56. *On Limit Spaces and the Double Weak Limit. II*

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§1. Introduction. Continuing our study on the limit spaces, in §2, let us show the difference between the (principal ideal) limit space and the topological space by the construction of some concrete limit spaces which characterize the generalized double weak limits (itself or with the restriction on sign) expressed by the filter. For terminologies, notations and references, see the paper [0]. Example II-1 shows the topological space J_w which characterizes the generalized double weak limit. Example II-2 shows the limit space J_{pw} defined on $\tilde{J}$ (with the restriction on sign at $x=0$) not to be the principal ideal limit space. Example II-3 shows the non-topological principal ideal limit space J_{sep} (with the restriction on sign in $(-\infty, \infty)$) defined on $\tilde{J}$. Example II-4 shows the concrete form $J_\wedge=(\tilde{J}, \tau_\wedge)$ of $\wedge$ ideal not to be a limit space shown in [0] Example I-1. τ_w, τ^p_∞ in Examples II-1, II-3 are given by the construction of the base of the weakest filter which becomes the fundamental system of neighbourhoods or the one like it. τ_p in Example II-2 is given by the construction of the join of τ^p_δ's in a principal ideal limit spaces $(\tilde{J}, \tau^p_\delta)$ (with the restriction on sign in $(-\delta, \delta)$) [0] Lemma I-7. In the construction of J_{pw}, we use the thought like the depth in ranked space [9] p. 5. We show in the final part of §2 that $\bar{L}_2 \supseteq \tilde{J}$, $\bar{L}_2 \supseteq U_{\delta>0} \tilde{J}_\delta$ and $\bar{L}_2 \supseteq \tilde{J}_\infty$ hold in J_w, J_{pw} and J_{sep} respectively. Here $\tilde{J}_\delta \equiv \{\mathfrak{f}\,;\, \exists\{f_n\} \in \tilde{\mathfrak{f}} \in \tilde{J}$ such that $f_m \cdot f_n \geqslant 0$ for any $m, n>0$ and for *a.e.* $x \in (-\delta, \delta)\}$. Finally we show that the axioms of the separation (T_1) (T_2) are not satisfied by J_w, J_{pw} and J_{sep}. The neglect of the sign $\pm$ in double weak limit leads to this result in Examples II-1, II-2 which becomes a remark. The space L_2 in J_{sep} (shown in Example II-3) satisfies (T_1) and (T_2). This detailed investigation on $\tilde{J}$, J_w etc. contributes to the investigation on the generalized eigenvalue problem (concerning to continuous spectrum) appearing in [10].

§2. Examples of limit space with the form $(\tilde{J}, \tau)$. 2.1. Let $\tilde{\mathfrak{f}}$ (or $\mathfrak{f}$) be the equivalent class $[\{g_n\}; \{f_n - g_n\} \in \tilde{O}, \{g_n\} \in J]$ to $\{f_n\} \in J$. Let's construct the families of the sets (contained in $\tilde{J}$) $G(\mathfrak{f})$ and $G^p_\delta(\mathfrak{f})$ corresponding to $\mathfrak{f} \in \tilde{J}$.

Definition II-1. $G(\mathfrak{f})$ is the family of all sets $V(\mathfrak{f}\,;\, \varepsilon, \{\varphi_i\,;\, i=1, 2,$

$\cdots, p\}) \equiv [\mathfrak{g}\,; \{g_n\} \in \tilde{\mathfrak{g}}, \{f_n\} \in \tilde{\mathfrak{f}}$ such that $\lim_{n\to\infty}\left|\int (g_n^2 - f_n^2)\varphi_i dx\right| \leqslant \varepsilon,\ i=1,2, \cdots, p]$ dependent on $\varepsilon>0$, and on the finite set $\{\varphi_i \in B\,; i=1,2,\cdots,p\}$.

