# 41. On Axiom Systems of Ontology. II

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(Comm. by Kinjirô KUNUGI, M. J. A., Feb. 12, 1971)

It is well known that Leśniewski's original system of Ontology has the form of the following single axiom [1], [2].

T. $a \varepsilon b \equiv [\exists c]\ \{c \varepsilon a\} \wedge [c]\{c \varepsilon a \supset c \varepsilon b\} \wedge [cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\}$.

It is mentioned that the following four theses are inferentially equivalent to {A1, A2, A3, A4} by C. Lejewski [1].

A 1. $a \varepsilon b \supset [\exists c]\{c \varepsilon a\}$

A 2. $(a \varepsilon b \wedge c \varepsilon a) \supset c \varepsilon b$

A 3. $a \varepsilon b \wedge c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d$

A 4. $c \varepsilon a \wedge [d]\{d \varepsilon a \supset d \varepsilon b\} \wedge [de]\{d \varepsilon a \wedge e \varepsilon a \supset d \varepsilon e\} \supset a \varepsilon b$

In this paper, we shall prove that T and {A1, A2, A3, A4} are equivalent. Furthermore, we shall prove that A1 and A2 alone can serve as axiom system of Ontology.

**Lemma 1.** T *implies* A1, A2, A3 *and* A4.

The proof will be given in the form of suppositional proofs [1], [2].

T 1=A 1. $a \varepsilon b \supset [\exists c]\{c \varepsilon a\}$ (T)

T 2=A 2. $a \varepsilon b \wedge c \varepsilon a \supset c \varepsilon b$

Proof.
1 $a \varepsilon b$
2 $c \varepsilon a \supset$ } (premise)
3 $[c]\{c \varepsilon a \supset c \varepsilon b\}$ (T, 1)
4 $c \varepsilon a \supset c \varepsilon b$ (O$\Pi$: 3)
$c \varepsilon b$ (4, 2)

T 3=A 4. $c \varepsilon a \wedge [d]\{d \varepsilon a \supset d \varepsilon b\} \wedge [de]\{d \varepsilon a \wedge e \varepsilon a \supset d \varepsilon e\} \supset a \varepsilon b$

Proof.
1 $c \varepsilon a$
2 $[d]\{d \varepsilon a \supset d \varepsilon b\}$
3 $[de]\{d \varepsilon a \wedge e \varepsilon a \supset d \varepsilon e\} \supset$ } (premise)
4 $[\exists c]\{c \varepsilon a\}$ (D$\Sigma$: 1)
$a \varepsilon b$ (T, 4, 2, 3)

D 1. $x \varepsilon a^* b \equiv x \varepsilon a \wedge b \varepsilon x$ (rule of adding definition)

T 4=A 3. $a \varepsilon b \wedge c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d$

Proof.
1 $a \varepsilon b$
2 $c \varepsilon a$
3 $d \varepsilon a \supset$ } (premise)
4 $a \varepsilon b^* c$ (1, 2, D1)
5 $d \varepsilon b^* c$ (3, 4, T2)
$c \varepsilon d$ (D1, 5)

**Lemma 2.** A1, A2, A3 *and* A4 *imply* T.

AT 1. $a \varepsilon b \supset [\exists c]\{c \varepsilon a\}$

Proof. 1 $a \varepsilon b \supset$ (premise)

$[\exists c]\{c \varepsilon a\}$ (A1, 1)

AT 2. $a \varepsilon b \supset [c]\{c \varepsilon a \supset c \varepsilon b\}$

Proof. 1 $a \varepsilon b$ (premise)

2 $a \varepsilon b \wedge c \varepsilon a \supset c \varepsilon b$ (A2)

3 $a \varepsilon b \supset (c \varepsilon a \supset c \varepsilon b)$ (2)

4 $c \varepsilon a \supset c \varepsilon b$ (3, 1)

$[c]\{c \varepsilon a \supset c \varepsilon b\}$ (D$\Pi$: 4)

AT 3. $a \varepsilon b \supset [cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\}$

Proof. 1 $a \varepsilon b \supset$ (premise)

