The coincidence of some topologies on the unit ball of the Fourier - Stieltjes algebra of weighted foundation semigroups

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1 Introduction

In our earlier paper [**10**], for a foundation $*-$semigroup S with an identity and with a Borel-measurable weight function $w \leq 1$, we proved that on the unit ball of $\mathcal{P}(S, w)$, the cone of $w-$bounded continuous positive definite functions on S, the weak topology coincides with the compact open topology. In the present paper, through some C^*-algebras techniques, we shall extend this result to the unit ball of the Fourier-Stieltjes algebra $\mathcal{F}(S, w)$ of a foundation semigroup S with a Borel measurable weight function w. Indeed, we shall establish our conjecture in [**10**] even in the more general setting of the Fourier-Stieltjes algebra $\mathcal{F}(S, w)$ for any Borel measurable weight function w. It should be noted that the family of foundation semigroups is quite extensive, for which locally compact groups and discrete semigroups are elementary examples. For further examples we refer to Appendix B of [**13**].

2 Preliminaries

Throughout this paper, S will denote a locally compact, Hausdorff topological semigroup with an identity. A topological semigroup S is called a $*-$semigroup if there is a continuous mapping $* : S \to S$ such that $(x^*)^* = x$ and $(xy)^* = y^*x^*$ for all $x, y \in S$. A function w of S into the set of positive real numbers) is called a

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weight function on S if $w(xy^*) \leq w(x)w(y)$ for all $x, y \in S$. A function $f : S \to \mathbb{C}$ ($\mathbb{C}$ denotes the set of complex numbers) is called $w-$bounded if there is a positive number k such that $\mid f(x) \mid \leq kw(x)(x \in S)$. A complex-valued function φ on S is called positive definite whenever

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_i\bar{c}_j\varphi(x_i x_j^*) \geq 0$$

for all choices $\{x_1, \ldots, x_n\}$ from S and $\{c_1, \ldots, c_n\}$ from $\mathbb{C}$. We denote by $\mathcal{P}(S, w)$ the set of $w-$bounded continuous positive definite functions on S. A $*$-representation of S by bounded operators on a Hilbert space $\mathcal{H}$ is a homomorphism: $x \to \pi(x)$ of S into $\mathcal{L}(\mathcal{H})$, the space of all bounded linear operators on $\mathcal{H}$, such that $\pi(x^*) = (\pi(x))^*$ for all $x \in S$ and $\pi(e)$ is the identity operator on $\mathcal{H}$. A representation π is called cyclic if there is a (cyclic) vector $\xi \in \mathcal{H}$ such that the set $\{\pi(x)\xi : x \in S\}$ is dense in $\mathcal{H}$, and π is called $w-$bounded if there is a positive number k such that $\|\pi(x)\| \leq kw(x)(x \in S)$. Note that a $*-$representation π is $w-$bounded if and only if $\|\pi(x)\| \leq w(x)(x \in S)$. For further information on the representation theory of topological $*-$semigroups we refer the reader to [**2**], [**8**], and [**9**].

We recall (see, for example [**2**]) that, on a topological semigroups S with a weight function w such that w and $1/w$ are locally bounded (i.e. bounded on compact subsets of S) $M(S, w)$ denotes the set of all complex, regular, signed measures μ (not necessarily bounded), of the form $\mu = \mu_1 - \mu_2 + i(\mu_3 - \mu_4)$ where μ_i is a positive regular measure on S with $w \in L^1(S, \mu_i)$ for $i = 1, 2, 3, 4$. Note that for an element $\mu \in M(S, w)$ and a Borel set B, $\mu(B)$ is well-defined in the case when B is relatively compact. For every $\mu \in M(S, w)$, the equation

$$\int_S f d(w.\mu) = \int_S f w d\mu \quad (f \in C_b(S)),$$

where $C_b(S)$ denotes the space of all continuous bounded complex-valued functions on S, defines a measure $w.\mu \in M(S)$, the space of all bounded regular complex measures on S. With the norm $\|\mu\|_w = \|w.\mu\| \quad (\mu \in M(S, w))$, where $\|w.\mu\|$ denotes the total variation of $w.\mu \in M(S)$, $M(S, w)$ defines a Banach lattice, and with the convolution product given by

$$(\mu * \nu)(f) = \int_S \int_S f(xy) d\mu(x) d\nu(y) \qquad (\mu,\ \nu \in M(S, w),\ f \in C_{00}(S)) \tag{1}$$

defines a Banach algebra, where $C_{00}(S)$ denotes the space of all functions in $C_b(S)$ with compact supports. By part (iii) of Theorem 4.6 of [**8**] we conclude that (1) is also valid for every $w-$bounded, Borel measurable function f on S.