According to the corollary of Lemma I-5 in [0] the definition of $V(\mathfrak{f}\,; \varepsilon, \{\varphi_i\,; i=1,2,\cdots,p\})$ is independent of the choice of $\{g_n\}$, $\{f_n\}$ in $\tilde{\mathfrak{g}}, \tilde{\mathfrak{f}}$.

Definition II-2. $G_\delta^p(\mathfrak{f})$ is the family of all sets $V_p(\mathfrak{f}\,; \varepsilon, \delta, \{\varphi_i\,; i=1,2,\cdots,p\}) \equiv [\mathfrak{g}\,; \exists\{g_n\} \in \tilde{\mathfrak{g}}, \exists\{f_n\} \in \tilde{\mathfrak{f}}$ such that $\lim_{n\to\infty}\left|\int (g_n^2 - f_n^2)\varphi_i dx\right| \leqslant \varepsilon$, $i=1,2,\cdots,p$, and $\exists\{g_n^0\} \in \tilde{\mathfrak{g}}$, $\exists\{f_n^0\} \in \tilde{\mathfrak{f}}$ such that $g_n^0(x)\cdot f_n^0(x) \geqslant 0$ for $a.e.\ x \in (-\delta, \delta)]$ dependent on $\varepsilon>0$, on $\delta>0$ and on the finite set $\{\varphi_i \in B\,; i=1,2,\cdots,p\}$.

Since, for any $A, B \in G(\mathfrak{f})$ $(G_\delta^p(\mathfrak{f}))$, there exists $C \in G(\mathfrak{f})$ $(G_\delta^p(\mathfrak{f}))$ such that $A \cap B \supseteq C$ holds, $G(\mathfrak{f})$ $(G_\delta^p(\mathfrak{f}))$ consisting of the non-void sets becomes the base of a filter.

2.2. Example II-1. Let $\tau_w\mathfrak{f}(\mathfrak{f} \in \tilde{J})$ be the set consisting of the filters finer than (or equal to) the one with the base $G(\mathfrak{f})$ in Definition II-1. $\tau_w\mathfrak{f}$ satisfies (L^1) (L^2) and (L^3) from [0] Lemma I-7, for $\mathfrak{f}$ is contained in all elements of $G(\mathfrak{f})$. The pair $(\tilde{J}, \tau_w)$ is denoted by J_w.

Theorem II-1. *J_w satisfies (L^4).*

Proof. If V is the set contained in the weakest filter in $\tau\mathfrak{f}$, there exists a $V(\mathfrak{f}\,; \varepsilon, \{\varphi_i\,; i=1,2,\cdots,p\}) \in G(f)$ such that $V \supseteq V(\mathfrak{f}\,; \varepsilon, \{\varphi_i\,; i=1,2\cdots,p\})$ holds.

Let $W(\in[\mathfrak{B}(\mathfrak{f})])$ be $V(\mathfrak{f}\,; \varepsilon/2, \{\varphi_i\,; i=1,2,\cdots,p\})$ and $\mathfrak{h}$ be an arbitrary fixed element of W. Since $V \supseteq W$ holds, and since $\lim_{n\to\infty}\left|\int (g_n^2 - f_n^2)\varphi_i dx\right| \leqslant \lim_{n\to\infty}\left|\int (g_n^2 - h_n^2)\varphi_i dx\right| + \lim_{n\to\infty}\left|\int (h_n^2 - f_n^2)\varphi_i dx\right| \leqslant \varepsilon/2 + \varepsilon/2 \leqslant \varepsilon$ $(i=1,2,\cdots,p)$ holds for any $\{f_n\} \in \tilde{\mathfrak{f}}$, any $\{h_n\} \in \tilde{\mathfrak{h}} \in W$ and any $\{g_n\} \in \tilde{\mathfrak{g}} \in V(\mathfrak{h}\,; \varepsilon/2, \{\varphi_i\,; i=1,2,\cdots,p\})$, $V \supseteq V(\mathfrak{h}\,; \varepsilon/2, \{\varphi_i\,; i=1,2,\cdots,p\})$ holds for any $\mathfrak{h} \in W$. Then $V \in \tau_w\mathfrak{h}$ holds for any $\mathfrak{h} \in W \subseteq V$, and J_w satisfies (L^4).

