Nonexistence of weak solutions for evolution problems on $\mathbb{R}^n$

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Abstract

We study the nonexistence of global weak solutions for equations of the following type:

$$u_{tt} - \Delta\ u + g(t)\ u_t = |u|^p \tag{1}$$

where $g(t)$ behaves like $t^\beta,\ 0 \le \beta < 1$. Then the situation is extended to systems of equations of the same type, and more general equation than (1).

1 Introduction

This article discusses the following problem

$$u_{tt} - \Delta\ u + g(t)\ u_t = |u|^p \tag{2}$$

for $(t,x) \in (0,+\infty) \times \mathbb{R}^n$, which the initial conditions are defined as

$$u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), \quad x \in \mathbb{R}^n \tag{3}$$

where $p > 1$, $g(t)$ is a function behaving like $t^\beta,\ 0 \le \beta < 1$. We provide conditions relating the space dimension n with parameters β, and p for which every global solution of (2) is trivial.

In [3], M. Qafsaoui and M. Kirane showed that the critical exponent for the semilinear wave equation with linear damping

$$u_{tt} + (-1)^m|x|^\alpha \Delta^m u + u_t = \ f(t,x)|u|^p + w(t,x),\ t > 0,\ x \in \mathbb{R}^n$$

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is $1+\dfrac{m(\lambda+2)-\alpha}{n}$, where $\alpha < m(\lambda+2),\ \lambda > 0$. In a recent paper [10], Todorova and Yordanov deal with the problem

$$u_{tt} - \Delta u + u_t = \ |u|^p.$$

They gave a Fujita's type results. For their proof, they used the fondamental solution of $(\partial_{tt} - \Delta_x + \partial_t)^k$ and a series of two propositions and four lemmas. However, they did not decide for the critical case $p_c = 1+\dfrac{2}{n}$. In [11], Qi S. Zhang uses a different and much shorter approach, he proves a blow up result more general than the interesting blow up result in [10]. He also showed that the critical exponent belongs to the blow up case. This problem had been left open by Todorova and Yordanov. Here, we present a brief and versatile proof of (2) based on Mitidieri, Pohozaev, Tesei and Véron [7], [8], [9] methods. This consists in a judicious choice of the test function in the weak formulation of the sought for solution of (2). The same method is applied for the more general equation:

$$u_{tt} + (-1)^m \Delta^m\ u + g(t)\ u_t = f(t,x)|u|^p + w(t,x), \quad t > 0,\ x \in \mathbb{R}^n, \tag{4}$$

where $\Delta^m,\ m \geq 1$ is the m-iterated Laplacian, $g(t)$ behaves like $t^\beta,\ 0 \leq \beta < 1$, $w(t,x)$ is a given function, and $f(t,x)$ is a given function behaving, like $t^\sigma |x|^\delta$, and the system:

$$\begin{cases} u_{tt} - \Delta\ u + g(t)\ u_t = |v|^p \\ v_{tt} - \Delta\ v + f(t)\ v_t = |u|^q \end{cases} \tag{5}$$

subjected to the conditions

$$u(0,x) = u_0(x),\ v(0,x) = v_0(x),\ u_t(0,x) = u_1(x),\ v_t(0,x) = v_1(x).$$

2 Notations and Definitions

Definition. 2.1. *A weak solution u of the differential equation* (2) *on $\mathbb{R}^+ \times \mathbb{R}^n$ with initial data $u(.,0) = u_0(.)$ and $u_t(.,0) = u_1(.)$ belonging to $L^1_{loc}(\mathbb{R}^n)$, is a locally integrable function $u \in L^p_{loc}\Big(\mathbb{R}^+ \times \mathbb{R}^n\Big)$ which satisfies*

$$\int\int |u|^p\zeta = \int_{\mathbb{R}^n} u_0(x)\zeta_t(0,x)\ dx + \int\int u\zeta_{tt} - \int\int ug(t)\zeta_t - \int\int ug'(t)\zeta - \int\int u\Delta\zeta$$
$$- \int_{\mathbb{R}^n} u_0(x)\zeta(0,x)\ dx - \int_{\mathbb{R}^n} u_1(x)\zeta(0,x)\ dx$$

for any smooth nonnegative test function ζ.

3 Statement and proof of the main result

In this section, we give the critical exponent for the equation (2). More precisely, we have the following result:

Theorem 3.1. *Assume that*

1. $u_0, u_1 \in L^1(\mathbb{R}^n)$ *such that* $\int_{\mathbb{R}^n} (u_0 + u_1)\ dx \geq 0$,

2. $1 < p \leq p_c = \dfrac{n+2}{n+2\beta}$,

hold, then there exist no weak solution u *to* (2) *defined on* $\mathbb{R}^+ \times \mathbb{R}^n$.

