59. **Notes on Regular Semigroups**

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In this note we shall give ideal-theoretical characterizations of regular semigroups whose left and/or right ideals are two-sided. Some ideal-theoretical characterizations of the class of regular semigroups were given in the author's recent paper [3].

For the notation and terminology we refer to A. H. Clifford and G. B. Preston's book [1].

**Theorem 1.** For a semigroup $S$ the following conditions are pairwise equivalent.

1. $S$ is a regular semigroup whose left ideals are two-sided.
2. $B \cap L = BL$ for every bi-ideal $B$ and every left ideal $L$ of $S$.
3. $L \cap Q = QL$ for each left ideal $L$ and each quasi-ideal $Q$ of $S$.

Proof. (1) implies (2). Suppose that $S$ is a regular semigroup whose left ideals are two-sided. Then by a recent result of the author [2] every bi-ideal $B$ of $S$ may be represented in the form

$$B = RI,$$

where $R$ is a suitable right ideal and $I$ is a suitable two-sided ideal of $S$.

Next applying the well known regularity criterion due to L. Kovács and K. Iséki (see [1], p. 34) we obtain

$$B \cap L = RI \cap L = RIL = BL$$

for every bi-ideal $B$ and every left ideal $L$ of $S$.

(2) implies (3). This is evident because every quasi-ideal of an arbitrary semigroup $S$ is a bi-ideal of $S$.

(3) implies (1). Let $S$ be a semigroup with property (3). Then in case $Q = R$, $R$ is an arbitrary right ideal of $S$, (3) implies that $S$ is regular. Secondly in case $L = S$, $Q = L$, $L$ is an arbitrary left ideal of $S$, condition (3) implies

$$L = L \cap S = LS,$$

that is, any left ideal $L$ is also a right ideal of $S$.

The proof of our Theorem 1 is complete.

We state the left-right dual of Theorem 1.

**Theorem 2.** For a semigroup $S$ the following assertions are mutually equivalent.

4. $S$ is regular and each right ideal of $S$ is two-sided.
5. $B \cap R = RB$ for any bi-ideal $B$ and for any right ideal $R$ of $S$.
6. $Q \cap R = RQ$ for every right ideal $R$ and every quasi-ideal $Q$.
of $S$.

Next we formulate a criterion for a semigroup to be a semilattice of groups. This is a sharpening of an earlier criterion due to A. H. Clifford and G. B. Preston [1] and its proof is quite similar to that of Theorem 2 in the author’s paper [4], and we omit it.

Theorem 3. A semigroup $S$ is a semilattice of groups if and only if $S$ is regular and every one-sided ideal of $S$ is two-sided.

Finally Theorem 1, Theorem 2, and Theorem 3 imply the following result.

Theorem 4. An arbitrary semigroup $S$ is a semilattice of groups if and only if it satisfies both the conditions (i) and (j) of Theorem 1 and Theorem 2, where $i=1, 2$, or $3$ and $j=4, 5$, or $6$.

References