Hence J_w becomes a topological space which is called double weak topological space. Because if $\lim_{n\to\infty}\int f_n^2\varphi dx$ becomes finite and definite for all $\varphi \in B$ $(f_n \in L^2_{(-\infty,\infty)})$, the filter with the base $[\{f_n\,; n \geqslant i\}\,; i=1,2,\cdots]$ is finer than the weakest filter $\mathfrak{F}_0$ in $\tau_w\mathfrak{f}$, where $\mathfrak{f}$ is the equivalent class to $\{f_n\} \in J$. Namely, for any $F \in (\mathfrak{F}_0)$, there exist i_0 such that $F \supseteq \{f_n\,; n \geqslant i\}$ for any $i \geqslant i_0$.

2.3. In the following we give the limit space like J_w by using τ non-uniformly dependent on x (in f_n's domain).

Example II-2. Let $\tau_\delta^p\mathfrak{f}$ be the set of all filters finer than (or equal to) the one with the base $G_\delta^p(\mathfrak{f})$ (see Definition II-2), and let $\tau_p\mathfrak{f} = U_{\delta>0}\tau_\delta^p\mathfrak{f}$. The pair $(\tilde{J}, \tau_p)$ is denoted by J_{pw}. It can be shown that

J_{pw} satisfies (L^1) (L^2) but does not satisfy (L^3). This τ_p differs from τ_w in $x=0$.

Theorem II-2. *J_{pw} does not satisfy (L^3).*

Proof. Let $\{f_n\}(\in J)$ be the sequence satisfying $f_n(x)>0$ for any n and for any x contained in a neighbourhood of zero, and $\mathfrak{f}$ (or $\tilde{\mathfrak{f}}$)$(\in \tilde{J})$ be the equivalent class of $\{f_n\}$. If $\mathfrak{F}$ is contained in $U_{\delta>0}\tau_\delta^p\mathfrak{f}$, $\mathfrak{F}$ is contained in $\tau_\delta^p\mathfrak{f}$ for a given $\delta>0$. Since there exists an element B_A in $G_{\delta/2}^p(\mathfrak{f})$ which truely contains any given element A in $G_\delta^p(\mathfrak{f})$, and since there is no element in $G_\delta^p(\mathfrak{f})$ which contains any given element B in $G_{\delta/2}^p(\mathfrak{f})$, the weakest $\tilde{\mathfrak{F}}_0\in\tau_{\delta/2}^p\mathfrak{f}$ satisfies $\tilde{\mathfrak{F}}_0<\mathfrak{F}$ (truely finer) for any $\mathfrak{F}\in\tau_\delta^p\mathfrak{f}$. Then, the weakest filter is not contained in $U_{\delta>0}\tau_\delta^p\mathfrak{f}$, and J_{pw} does not satisfy (L^3).

Theorem II-3. *J_{pw} satisfies (L^1), (L^2).*

Proof. $(\tilde{J},\tau_\delta^p)$ for any $\delta>0$ satisfies (L^1), (L^2) and (L^3) from [0] Lemma I-7. Since $\tau_{\delta'}\mathfrak{f}\supset\tau_\delta\mathfrak{f}$ holds from the proof of Theorem II-2 for any pair (δ,δ') satisfying $\delta>\delta'>0$ and for any $\mathfrak{f}\in\tilde{J}$, (L^1) and (L^2) are satisfied by J_{pw}. Let us show it in the following.

(1) If $\mathfrak{F}_1\in\tau_p\mathfrak{f}$, $\mathfrak{F}_1\in\tau_\delta^p\mathfrak{f}$ for a given $\delta>0$. Then, if $\mathfrak{F}\geqslant\mathfrak{F}_1$, $\mathfrak{F}\in\tau_\delta^p\mathfrak{f}\subseteq\tau_p\mathfrak{f}$.