Proof. Let u be such a weak solution to (2) and ζ be a smooth test function which will be specified later. We have from the definition of the weak solution

$$\int\int |u|^p\zeta + \int_{\mathbb{R}^n} u_0(x)\zeta(0,x)\ dx + \int_{\mathbb{R}^n} u_1(x)\zeta(0,x)\ dx = \int_{\mathbb{R}^n} u_0(x)\zeta_t(0,x)\ dx + \int\int u\zeta_{tt}$$
$$-\int\int ug(t)\zeta_t - \int\int ug'(t)\zeta - \int\int u\Delta\zeta.$$

If ζ is chosen such that

$$\int_{\mathbb{R}^n} u_0(x)\zeta_t(0,x)\ dx = 0 \tag{6}$$

and

$$\int\int \zeta^{-\frac{1}{p-1}}\left(|\zeta_{tt}|^{\frac{p}{p-1}} + |g\zeta_t|^{\frac{p}{p-1}} + |\Delta\zeta|^{\frac{p}{p-1}} + |g'\zeta|^{\frac{p}{p-1}}\right) < \infty$$

then

$$\int\int |u|^p\zeta + \int_{\mathbb{R}^n} u_0(x)\zeta(0,x)\ dx + \int_{\mathbb{R}^n} u_1(x)\zeta(0,x)\ dx \leq \int\int |u||\zeta_{tt}|$$
$$+\int\int |u|g|\zeta_t| + \int\int ug'\zeta + \int\int |u||\Delta\zeta|. \tag{7}$$

By applying Hölder's inequality, with parameters p and p', to the right hand side of inequality (7), we obtain

$$\int\int |u|^p\zeta + \int_{\mathbb{R}^n} (u_0+u_1)\zeta(0,x)\ dx \leq \left(\int\int |u|^p\zeta\right)^{\frac{1}{p}}\left[\left(\int\int |\zeta_{tt}|^{p'}\zeta^{-\frac{p'}{p}}\right)^{\frac{1}{p'}}\right.$$
$$\left.+\left(\int\int (g|\zeta_t|)^{p'}\zeta^{-\frac{p'}{p}}\right)^{\frac{1}{p'}} + \left(\int\int (g'\zeta)^{p'}\zeta^{-\frac{p'}{p}}\right)^{\frac{1}{p'}} + \left(\int\int |\Delta\zeta|^{p'}\zeta^{-\frac{p'}{p}}\right)^{\frac{1}{p'}}\right] = \tag{8}$$
$$\left(\int\int |u|^p\zeta\right)^{\frac{1}{p}}\left[\mathcal{A}_{p,\zeta_{tt}} + \mathcal{B}_{p,\zeta_t} + \mathcal{C}_{p,\zeta} + \mathcal{D}_{p,\zeta}\right],$$

where $p' = \dfrac{p}{p-1}$. An application of the ε-Young's inequality to the right hand side of (8), yields for some $C(\varepsilon) > 0$,

$$\int\int |u|^p\zeta + \int_{\mathbb{R}^n} (u_0+u_1)\zeta(0,x)\ dx \le C(\varepsilon)\Big(\mathcal{A}_{p,\zeta_{tt}} + \mathcal{B}_{p,\zeta_t} + \mathcal{C}_{p,\zeta} + \mathcal{D}_{p,\zeta}\Big)^{\frac{p}{p-1}}. \quad (9)$$

Now we take $\zeta(t,x) = \phi\Big(\dfrac{t^2+|x|^4}{R^4}\Big)$ where $\phi \in C_c^\infty(\mathbb{R}^+)$ satisfies $0 \le \phi \le 1$ and

$$\phi(r) = \begin{cases} 0 & \text{if } \ r \ge 2, \\ 1 & \text{if } \ 0 \le r \le 1. \end{cases} \quad (10)$$