We also recall (see, for example. [**6**] or [**1**]) that $M_a(S)$ (or $\tilde{L}(S)$) denotes the set of all measures $\mu \in M(S)$ for which the mappings $x \longmapsto \bar{x}^* \mid \mu \mid$ and $x \longmapsto \mid \mu \mid^* \bar{x}$ (where $\bar{x}$ denotes the point mass at x for $x \in S$) from S to $M(S)$ are weakly continuous. As in [**8**], we can define $M_a(S, w)$ (or $\tilde{L}(S, w)$) as the set of measures μ in $M(S, w)$ for which $w.\mu$ is in $M_a(S)$. Then $M_a(S, w)$ is a closed, two-sided $L-$ideal of $M(S, w)$. Finally, we call S a foundation semigroup if $\cup\{\mathrm{supp}(\mu) : \mu \in M_a(S)\}$ is dense in S.

3 The results

Before stating our first result we note that if S is a foundation $*-$semigroup with an identity and with a locally bounded Borel measurable weight function w, then $M_a(S,w)$ defines a Banach $*-$algebra with a bounded approximate identity (see, [**8**, Proposition 4.7]). We denote the enveloping C^*-algebra of the Banach $*-$algebra $M_a(S,w)$ by $C^*(S,w)$ (see Proposition 2.7.1 of [**5**]).

Lemma 1. *Let S be a foundation $*-$semigroup with an identity and with a Borel measurable weight function w. Then the mapping: $\varphi \longmapsto p_\varphi$ given by the equation*

$$p_\varphi(\mu) = \int_S \varphi(x)d\mu(x) \quad (\varphi \in \mathcal{F}(S,w), \mu \in M_a(S,w)) \tag{2}$$

defines an isometric isomorphism between $\mathcal{F}(S,w)$ and $C^*(S,w)'$.

Proof. Recall that every φ in $\mathcal{F}(S,w)$ can be expressed as $\varphi = \varphi_1 - \varphi_2 + i(\varphi_3 - \varphi_4)$ with $\varphi_i \in \mathcal{P}(S,w)(1 \leq i \leq 4)$ and also by Proposition 2.1 of [**14**] every functional p in $C^*(S,w)'$ can be represented as $p = p_1 - p_2 + i(p_3 - p_4)$ where $p_i(1 \leq i \leq 4)$ is a positive functional. So an application of Lemma 2.3 of [**9**] together with the Corollary 11.3.8 of [**11**] completes the proof. ∎

Before turning the next lemma we need to recall some results from the theory of C^*-algebras. By ([**5**], p.243) for every functional $f \in A'$ of a C^* algebra A, there is a unique positive functional $|f|$ of A such that $\|f\| = \||f|\|$ with $|f(x)|^2 \leq \|f\||f|(xx^*)(x \in A)$. Moreover, the map $f \longmapsto |f|$ defines a continuous map from (A', τ) into $(A', \sigma(A', A))$, where τ denotes the locally convex topology on A' defined by the systems of neighbourhoods

$$\mathcal{W}(f_0; \varepsilon, x_1, \ldots, x_n) = \{f \in A' : |f(x_j) - f_0(x_j)| < \varepsilon \quad 1 \leq i \leq n, |\|f\| - \|f_0\|| < \varepsilon\}$$

$(f_0 \in A', \varepsilon > 0)$ and $\{x_1, \ldots, x_n\} \subseteq A\}$ (cf. Proposition 1) of [**4**]). We also note that the given isomorphy in Lemma 1 between $\mathcal{F}(S,w)$ and $C^*(S,w)'$ permits us to define a norm $\varphi \longmapsto \|\varphi\|$ and an "absolute value" $\varphi \longmapsto |\varphi|$ on $\mathcal{F}(S,w)$.