(2) If $\mathfrak{F}_1,\mathfrak{F}_2\in\tau_p\mathfrak{f}$, $\mathfrak{F}_1,\mathfrak{F}_2\in\tau_\delta^p\mathfrak{f}$ for a given $\delta>0$, and $\mathfrak{F}_1\cap\mathfrak{F}_2\in\tau_\delta^p\mathfrak{f}\subseteq\tau_p\mathfrak{f}$.

(3) $[\mathfrak{f}]\in\tau_\delta^p\mathfrak{f}\subseteq\tau_p\mathfrak{f}$.

2.4. Example II-3. Let J_{sep} be the limit space $(\tilde{J},\tau_\infty^p)$ (see Defi-. nition II-2). J_{sep} becomes the principal ideal limit space from [0] Lemma I-7.

Let $E(x)=\exp(-x^2)$, $E_K^{(1)}(x)=\begin{cases}\exp(-x^2) & \text{for } |x|\leqslant K,\\ 0 & \text{for } |x|>K\end{cases}$ and

$$E_K^{(2)}(x)=\begin{cases}\exp(-x^2) & \text{for } |x|\leqslant K\\ -\exp(-x^2) & \text{for } |x|>K.\end{cases}$$

Let $\{E(x)\}$, $\{E_K^{(i)}(x)\}$ $i=1,2$ be the sequences $\{E(x),E(x),\cdots\}$, $\{E_K^{(i)}(x), E_K^{(i)}(x),\cdots\}$ $i=1,2$, (contained in J), and $E(x), E_K^{(i)}(x)$ (or $\tilde{E}(x),\tilde{E}_K^{(i)}(x)$) $i=1,2$ be their equivalent classes respectively. Let $V_p(E(x);\varepsilon,\infty,\{\varphi_i\in B;i=1,2,\cdots,p\})$ be the set shown in Definition II-2.

Theorem II-4. *J_{sep} does not satisfy (L^4).*

Proof. $E_{K_1}^{(1)}(x)$ and $E_{K_2}^{(2)}(x)$ are also contained in the sets $V_p(E(x);\varepsilon,\infty,\{\varphi_i\in B;i=1,2,\cdots,p\})$, $V_p(E_{K_2}^{(1)}(x);\varepsilon,\infty,\{\varphi_i\in B;i=1,2,\cdots,p\})$ respectively for sufficiently large K_1 and K_2. If $\{g_n^0(x)\}$ and $\{f_n^0(x)\}$ are the sequences $\{g_n^0(x)\}\in\tilde{E}_{K_3}^{(2)}(x)$ and $\{f_n^0(x)\}\in\tilde{E}(x)$ satisfying $g_n^0(x)\cdot f_n^0(x)\geqslant 0$ for *a.e.* $x\in(-\infty,\infty)$ regardless of n, then $\int_{(-\infty,-K_3)\cup(K_3,\infty)}\{|g_n^0(x)+\exp(-x^2)|+|f_n^0(x)-\exp(-x^2)|\}^2dx>\int_{(-\infty,-K_3)\cup(K_3,\infty)}\exp(-2x^2)dx$ must hold from $\mathrm{Max}\{|g_n^0(x)+\exp(-x^2)|,|f_n^0(x)-\exp(-x^2)|\}\geqslant\exp(-x^2)$ in

