100. *On the Behaviour of an Inverse Function of a Meromorphic Function at its Transcendental Singular Point, II.*

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In my former paper[1], I have proved a theorem concerning the behaviour of an inverse function of a meromorphic function at its transcendental singularity $w=0$.

In the proof of $\lim_{r\to\infty} A(r)=\infty$, I have tacitly assumed that the boundary of $\varDelta$ has a branch tending to infinity. I will here give a proof of $\lim_{r\to\infty} A(r)=\infty$ in the general case, where the boundary of $\varDelta$ has a branch tending to infinity or it has no such a branch, but it consists of only closed curves. We will follow Mr. K. Kunugui's proof (in the paper cited in my former paper), but simplify it at some points.

Proof of $\lim_{r\to\infty} A(r)=\infty$.

Case I, where for some w_0 $(0<|w_0|<\rho)$, $w_0=f(z)$ has only finite number of roots in $\varDelta$. Then by Mr. K. Noshiro's extension of Iversen-Valiron's theorem (in the paper cited in my former paper), there exists a curve Γ_1 in $\varDelta$ tending to infinity, such that $\lim f(z)=w_0$, when z tends to ∞ along Γ_1. Also, as we have remarked in my former paper, there exists a curve Γ in $\varDelta$ tending to infinity, such that $\lim f(z)=0$, when z tends to ∞ along Γ. As before, let $\{a_i^{(r)}\}$ be the part of the boundary of $\varDelta_r$ which lies on $|z-z_0|=r$ and let $a_{i_0}^{(r)}$ intersect Γ. There occurs two cases: (i) two end-points of $a_{i_0}^{(r)}$ lie on the boundary of $\varDelta$ or (ii) the circle $|z-z_0|=r$ does not interesect the boundary of $\varDelta$. In the case (i), we have as before $L(r)\geqq\frac{\rho}{2}$ for $r\geqq r_1$, and in the case (ii), since $|z-z_0|=r$ intersects Γ and Γ_1, $L(r)\geqq\frac{|w_0|}{2}$ for $r\geqq r_2>r_1$. In either case, $L(r)\geqq l$ for $r\geqq r_2$, where $l=\text{Min.}\left(\frac{\rho}{2}, \frac{|w_0|}{2}\right)$, whence we conclude as before $\lim_{r\to\infty} A(r)=\infty$.

Case II, where $w=f(z)$ has infinitely many roots in $\varDelta$ for every w $(0<|w|<\rho)$. Let $n(r, w)$ be the number of roots of $w=f(z)$ in $\varDelta_r$, then $\lim_{r\to\infty} n(r, w)=\infty$.

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1) M. Tsuji: On the Behaviour of an Inverse Function of a Meromorphic Function at its Transcendental Singular Point. This Proc. **17** (1941), 414–417. During the correction of my former paper Mr. Y. Tumura's paper: Sur le problème de M. Kunugui appeared in this Proc. **17** (1941), 289–295 and its full detail will appear in the Japanese Journal of Mathematics, **18**, where a more general theorem as mine is proved, but in somewhat different definition of $A(r)$ and $S(r)$.

Now $A(r)=\int_{|w|<\rho} n(r,w)\,d\omega(w)$, where $d\omega(w)$ is the surface element at w. Since $n(r,w)$ is an increasing function of r, we have by Beppo Levi's theorem

$$\lim_{r\to\infty} A(r)=\int_{|w|<\rho} \lim_{r\to\infty} n(r,w)\,d\omega(w)=\infty \qquad \text{q. e. d.}$$

Hence our theorem in the former paper holds for any domain Δ.