Lemma 2. *Let S be a foundation $*-$semigroup with an identity and with a Borel measurable weight function w. For every $\varphi \in \mathcal{F}(S,w)$ there exists a $w-$bounded continuous cyclic $*-$representation π of S by bounded operators on a Hilbert space $\mathcal{H}$ with a cyclic vector ξ such that $\mid \varphi \mid (x) = \langle \pi(x)\xi, \xi\rangle$. Furthermore there exist some vector $\zeta \in \mathcal{H}$ such that $\varphi(x) = \langle \pi(x)\xi, \zeta\rangle (x \in S)$ and*

$$\|\varphi\| = \mid \varphi \mid (e) = \|\xi\|^2 = \|\zeta\|^2. \tag{3}$$

Proof. Let p be a positive functional on $C^*(S,w)$. By the GNS construction ([**12**], Theorem 3.3.3) there exists a cyclic $*-$representation $\tilde{\pi}$ of $C^*(S,w)$ by bounded operators on a Hilbert space $\mathcal{H}$ with a cyclic vector $\xi \in \mathcal{H}$ such that $p(\mu) = \langle \tilde{\pi}(\mu)\xi, \xi\rangle (\mu \in M_a(S,w))$. Now, by Theorem 5.2 of [**8**] there exists a $w-$bounded continuous cyclic $*-$representation π of S by operators on H such that

$$\langle \tilde{\pi}(\mu)\xi, \eta\rangle = \int_S \langle \pi(x)\xi, \eta\rangle d\mu(x) \quad (\mu \in M_a(S,w), \xi, \eta \in \mathcal{H}).$$

In particular, $p = p_\varphi$ with $\varphi(x) = \langle \pi(x)\xi, \xi\rangle (x \in S)$. From Lemma 2.2. of [**8**] and the fact that φ is $w-$bounded and continuous, it follows that φ is unique. Now an application of Proposition 2 of [**4**] reveals the equation 3. ■

Lemma 3. *Let S be a foundation $*-$semigroup with an identity and with a Borel measurable weight function w. Then every $\varphi \in \mathcal{F}(S, w)$ satisfies the following equation:*

$$| \varphi(x) - \varphi(xy) |^2 \leq \|\varphi\| w(x)[\|\varphi\| - 2Re \mid \varphi \mid (y) + \mid \varphi \mid (y^* y)] \quad (x, y \in S). \tag{4}$$

Proof. Let $\varphi \in \mathcal{F}(S, w)$. By Lemma 2 there exist a $w-$bounded continuous cyclic *- representation π with a cyclic vector ξ of S by bounded operators on a Hilbert space $\mathcal{H}$ and a vector $\zeta \in \mathcal{H}$ such that $\varphi(x) = \langle \pi(x)\xi, \zeta\rangle, \mid \varphi \mid (x) = \langle \pi(x)\xi, \xi\rangle$ and $\|\varphi\| = \mid \varphi \mid (e) = \|\xi\|^2 = \|\zeta\|^2$. For every $x, y \in S$ we have

$$\begin{aligned} | \varphi(x) - \varphi(xy) |^2 &= | \langle \pi(x)\xi, \zeta\rangle - \langle \pi(xy)\xi, \zeta\rangle |^2 \\ &= | \langle \pi(x)(\xi - \pi(y)\xi), \zeta\rangle |^2 \\ &\leq \|\pi(x)\|^2 . \|\xi - \pi(y)\xi\|^2 \|\zeta\|^2 \\ &\leq (w(x))^2 \|\varphi\| \langle \xi - \pi(y)\xi, \xi - \pi(y)\xi\rangle \\ &= \|\varphi\|(w(x))^2[\|\varphi\| - 2Re \mid \varphi \mid (y) + \mid \varphi \mid (y^* y)]. \end{aligned}$$

The proof is complete. ■

Before turning to the next result we recall that for locally compact Hausdorff spaces X and Y and a continuous function θ of X into Y and a complex regular measure μ on X, μ_θ (the image of μ under θ) given by $\mu_\theta(K) = \mu(\theta^{-1}(K))$ for every compact subset K of Y, defines a complex regular measure on Y (see [**3**] , Proposition 2.1.15).