$(-\infty, -K_3)\cup(K_3, \infty)$, and it is contradict to $\lim_{n\to\infty}\int_{(-\infty,-K_3)\cup(K_3,\infty)}\{|g_n^0(x)+\exp(-x^2)|+|f_n^0(x)-\exp(-x^2)|\}^2dx=0$ derived from $\{|g_n^0(x)-E_{K_3}^{(2)}(x)|+|f_n^0(x)-E(x)|\}\in\tilde{O}$. Then $g_n(x)\cdot f_n(x)\geqslant 0$ does not hold for $a.e.\ x\in(-\infty,\infty)$ regardless of n (for any $\{g_n(x)\}\in\tilde{E}_{K_3}^{(2)}(x)$ and any $\{f_n(x)\}\in\tilde{E}(x)$), and $E_{K_3}^{(2)}(x)$ is not contained in $V_p(E(x);\varepsilon,\infty,\{\varphi_i\in B;i=1,2,\cdots,p\})$ for any $K_3>0$. Since $V_p(E(x);\varepsilon,\infty,\{\varphi_i\in B;i=1,2,\cdots\cdots,p\})$ for any $\varepsilon>0$ contains $E_K^{(1)}(x)$ for any $K\geqslant K_1(\varepsilon)>0$, and since $V_p(E(x);\varepsilon,\infty,\{\varphi_i\in B;i=1,2,\cdots,p\})\not\supseteq V_p(E_K^{(1)}(x);\varepsilon',\infty,\{\varphi_i\in B;i=1,2,\cdots,p\})$(from $\ni E_K^{(2)}(x)$ for any $K>K_2(\varepsilon')>0$ etc.) holds for any $\varepsilon'>0$ and any $K>K_1(\varepsilon)$, we can easily see that J_{sep} does not satisfy $(\boldsymbol{L}^4)$.

2.5. Example II-4. Let $\mathfrak{F}_0(\mathfrak{f})$ be the weakest filter in $\tau_w\mathfrak{f}$ ($\mathfrak{f}\in\tilde{J}$), and $G^\wedge(\mathfrak{f})\equiv\{A_\alpha-\mathfrak{f};\ A_\alpha\in(\mathfrak{F}_0(\mathfrak{f}))\}$. Let $\tau_\wedge\mathfrak{f}$ be the set consisting of the filters finer than (or equal to) the one with the base $G^\wedge(\mathfrak{f})$. The pair (i.e. pure $\wedge$ ideal) $(\tilde{J},\tau_\wedge)$ is denoted by $J_\wedge$.

2.6. Definition II-3. $\delta^{1/2}$ is defined by the set of the equivalent classes consisting of the sequences contained in J satisfying $\lim_{n\to\infty}\int f_n^2\varphi dx=\varphi(0)$ for any $\varphi\in B$. $\nu^{1/2}$ is defined by the set of the equivalent classes consisting of the sequences contained in J satisfying $\lim_{n\to\infty}\int f_n^2\varphi dx=\lim_{T\to\infty}1/(2T)\cdot\int_{-T}^{T}\varphi dx$ for any $\varphi\in B$.

Let $\tilde{J}_\delta$ be the set $[\mathfrak{f};\exists\{f_n\}\in\tilde{\mathfrak{f}}\in\tilde{J}$ such that $f_m\cdot f_n\geqslant 0$ holds for any $m,n>0$ and for $a.e.\ x\in(-\delta,\delta)]$. We can easily show the concrete element of $\delta^{1/2}$ and $\nu^{1/2}$ contained in $\tilde{J}_\infty$. Namely, let

$$f_n^0(x)=\begin{cases}\sqrt{n} & \text{for } |x|\leqslant 1/(2n)\\ 0 & \text{for } |x|>1/(2n)\end{cases}\quad\text{and let}\quad g_n^0(x)=\begin{cases}1/\sqrt{n} & \text{for } |x|\leqslant n/2\\ 0 & \text{for } |x|>n/2.\end{cases}$$

The equivalent class of $\{f_n^0(x)\}$ is contained in $\delta^{1/2}\cap\tilde{J}_\infty$, and the equivalent class of $\{g_n^0(x)\}$ is contained in $\nu^{1/2}\cap\tilde{J}_\infty$.

