145. *On Some Results Involving Jacobi Polynomials and the Generalized Function $\tilde{\omega}_{\mu_1,\cdots,\mu_n}(x)$. II*

By R. S. DAHIYA

(Comm. by Kinjirô KUNUGI, M. J. A., Sept. 12, 1970)

4. Particular cases. In (3.1) and (3.2), taking $n=2$; we get

(4.1) $x^{\rho}\tilde{\omega}_{\mu_1,\mu_2}[\lambda x^{h/2}]$

$$=\frac{h^{-\beta-1}}{2\pi}\sum_{r=0}^{\infty}\frac{(\alpha+\beta+2r+1)\Gamma(\alpha+\beta+r+1)}{\Gamma(\alpha+r+1)}\sum_{i,-i}\frac{1}{i}G^{h+1,h+3}_{2h+5,2h+1}$$
$$\times\left(\frac{16e^{i\pi}}{\lambda^2}\,\middle|\,\begin{matrix}\left(\frac{3}{4}-\frac{\mu_1}{2}\right),\ \frac{3}{4}-\frac{\mu_2}{2},\ \Delta(h,\rho-r+1),1,\\ \Delta(h,\rho+\alpha+1),1,\end{matrix}\right.$$
$$\left.\begin{matrix}\frac{3}{4}+\frac{\mu_1}{2},\frac{3}{4}+\frac{\mu_2}{2},\ \Delta(h,\beta+\rho+\alpha+r+2)\\ \Delta(h,\rho+1)\end{matrix}\right)P_r^{(\alpha,\beta)}(1-2x),$$

where h is a positive number and $\tilde{\omega}_{\mu_1,\mu_2}(x)$ is a Watson's Fourier Kernal [1]. $R(\rho)\geq -1$.

(4.2) $x^{\rho}\tilde{\omega}_{\mu_1,\mu_2}[\lambda x^{-h/2}]$

$$=\frac{h^{-\beta-1}}{2\pi}\sum_{r=0}^{\infty}\frac{(\alpha+\beta+2r+1)\Gamma(\alpha+\beta+r+1)}{\Gamma(\alpha+r+1)}\sum_{i,-i}\frac{1}{i}G^{h+1,h+3}_{2h+5,2h+1}$$
$$\times\left(\frac{16e^{i\pi}}{\lambda^2}\,\middle|\,\begin{matrix}\left(\frac{3}{4}-\frac{\mu_1}{2}\right),\frac{3}{4}-\frac{\mu_2}{2},\ \Delta(h,\alpha-\rho),1,\\ \Delta(h,r-\rho),1,\end{matrix}\right.$$
$$\left.\begin{matrix}\frac{3}{4}+\frac{\mu_1}{2},\frac{3}{4}+\frac{\mu_2}{2},\Delta(h,\ -\rho)\\ \Delta(h,\ -\alpha-\beta-\rho-r-1)\end{matrix}\right)P_r^{(\alpha,\beta)}(1-2x),$$

where h is a positive number and $R(\rho)\geq -1$. Further setting $\mu_1=\mu_2-1=\mu$ in (4.1) and (4.2) will give the following results for Bessel functions.

(4.3) $x^{\rho}J_{2\mu+1}(2\lambda^{1/2}x^{h/4})$

$$=\frac{h^{-\beta-1}}{2\pi}\sum_{r=0}^{\infty}\frac{(\alpha+\beta+2r+1)\Gamma(\alpha+\beta+r+1)}{\Gamma(\alpha+r+1)}\sum_{i,-i}\frac{1}{i}G^{h+1,h+3}_{2h+5,2h+1}$$
$$\times\left(\frac{16e^{i\pi}}{\lambda^2}\,\middle|\,\begin{matrix}\left(\frac{3}{4}-\frac{\mu}{2}\right),\frac{5}{4}-\frac{\mu}{2},\ \Delta(h,\rho-r+1),1,\\ \Delta(h,\rho+\alpha+1),1,\end{matrix}\right.$$
$$\left.\begin{matrix}\frac{3}{4}+\frac{\mu}{2},\frac{1}{4}+\frac{\mu}{2},\ \Delta(h,\beta+\rho+\alpha+r+2)\\ \Delta(h,\rho+1)\end{matrix}\right)P_r^{(\alpha,\beta)}(1-2x),$$

where h is a positive number and $R(\rho)\geq -1$.