Lemma 4. *Let S be a topological semigroup with a Borel measurable weight function w such that w and $\frac{1}{w}$ are locally bounded and let θ be a continuous function on S. If μ in $M_a(S, w)$ is with compact support then μ_θ is in $M_a(S, w)$.*

Proof. Let K denote the compact support of μ. Since w is locally bounded, there exists $M_\theta > 0$ such that $\frac{w(\theta(x))}{w(x)} \leq M_\theta$ for all $x \in K$. Let C be a compact subset of S, then

$$\begin{aligned} | w\mu_\theta(C) | &= \left| \int_{K \cap C} \chi_C(x) w(x) d\mu_\theta(x) \right| = \left| \int_{K \cap C} \chi_C(\theta(x)) w(\theta(x) d\mu(x) \right| \\ &\leq \int_{K \cap C} \frac{w(\theta(x))}{w(x)} d \mid w.\mu \mid (x) \leq M_\theta \mid \omega.\mu \mid (C). \end{aligned}$$

Thus $w\mu_\theta << | w.\mu |$. Since $w.\mu \in M_a(S)$, it follows that $\mu_\theta \in M_a(S, w)$. ■

The following theorem is indeed the main result of this paper. We shall first introduce a topology τ_w on $\mathcal{F}(S,w)$. For $\varphi_0 \in \mathcal{F}(S,w)$ of a foundation semigroup S with identity and with a Borel measurable weight function w, and $\varepsilon > 0$ and $\mu_1, \ldots, \mu_n \in M_a(S,w)$ we define

$$\mathcal{W}(\varphi_0; \varepsilon; \mu_1, \ldots, \mu_n) = \{\varphi \in \mathcal{F}(S,w) : \\ \mid \mu_j(\varphi) - \mu_j(\varphi_0) \mid < \varepsilon, 1 \leq j \leq n, \mid \|\varphi\| - \|\varphi_0\| \mid < \varepsilon\}. \quad (5)$$

Let τ_w denote the topology on $\mathcal{F}(S,w)$ for which the sets of the form (5) define an open base.

Theorem 5. *Let S be a foundation $*-$semigroup with an identity and with a Borel measurable weight function w such that w and $\frac{1}{w}$ are locally bounded. Then the topology τ_w of $\mathcal{F}(S,w)$ is stronger than its compact open topology.*

Proof. Given $\varphi_0 \in \mathcal{F}(S,w)$, $\varepsilon > 0$ and a compact subset K of S, we have to show that there exist a positive number δ and a finite subset $\{\lambda_1, \ldots, \lambda_n\}$ of $M_a(S,w)$ such that $\mathcal{W}(\varphi_0; \delta, \lambda_1, \ldots, \lambda_n) \subseteq \mathcal{U}(\varphi_0, \varepsilon, K)$, where

$$\mathcal{U}(\varphi_0, \varepsilon, K) = \{\varphi \in \mathcal{F}(S,w) : \ \mid \varphi(x) - \varphi_0(x) \mid < \varepsilon \ \text{ for all } \ x \in K\}.$$

To this end, we choose a positive number M such that $w(x) \leq M$ for all $x \in K$. Let $\alpha = \min(1, \frac{\varepsilon^2}{567M^2(1+\|\varphi_0\|)})$. By the continuity of $\mid \varphi_0 \mid$ at e, there exists a neighbourhood V of e with compact closure such that $V^*V \subseteq V$ ($V^* = \{x^* : x \in V\}$) and

$$\Big| \mid \varphi_0 \mid (y) - \|\varphi_0\| \Big| = \Big| \mid \varphi_0 \mid (y) - \mid \varphi_0 \mid (e) \Big| < \alpha, \quad (y \in V). \quad (6)$$

So

$$\Big| \mid \varphi_0 \mid (y^*y) - \|\varphi_0\| \Big| < \alpha \quad (y \in V). \quad (7)$$