Since $\partial_t\zeta(t,x) = 2tR^{-4}\phi'\Big(\dfrac{t^2+|x|^4}{R^4}\Big)$, the estimate (6) holds. In order to estimate the right hand side of (9) we consider the scale of variables

$$t = R^2\ \tau; \quad x = R\ y. \quad (11)$$

Denoting $\Omega = \Big\{(t,x) \in \mathbb{R}^+ \times \mathbb{R}^n : t^2 + |x|^4 \le R^4\Big\}$. With such choice of ζ and by using the scaled variables (11) we get from (9) that

$$\int\int_\Omega |u|^p + \int (u_0+u_1) \le C\Big(R^{\lambda_1} + R^{\lambda_2} + R^{\lambda_3} + R^{\lambda_4}\Big) \quad (12)$$

for R sufficiently large and the constant C is positive and independent of R, and

$$\lambda_1 = 2+n-\frac{4p}{p-1}; \quad \lambda_2 = 2+n+\frac{(2\beta-2)p}{p-1};$$
$$\lambda_3 = 2+n+\frac{(2\beta-2)p}{p-1}; \quad \lambda_4 = 2+n-\frac{2p}{p-1}.$$

Since R is large and $\lambda_1 < \lambda_4 < \lambda_3 = \lambda_2$ then inequality (12) can be rewritten as

$$\int\int_\Omega |u|^p + \int (u_0+u_1) \le 4C\ R^{\lambda_2}. \quad (13)$$

Now, if $\lambda_2 < 0$, ie

$$p < \frac{n+2}{n+2\beta} \quad (14)$$

then it follows from (13) by letting $R \to \infty$ that $\int\int_\Omega |u|^p + \int (u_0+u_1) = 0$ and hence $u \equiv 0$. This proves Theorem 3.1 in the case of $p < \dfrac{n+2}{n+2\beta}$.

Next, consider the case $\lambda_2 = 0$ and let $\mathcal{M}$ denote the restriction to $B_{t,R} = \Big\{(t,x) \in \mathbb{R}^+ \times \mathbb{R}^n : R^4 < t^2 + |x|^4 < 2R^4\Big\}$ of $\Big(\mathcal{A}_{p,\zeta_{tt}} + \mathcal{B}_{p,\zeta_t} + \mathcal{C}_{p,\zeta} + \mathcal{D}_{p,\zeta}\Big)$. We have

$$\int (u_0+u_1)\zeta + \int\int_\Omega |u|^p\zeta \le C\mathcal{M}\Big(\int\int_{B_{t,R}} |u|^p\zeta\Big)^{\frac{1}{p}}. \quad (15)$$

By letting $R \to \infty$, the inequality (13) with $\lambda_2 = 0$ leads to

$$\int\int_{\mathbb{R}^+\times\mathbb{R}^n} |u|^p < \infty.$$

Since $\lambda_2 = 0$, it follows from (8) that

$$\int\int_{\mathbb{R}^+\times\mathbb{R}^n} |u|^p\zeta \leq \left(\int\int_{B_{t,R}} |u|^p\zeta\right)^{\frac{1}{p}} \left(\mathcal{A}_{p,\zeta_{\tau\tau}} + \mathcal{B}_{p,\zeta_\tau} + \mathcal{C}_{p,\zeta} + \mathcal{D}_{p,\zeta}\right).$$

Since

$$\int\int_{\mathbb{R}^+\times\mathbb{R}^n} |u|^p < \infty$$

we get

$$\lim_{R\to+\infty} \int\int_{B_{t,R}} |u|^p\zeta = 0.$$

Now, letting $R \to \infty$ in (15), we obtain

$$\int (u_0 + u_1)\ dx + \int\int |u|^p = 0 \Longrightarrow u = 0.$$

This ends the proof of Theorem 3.1.

4 Remarks

Remark 4.1. *We notice that, in the case where $\beta = 0$, we retrieve the critical exponent $p_{dw} = 1 + \frac{n}{2}$ obtained by Todorova and Yordanov [10].*

The following remark is devoted to some generalization of equation (2).

Remark 4.2. *We can treat, in the same manner, the more general equation with linear damping*

$$u_{tt} + (-1)^m\Delta^m u + g(t)u_t = f(x,t)|u|^p + w(t,x), \quad t > 0, x \in \mathbb{R}^n, \tag{16}$$

subjected to the initial conditions

$$u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), \quad x \in \mathbb{R}^n,$$

where $\Delta^m, m \geq 1$, is the $m-$iterated Laplacian, $p > 1$, $f(t,x) \geq 0$ is a given function behaving like $t^\sigma|x|^\delta$, $w(t,x)$ is a given function, and $g(t)$ behaves like $t^\beta, 0 \leq \beta < 1$.