2 $a \varepsilon b \wedge c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d$ (A3)

3 $c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d$ (2, 1)

$[cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\}$ (D$\Pi$: 3)

AT 4. $a \varepsilon b \supset [\exists c]\{c \varepsilon a\} \wedge [c]\{c \varepsilon a \supset c \varepsilon b\}$
$\wedge [cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\}$ (AT1, AT2, AT3)

AT 5. $[\exists c]\{c \varepsilon a\} \wedge [c]\{c \varepsilon a \supset c \varepsilon b\} \wedge [cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\} \supset a \varepsilon b$

Proof. 1 $[\exists c]\{c \varepsilon a\}$

2 $[c]\{c \varepsilon a \supset c \varepsilon b\}$ (premise)

3 $[cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\} \supset$

4 $c_1 \varepsilon a$ (O$\Sigma$: 1)

5 $c_1 \varepsilon a \wedge [c]\{c \varepsilon a \supset c \varepsilon b\}$
$\wedge [cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\} \supset a \varepsilon b$ (A4)

$a \varepsilon b$ (5, 4, 2, 3)

AT 6=T. $a \varepsilon b \equiv [\exists c]\{c \varepsilon a\} \wedge [c]\{c \varepsilon a \supset c \varepsilon b\}$
$\wedge [cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\}$ (AT4, AT5)

**Theorem 1.** T *is equivalent to* {A1, A2, A3, A4}.

The proof is clear from Lemma 1 and Lemma 2.

**Lemma 3.** A2 *implies* A3.

The lemma has been showed by Tarski in 1912, see Slupecki [2].

**Lemma 4.** A1 *and* A2 *imply* A4.

We shall use the extensionality rule:

ER1. $[x]\{x \varepsilon X \equiv x \varepsilon Y\} \supset [\varphi]\{\varphi(X) \equiv \varphi(Y)\}$.

Let D2 be the definition:

D 2. $\psi\langle X\rangle(x) \equiv x \varepsilon X$

Proof. 1 $c \varepsilon a$

2 $[d]\{d \varepsilon a \supset d \varepsilon b\}$ (premise)

3 $[de]\{d \varepsilon a \wedge e \varepsilon a \supset d \varepsilon e\}$

4 $[\exists f]\{f \varepsilon c\}$ (A1, 1)

5 $f_1 \varepsilon c$ (O$\Sigma$: 4)

6 $f_1 \varepsilon a$ (A2, 1, 5)

| | | |
|---|---|---|
| 7 | $f_1 \varepsilon b$ | (2, 6) |
| 8 | $x \varepsilon f_1 \supset x \varepsilon a$ | (A2, 6) |
| 9 | $x \varepsilon a \supset x \varepsilon f_1$ | (3, 6) |
| 10 | $x \varepsilon f_1 \equiv x \varepsilon a$ | (8, 9) |
| 11 | $[x]\{x \varepsilon f_1 \equiv x \varepsilon a\}$ | ($\mathrm{D}\Pi$ : 10) |
| 12 | $[\varphi]\{\varphi(f_1) \equiv \varphi(a)\}$ | (ER1, 11) |
| 13 | $\psi\langle b\rangle(f_1) \equiv \psi\langle b\rangle(a)$ | ($\mathrm{O}\Pi$ : 12) |
| 14 | $\psi\langle b\rangle(f_1)$ | (D2, 7) |
| 15 | $\psi\langle b\rangle(a)$ | (13, 14) |
| | $a \varepsilon b$ | (D2, 15) |

Therefore A4 is proved.

**Theorem 2.** A1 *and* A2 *alone can serve as an axiom system of Ontology.*

It is seen from Lemma 3 and Lemma 4 that A1 and A2 imply A3 and A4. T is equivalent to {A1, A2, A3, A4} from Theorem 1. Hence the proof is complete.

## References

[1] C. Lejewski: On Leśniewski's ontology. Ratio, **1**, 150–176 (1958).
[2] J. Slupecki: S. Leśniewski's calculus of names. Studia Logica (Warszawa), **3**, 7–65 (1955).