Choose a positive measure $\mu \in M_a(S,w)$ with $e \in supp(\mu) \subset \overline{V}$ and $\|\mu\| = 1$. Let $x \in K$. Then an application of the inequality (4) with the aid of the Hölder inequality gives

$$\begin{aligned} \mid \delta_x * \mu(\varphi) - \varphi(x) \mid &= \Big| \int_S [\varphi(xy) - \varphi(x)] d\mu(y) \Big| \\ &\leq \int_S \mid \varphi(xy) - \varphi(x) \mid d\mu(y) \\ &\leq 2M\|\varphi\|^{1/2} \Big(\int_V [\|\varphi\| - 2Re \mid \varphi \mid (y) + \mid \varphi \mid (y^*y)] d\mu \Big)^{1/2} \\ &\leq 2M\|\varphi\|^{1/2} \Big| \int_V [\|\varphi\| - 2\varphi(y) + \mid \varphi \mid (y^*y)] d\mu \Big|^{1/2}. \end{aligned} \quad (8)$$

Since for $A = C^*(S,w)$ the mapping $\varphi \longmapsto \mid \varphi \mid$ is continuous from (A', τ_w) into $(A', \sigma(A',A))$, it follows that there exist a finite subset $\{\mu_1, \ldots, \mu_m\}$ of $M_a(S,w)$ and η with $0 \leq \eta < \alpha$ such that $\varphi \in \mathcal{W}(\varphi_0; \eta, \mu_1, \ldots, \mu_m)$ implies that $|\varphi| \in \mathcal{W}(|\varphi_0|; \alpha, \mu, \mu_\theta)$, where θ denotes the continuous map on S given by $\theta(y) = y^*y(y \in$

S). Let $\varphi \in \mathcal{W}(\varphi_0; \eta, \mu_1, \ldots, \mu_m)$. By (6) and (7) we have

$$\begin{aligned}
&\Big| \int_V [\|\varphi\| - 2 \mid \varphi \mid (y) + \mid \varphi \mid (y^* y)] d\mu(y) \Big| \\
&\qquad \le \Big| \int_V (\|\varphi\| - \|\varphi_0\|) d\mu(y) \Big| + 2 \Big| \int_V [\|\varphi_0\| - \mid \varphi_0 \mid (y)] d\mu(y) \Big| \\
&\qquad + \Big| 2 \int_V [\mid \varphi_0 \mid (y) - \mid \varphi \mid (y)] d\mu(y) \Big| + \Big| \int_V [\mid \varphi_0 \mid (y^* y) - \mid \varphi \mid (y^* y)] d\mu \Big| \\
&\qquad + \Big| \int_V [\mid \varphi_0 \mid (y^* y) - \|\varphi_0\|] d\mu \Big| \\
&< \alpha + 2\alpha + 2\alpha + \Big| \int_S [\mid \varphi_0 \mid (y) - \mid \varphi \mid (y)] d\mu_\theta \Big| + \alpha \\
&\qquad < 7\alpha.
\end{aligned}$$

Thus for every $x \in K$ and every $\varphi \in \mathcal{W}(\varphi_0, \eta, \mu_1, \ldots, \mu_m)$ we have

$$\mid \delta_x * \mu(\varphi) - \varphi(x) \mid < 2M(7\|\varphi\|\alpha)^{1/2} < \frac{\varepsilon}{3}. \tag{9}$$

Let $\|.\|'_w$ denote the norm on $A = C^*(S, w)$. Since $\|\mu\|'_w \le \|\mu\|_w$ for every $\mu \in M_a(S, w)$, the mapping : $x \longmapsto \delta_{x^*}\mu$ from S into $M_a(S, w)$ is $\| \cdot \|'_w$−continuous. So by the compactness of K there exists a finite subset $\{x_1, \ldots, x_l\}$ of K such that for every $x \in K$ there exists $x_j (1 \le j \le l)$ with $\|\delta_x * \mu - \delta x_j * \mu\|'_w \le \beta$, where $\beta = \min(1, \frac{\varepsilon}{9(1+\|\varphi_0\|)})$. Put $\nu_i = \delta x_i * \mu (1 \le i \le l)$. Let $\varphi \in \mathcal{W}(\varphi_0; \beta, \nu_1, \ldots, \nu_l)$ and x be an arbitrary and fixed point of K. Then there exists $\nu_j (1 \le j \le l)$ such that $\|\delta_x * \mu - \nu_j\|'_w < \beta$. Thus