Theorem II-5. (a) *The closure of L_2 in J_w contains $\tilde{J}$.* (b) *The closure of L_2 in J_{sep} contains $\tilde{J}_\infty$, and the closure of L_2 in J_{pw} contains $U_{\delta>0}\tilde{J}_\delta$.*

Proof. (a) Let $\mathfrak{f}$ (or $\tilde{\mathfrak{f}}$) be the element in $\tilde{J}$ equivalent to an arbitrary element $\{f_n;n=1,2,\cdots\}\in J$, and f_n be the equivalent class to $\{f_n,f_n,\cdots\}\in J$. Since, for any $\varepsilon>0$, there exists a positive integer N_0 such that $f_{n_0}\in V(\mathfrak{f};\varepsilon,\{\varphi_i;i=1,2,\cdots,p\})$ holds for $n_0>N_0$, the weakest filter $\mathfrak{F}_0\in\tau_w\mathfrak{f}$ satisfies $F\cap L_2\neq\phi$ for all $F\in(\mathfrak{F}_0)$, and $\bar{L}_2\supseteq\tilde{J}$ holds in J_w.

(b) Since, for any $\varepsilon>0$ and for any element $\mathfrak{f}\in\tilde{J}_\delta$, there exists a positive integer N_0 such that $f_{n_0}\in V_p(\mathfrak{f};\varepsilon,\delta,\{\varphi_i;i=1,2,\cdots,p\})$ holds for $n_0>N_0$, all element F of the weakest filter $\mathfrak{F}_0\in\tau_\delta^p\mathfrak{f}(\mathfrak{f}\in\tilde{J}_\delta)$ satisfies $F\cap L_2\neq\phi$. Then $\bar{L}_2\supseteq\tilde{J}_\infty$ holds in J_{sep}, $\bar{L}_2\supseteq\tilde{J}_\delta$ holds in $(\tilde{J},\tau_\delta^p)$ and $\bar{L}_2\equiv U_{\delta>0}(\bar{L}_2$ by $\tau_\delta^p)\supseteq U_{\delta>0}\tilde{J}_\delta$ holds in J_{pw}. Because $\tilde{J}_{\delta_1}\supseteq\tilde{J}_{\delta_2}$ holds for

$0<\delta_1<\delta_2$, and $\bar{L}_2$ in $(\tilde{J}, \tau^p_{\delta_1})$ contains $\bar{L}_2$ in $(\tilde{J}, \tau^p_{\delta_2})$ for $0<\delta_1<\delta_2$ from $\tau^p_{\delta_1}\mathfrak{f} \supseteq \tau^p_{\delta_2}\mathfrak{f}$.

§ 3. The axioms of separation.

Theorem II-6. *The topology τ_w (or limit τ_p) in $\tilde{J}$ does not satisfy the axioms of separation (T_1) and (T_2) (see* [0] §1, **1.1.***).*

Proof. Let $u(x)$ $(-\infty<x<+\infty)$ be a square integrable function satisfying $u(x)>0$ in a set $(-M, M)-(-M/2, M/2)$, where $M>0$. Choose a sequence $\{u^{(1)0}_n\} \in \tilde{\mathfrak{u}}^{(1)} \in \tilde{J}$ satisfying $|u^{(1)0}_n(x)|>|u(x)|$ in $(-\infty, \infty)-(-M/2, M/2)$ for any n, and satisfying $u^{(1)0}_n(x)\equiv 0$ in $(-M/2, M/2)$ for any n. Next let $\{u^{(2)0}_n\} \in \tilde{\mathfrak{u}}^{(2)} \in \tilde{J}$ be the sequence consisting of the functions satisfying $(u^{(1)0}_n)^2=(u^{(2)0}_n)^2$. Even if $u^{(1)0}_n \neq u^{(2)0}_n$ holds on a set $(-M, M)-(-M/2, M/2)$ regardless of n, $\tau_w\mathfrak{u}^{(1)}=\tau_w\mathfrak{u}^{(2)}$ holds. Then $[\mathfrak{u}^{(1)}] \in \tau_w\mathfrak{u}^{(2)}$ holds, and (T_1) ([0] §1. **1.1.**) is not satisfied in J_w. Here $[\mathfrak{u}^{(1)}]$ means the filter $(\in \tau_w\mathfrak{u}^{(1)})$ with the base $\{\mathfrak{u}^{(1)}\}$. Since $\tau_w\mathfrak{u}^{(1)} \cap \tau_w\mathfrak{u}^{(2)}=\tau_w\mathfrak{u}^{(1)} \neq \phi$ also holds, then (T_2) is not satisfied. By using the same $\mathfrak{u}^{(1)}$ and $\mathfrak{u}^{(2)}$, we can easily prove that τ_p does not satisfy (T_1) and (T_2).

