_4$. We denote this deduction by

$$AOO_2 + EO_2 \rightarrow AEO_4.$$

Under such a consideration, we have the followings:

$$OAO_3 + EO_2 \rightarrow EAO_4,$$
$$AII_3 + II_2 \rightarrow AII_1,$$
$$AII_3 + AI_2 \rightarrow AAI_1,$$
$$AEE_2 + EE_2 \rightarrow AEE_4,$$
$$EIO_1 + AI_2 \rightarrow EAO_3,$$
$$EIO_1 + EE_2 \rightarrow EIO_2.$$

Further, we have

$$AOO_2 + EO_1 \rightarrow AEO_2,$$
$$OAO_3 + EO_1 \rightarrow EAO_3,$$
$$EIO_1 + AI_1 \rightarrow EAO_1,$$

$$AII_3 + AI_1 \to AAI_3.$$

Some of above results imply

$$EIO_2 \to ENONI_1 \to EAE_1,$$
$$EAE_1 \to NEANE_3 \to IAI_3,$$
$$AII_1 \to NIINA_3 \to EIO_3,$$
$$EIO_1 \to ENONI_2 \to EAE_2.$$

Next we have

$$EIO_2 + AI_2 \to EAO_4,$$
$$EIO_2 + II_2 \to EIO_4,$$
$$IAI_3 + AI_2 \to AAI_4,$$
$$IAI_3 + II_2 \to IAI_4.$$

Therefore, we have the following:

Theorem 1. *Under the axiom systems of Lukasiewicz, we have*

AAA_1	AOO_2	OAO_3	AEE_4
EAE_1	EIO_2	IAI_3	AAI_4
AAI_1	EAE_2	EAO_3	EIO_4
EAO_1	AEE_2	AAI_3	IAI_4
EIO_1	EAO_2	AII_3	AEO_4
AII_1	AEO_2	EIO_3	EAO_4

An axiom system of I. Bocheński [1] *is given* by L1), L2), L3), *and*

B4) EIO_1.

We can easily obtain L4) AII_3 as follows:

$$EIO_1 \to NOINE_3 \to AII_3.$$

By theorem 1, it is seen that the axiom systems of I. Bocheński implies all moods of syllogism in theorem 1.

By the same technique, we can obtain various axiom systems. For examples,

$$OAO_3 \to NOANO_1 \to AAA_1,$$
$$AII_3 \to NIINA_1 \to EIO_1$$

imply an axiom system. As examples of the second group, we have

$$AOO_2 \to ANONO_1 \to AAA_1,$$
$$EAE_2 \to ENENA_1 \to EIO_1,$$
$$EIO_2 \to NOINE_3 \to AII_3.$$

Therefore we have the following:

Theorem 2. *The Aristotle traditional logic is given by each of the following axiom system:*

1) L1), L2), OAO_3, AII_3,

2) L1), L2), AOO_2, EAE_2,

3) L1), L2), AOO_2, EIO_2.

References

[1] I. M. Bocheński: A precis of mathematical logic. Dordrecht (1959).
[2] N. Kretzmann: Elements of Formal Logic. Indianapolis (1965).
[3] J. Lukasiewicz: Elements of Mathematical Logic. Oxford (1963).
[4] ——: Aristotle's Syllogistic. Oxford (1963).
[5] A. N. Prior: Formal Logic. Oxford (1962).