$$\begin{aligned}
\mid \delta_x * \mu(\varphi) - \delta_x * \mu(\varphi_0) \mid &\le \mid \delta_x * \mu(\varphi) - \nu_j(\varphi) \mid + \mid \nu_j(\varphi) - \nu_j(\varphi_0) \mid \\
&+ \mid \nu_j(\varphi_0) - \delta_x * \mu(\varphi_0) \mid \\
&\le \|\delta_x * \mu - \nu_j\|'_w \|\varphi\| + \mid \nu_j(\varphi) - \nu_j(\varphi_0) \mid \\
&+ \|\nu_j - \delta_x * \mu\|'_w \|\varphi\| \\
&< 2\beta\|\varphi\| + \beta. \\
&< 2\beta(\|\varphi_0\| + \beta) + \beta \\
&< 2\beta(\|\varphi_0\| + 1) + \varepsilon/9 \\
&< \frac{2\varepsilon}{9} + \frac{\varepsilon}{9} = \frac{\varepsilon}{3}.
\end{aligned} \tag{10}$$

Let $n = m + l$. Put $\lambda_i = \mu_i$ for $1 \le i \le m$, and $\lambda_{m+k} = \nu_k$ for $1 \le k \le l$. For $\varphi \in \mathcal{W}(\varphi_0; \frac{\varepsilon}{3}, \lambda_1, \ldots, \lambda_n)$ and every $x \in K$ from (9) and (10) it follows that

$$\begin{aligned}
\mid \varphi(x) - \varphi_0(x) \mid &\le \mid \varphi(x) - \delta_x * \mu(\varphi) \mid \\
&+ \mid \delta_x * \mu(\varphi) - \delta_x * \mu(\varphi_0) \mid \\
&+ \mid \delta_x * \mu(\varphi_0) - \varphi_0(x) \mid \\
&< \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \varepsilon.
\end{aligned}$$

Thus the proof is complete. ∎

As an immediate consequence of the above result we obtain the following corollary.

Corollary 6. Let S be a foundation $*-$semigroup with identity and with a Borel measurable w such that w and $\frac{1}{w}$ are locally bounded. Then on the unit ball of $\mathcal{F}(S, w)$, the weak$*-$topology the compact open topology are identical.

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References

[1] A. C. Baker and J. W. Baker; *Algebra of measures on locally compact semigroups III*, J. London Math. Soc. 4 (1972) 685-695.

[2] J. W. Baker; *Measure algebras on semigroups,* Analytical and Topological Theory of Semigroups (K.H. Hofmann et al.), de Gruyter, 1990, pp. 221-252.

[3] C. Berg, J. P. R. Christensen and P. Ressel; *Harmonic analysis on semigroups* Springer Verlag, 1984 .

[4] A. Derighetti; *Some results on the Fourier-Stieltjes algebras of a locally compact group,* Math. Helv. 45 (1970), 219-228.

[5] J. Dixmier; *C^*-algebras,* North-Holand publishing company 1970.

[6] H. A. M. Dzinotyiweyi; *The Analogue of the Group Algebra for Topological Semigroups*, Pitman, Boston, Mass., London, 1984.

[7] E. G. Effros; *Order ideals in a C^*-algebra and its dual,* Duke Math. J. 30 (1963), 391-411.

[8] M. Lashkarizadeh Bami; *Representations of foundation semigroups and their algebras*, Canadian J. Math. 37 (1985), 29-47.

[9] M. Lashkarizadeh Bami; *Bochner's theorem and the Hausdorff moment theorem on foundation topological semigroups*, Canadian J. Math. 37 (1985) 785-809.

[10] M. Lashkarizadeh Bami; *On various types of convergence of positive definite functions on foundation semigroups,* Math. Proc. Camb. Phil. Soc. 111 (1992), 325-330.

[11] T.W. Palmer, *Banach algebras and the general theory of $*-$algebras,* Vol.II, $*-$algebras, Cambridge University Press, Cambridge, 2001.

[12] G. K. Pedersen; *C^*-algebras and their automorphism groups*, Academic Press Inc, London 1979.

[13] G. L. G. Sleijpen; *Convolution measure algebras on semigroups,* Ph.D. Thesis, Katholike Universiteit, The Netherlands, 1976.

[14] M. Takasaki; *Theory of operator Algebras I,* Springer Verlag, New York Inc 1979.

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