$$(4.4)\quad x^{\rho}J_{2\mu+1}(2\lambda^{1/2}x^{-h/4}) = \frac{h^{-\beta-1}}{2\pi}\sum_{r=0}^{\infty}\frac{(\alpha+\beta+2r+1)\Gamma(\alpha+\beta+r+1)}{\Gamma(\alpha+r+1)}\sum_{i,-i}\frac{1}{i}G_{2h+5,2h+1}^{h+1,h+3}\left(\frac{16e^{i\pi}}{\lambda^2}\middle|\begin{matrix}\frac{3}{4}-\frac{\mu}{2},\frac{5}{4}-\frac{\mu}{2},\Delta(h,\alpha-\rho),1,\frac{3}{4}+\frac{\mu}{2},\frac{1}{4}+\frac{\mu}{2},\Delta(h,-\rho)\\ \Delta(h,r-\rho),1,\Delta(h,-\alpha-\beta-\rho-r-1)\end{matrix}\right)P_r^{(\alpha,\beta)}(1-2x),$$

where h is a positive number and $R(\rho)\geq -1$. Now if we take $n=1$ in (3.1) and (3.2), we obtain

$$(4.5)\quad x^{\rho+h/4}J_{\mu}(\lambda x^{h/2}) = \frac{h^{-\beta-1}}{\sqrt{2\lambda\pi}}\sum_{r=0}^{\infty}\frac{(\alpha+\beta+2r+1)\Gamma(\alpha+\beta+r+1)}{\Gamma(\alpha+r+1)}\sum_{i,-i}\frac{1}{i}G_{2h+3,2h+1}^{h+1,h+2}\left(\frac{4e^{i\pi}}{\lambda^2}\middle|\begin{matrix}\frac{3}{4}-\frac{\mu}{2},\Delta(h,\rho-r+1),1,\frac{3}{4}+\frac{\mu}{2},\Delta(h,\beta+\rho+\alpha+r+2)\\ \Delta(h,\rho+\alpha+1),1,\Delta(h,\rho+1)\end{matrix}\right)P_r^{(\alpha,\beta)}(1-2x),$$

$$(4.6)\quad x^{\rho-h/4}J_{\mu}(\lambda x^{-h/2}) = \frac{h^{-\beta-1}}{\sqrt{2\lambda\pi}}\sum_{r=0}^{\infty}\frac{(\alpha+\beta+2r+1)\Gamma(\alpha+\beta+r+1)}{\Gamma(\alpha+r+1)}\sum_{i,-i}\frac{1}{i}G_{2h+3,2h+1}^{h+1,h+2}\left(\frac{4e^{i\pi}}{\lambda^2}\middle|\begin{matrix}\frac{3}{4}-\frac{\mu}{2},\Delta(h,\alpha-\rho),1,\frac{3}{4}+\frac{\mu}{2},\Delta(h,-\rho)\\ \Delta(h,r-\rho),1,\Delta(h,-\alpha-\beta-\rho-r-1)\end{matrix}\right)P_r^{(\alpha,\beta)}(1-2x),$$

where h is a positive integer and $R(\rho)\geq -1$. From (4.5) and (4.6), we obtain

$$(4.7)\quad \int_0^1 x^{\rho+h/4}(1-x)^{\beta}P_n^{(\alpha,\beta)}(1-2x)J_{\mu}(\lambda x^{h/2})dx = \frac{h^{-\beta-1}\Gamma(\beta+n+1)}{\pi\sqrt{2\lambda}\,\Gamma(n+1)}\sum_{i,-i}\frac{1}{i}G_{2h+3,2h+1}^{h+1,h+2}\left(\frac{4e^{i\pi}}{\lambda^2}\middle|\begin{matrix}\frac{3}{4}-\frac{\mu}{2},\Delta(h,\rho-\alpha-n+1),1,\frac{3}{4}+\frac{\mu}{2},\Delta(h,\beta+\rho+n+2)\\ \Delta(h,\rho+1),1,\Delta(h,\rho-\alpha+1)\end{matrix}\right),$$

$$(4.8)\quad \int_0^1 x^{\rho-h/4}(1-x)^{\beta}P_n^{(\alpha,\beta)}(1-2x)J_{\mu}(\lambda x^{-h/2})dx$$
$$=\frac{h^{-\beta-1}\Gamma(\beta+n+1)}{\pi\sqrt{2\lambda}\,\Gamma(n+1)}\sum_{i,-i}\frac{1}{i}\,G^{h+1,h+2}_{2h+3,2h+1}$$
$$\times\left(\frac{4e^{i\pi}}{\lambda^2}\middle|\begin{matrix}\frac{3}{4}-\frac{\mu}{2},\ \Delta(h,-\rho),1,\ \frac{3}{4}+\frac{\mu}{2},\ \Delta(h,\alpha-\rho)\\ \Delta(h,\alpha-\rho+n),1,\quad \Delta(h,-\beta-\rho-n-1)\end{matrix}\right),$$

where h is a positive number and $R(\rho)\geq -1$.

Reference

[1] Watson, G. N.: Some self-reciprocal functions. Quart. J. Math. (Oxford), **2**, 298–309 (1931).