15. Axiom Systems of Aristotle Traditional Logic. III

By Yasuo SETÔ

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In this paper, we shall give a new axiom system of Aristotle traditional logic. Some axiom systems have been obtained by J. Lukasiewicz [4], [5], I. Bochenski [1], N. Kretzmann [3], K. Iséki [2], and S. Tanaka [6]. K. Iséki has given a method to find its axiom systems. For the detail, see [2]. In this paper, we use the following notations. For the categorical sentences,

1) Aab: Every a is b,
2) Iab: At least one a is b,
3) Oab: At least one a is not b,
4) Eab: No a is b.

For the functors,

1) C: implication, 2) N: negation, 3) K: conjunction.

Then we have

$D1$. $Eab = NIab$,

$D2$. $Oab = NAab$.

For the moods and figures,

1) XY_1: $CXabYab$, 2) XY_2: $CXabYba$,
3) XYZ_1: $CKXabYcaZcb$, 4) XYZ_2: $CKXabYcbZca$,
5) XYZ_3: $CKXabYacZcb$,
6) XYZ_4: $CKXabYbcZca$.

Under these notations, the Lukasiewicz axiom system is written as follows:

$L1$. Aaa,

$L2$. Iaa,

$L3$. AAA_1,

$L4$. AII_3.

The following deduction rules $T1$, $T2$, $T3$ from the classical propositional calculus are used in our discussion.

$T1$. $CK\alpha\beta\gamma \rightarrow CK\beta\alpha\gamma$,

$T2$. $CK\alpha\beta\gamma, C\gamma\delta \rightarrow CK\alpha\beta\delta$,

$T3$. $C\alpha\beta \rightarrow CN\beta N\alpha$.

For the simplicity, we shall write these as

$T1$. $\alpha\beta\gamma \rightarrow \beta\alpha\gamma$,

$T2$. $\alpha\beta\gamma + \gamma\delta \rightarrow \alpha\beta\delta$,

$T3$. $\alpha\beta \rightarrow N\beta N\alpha$.

In our previous notes, K. Iséki and S. Tanaka have given some important results related with this note. We shall prove the following

Theorem 1. *The Aristotle traditional logic is characterized by the following axiom system:*

*L*1. *Aaa*, *L*2. *Iaa*,

a thesis of AAA_1, AOO_2, OAO_3, *a thesis of* EAE_1, EIO_2, IAI_3, *and* II_2.

Proof. Let

G_1: AAA_1, AOO_2, OAO_3,

G_2: EAE_1, EIO_2, IAI_3,

G_5: EIO_1, EAE_2, AII_3,

G_7: AEE_4, EIO_4, IAI_4.

Then each group $G_i (i=1, 2, 5, 7)$ is equivalent (see [6]). Put $\alpha = Iab$, $\beta = A_{ac}$, $\gamma = Icd$, $\delta = Ibc$ in $T2$, then we have

$$I_{ab}A_{ac}I_{cb} + I_{cb}I_{bc} \to I_{ab}A_{ac}I_{bc} \to A_{ac}I_{ab}I_{bc} \text{ (by } T1) \to AII_3.$$

Hence we obtain G_5: AII_3, EIO_1, EAE_2.

Put $\alpha = I_{ab}$, $\beta = I_{ba}$ in $T3$, then by $D1$, we have

$$I_{ab}I_{ba} \to NI_{ba}NI_{ab} = E_{ba}E_{ab} \to EE_2.$$

Put $\alpha = E_{ab}$, $\beta = A_{ca}$, $\gamma = E_{cb}$, $\delta = E_{bc}$ in $T2$, then we have

$$E_{ab}A_{cb}E_{cb} + E_{cb}E_{bc} \to E_{ab}A_{ca}E_{bc} \to A_{ca}E_{ab}E_{bc} \text{ (by } T1) \to AEE_4.$$

Hence we have $G7$: AEE_4, EIO_4, IAI_4.

The set of $L1$, $L2$, AAA_1 (one of $G1$), and AII_3 (one of $G5$) is the well known Lukasiewicz axiom system. The set of $L1$, $L2$, AAA_1, and EIO_1 (one of $G5$) is Bochenski axiom system. The set of $L1$, $L2$, OAO_3 (one of $G1$), AII_3(one of $G5$), the set of $L1$, $L2$, AOO_2 (one of $G1$), EAE_2 (one of $G5$), and the set of $L1$, $L2$, AOO_2, EIO_1 (one of $G5$) are Iséki axiom systems (see [2]). The set of $L1$, $L2$, any one of $G1$ and any one of $G7$ is Tanaka axiom system (see [6]). Therefore the proof is complete.

References

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