From the difficulty like the above impossibility of the separation, the special decomposition and construction of the function (elements of the sequence) has been used by us in [7] p. 340.

Theorem II-7. *L_2 in J_{sep}* [1] p. 39 *satisfies (T_1) and (T_2).*

Proof. (I) Let's show here that $x=y$ hold from $[x] \in \tau y$.

Let $g \in L_2$ be an element in $\cap_{\{\varphi_i\}, \varepsilon>0} V_p(f; \varepsilon, \infty, \{\varphi_i; i=1, 2, \cdots, p\})$ $(f \in L_2)$, and let $\{g^0_n\} \in \tilde{g}$ and $\{f^0_n\} \in \tilde{f}$ satisfying $f^0_n(x)\cdot g^0_n(x) \geqslant 0$ for *a.e.* $x \in (-\infty, \infty)$. Since $\int (g^2-f^2)\varphi dx = \lim_{n\to\infty} \int (g^{02}_n - f^{02}_n)\varphi dx = 0$ holds for any $\varphi \in B$ from the corollary of Lemma 1-5, $g^2=f^2$ holds for *a.e.* $x \in (-\infty, \infty)$. Furthermore, since $g^0_n(x)\cdot f^0_n(x) \geqslant 0$ holds for *a.e* $x \in (-\infty, \infty)$, $g-f=0$ holds for *a.e.* $x \in (-\infty, \infty)$. Then $\cap_{\{\varphi_i\}, \varepsilon>0} V_p(f; \varepsilon, \infty, \{\varphi_i; i=1, 2, \cdots, p\}) \equiv f$ holds. Since $\cap_{A_\alpha \in \mathfrak{F}_0} A_\alpha \equiv f$ holds for the weakest filter $\mathfrak{F}_0$ in $\tau^p_\infty f$, it follows that $f=g(\in L_2)$ holds from $[g] \in \tau^p_\infty f$. Then L_2 in J_{sep} satisfies (T_1).

(II) Since $\cap_{\{\varphi_i\}, \varepsilon>0} V_p(f; \varepsilon, \infty, \{\varphi_i; i=1, 2, \cdots, p\}) \equiv f$ holds, $\cap_{A_\alpha \in \mathfrak{F}} A_\alpha = f$ holds for any $\mathfrak{F} \in \tau^p_\infty f$. Then, if $f \neq g$ (for $f, g \in L_2$) holds, $\tau^p_\infty g \cap \tau^p_\infty f = \phi$ holds, and L_2 in J_{sep} satisfies (T_2).

Let f^0_n be the function defined in § 2. 2.6. If $\mathfrak{f}_1$, $\mathfrak{f}_2$ are the equivalent classes of $\{f^0_n(x-1/(2n))\}$ and $\{f^0_n(x+1/(2n))\}$, $\bigcap_{\{\varphi_i\}, \varepsilon>0} V_p(\mathfrak{f}_1; \varepsilon, \{\varphi_i; i=1, 2, \cdots, p\}) \ni \mathfrak{f}_2$ and $\mathfrak{f}_1 \neq \mathfrak{f}_2$ in $\tilde{J}$. Then (T_1) and (T_2) are not satisfied in J_{sep}.

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