Our assumption on the initial conditions reads

$$\int_{\mathbb{R}^n} (u_0 + u_1)\ dx \geq 0, \quad \int\int w(t,x)dtdx \geq 0.$$

Taking in the weak formulation of the solution of (16) the test function ζ such that

$$\int_{\mathbb{R}^n} u_0(x)\ \zeta_t(0,x)\ dx = 0$$

this can be obtained by choosing $\zeta(t,x)=\phi\Big(\dfrac{t^2+|x|^{4m}}{R^4}\Big)$. We obtain as before

$$\int\int w\zeta+\int_{\mathbb{R}^n}(u_0+u_1)\ dx+\int f|u|^p\zeta\le C\Bigg[\Big(\int\int(f\zeta)^{-\frac{p'}{p}}|\zeta_{tt}|^{p'}\Big)^{\frac{1}{p'}}$$

$$+\Big(\int\int(f\zeta)^{-\frac{p'}{p}}(g|\zeta_t|)^{p'}\Big)^{\frac{1}{p'}}+\Big(\int\int(f\zeta)^{-\frac{p'}{p}}(g'\zeta)^{p'}\Big)^{\frac{1}{p'}}+\Big(\int\int(f\zeta)^{-\frac{p'}{p}}|\Delta^m\zeta|^{p'}\Big)^{\frac{1}{p'}}\Bigg] \tag{17}$$

Applying $\varepsilon-$Young's inequality to the right hand side to (17), we obtain for some $C(\varepsilon)>0$

$$\int\int w\zeta+\int_{\mathbb{R}^n}(u_0+u_1)\ dx+\int f|u|^p\zeta\le C(\varepsilon)\Big(\mathcal{A}_{p,\zeta_{tt}}+\mathcal{B}_{p,\zeta_t}+\mathcal{C}_{p,\zeta}+\mathcal{D}_{p,\zeta}\Big)^{\frac{p}{p-1}}.$$

Using the scale variables $\tau=R^{-2}t,\ y=R^{-\frac{1}{m}}x$, we obtain

$$1<p\le\frac{n+m(2+\lambda)}{n+2\beta m},$$

for the nonexistence of global solutions of equation (16).

5 Case of system of equations

In this section we consider nonnegative solutions to

$$\begin{cases}u_{tt}-\Delta\ u+g(t)\ u_t=|v|^p & (t,x)\in(0,\infty)\times\mathbb{R}^n\\ v_{tt}-\Delta\ v+f(t)\ v_t=|u|^q & (t,x)\in(0,\infty)\times\mathbb{R}^n\end{cases} \tag{18}$$

subjected to the conditions

$$u(0,x)=u_0(x)\ge 0,\quad u_t(0,x)=u_1(x)\ge 0$$

$$v(0,x)=v_0(x)\quad v_t(0,x)=v_1(x).$$

Theorem 5.1. *Assume that*

1. *$g(t)$ behaves like $t^\beta,\ 0\le\beta<1$*
2. *$f(t)$ behaves like $t^\alpha,\ 0\le\alpha<1$*
3. *$n\le -2\ \max(\alpha,\beta)+\dfrac{2}{pq-1}\ \max\Big(1-\beta+p(1-\alpha),\ 1-\alpha+q(1-\beta)\Big)$,*

then problem (18) *has only the trivial solution* $(u,v)=(0,0)$.

Proof. Set $\zeta(t,x) = \phi\Big(\dfrac{t^2+|x|^4}{R^4}\Big)$. Now multiplying equation of (18) by ζ and integrating over $Q_T=(0,T)\times\mathbb{R}^n$, we get

$$\begin{aligned}\int_{Q_T}|v|^p\zeta &= \int_{\mathbb{R}^n}u_0(x)\zeta_t(0,x)\,dx+\int_{Q_T}u\zeta_{tt}-\int_{Q_T}ug(t)\zeta_t-\int_{Q_T}ug'(t)\zeta-\int_{Q_T}u\Delta\zeta\\ &- \int_{\mathbb{R}^n}u_0(x)\zeta(0,x)\,dx-\int_{\mathbb{R}^n}u_1(x)\zeta(0,x)\,dx\end{aligned} \tag{19}$$

hence

$$\int_{Q_T}|v|^p\zeta\le\int_{Q_T}|u||\zeta_{tt}|+\int_{Q_T}|u|g(t)|\zeta_t|+\int_{Q_T}|u|g'(t)\zeta+\int_{Q_T}|u||\Delta\zeta|. \tag{20}$$

To estimate

$$\int_{Q_T}|u||\zeta_{tt}|,$$

we observe that it can be rewritten as

$$\int_{Q_T}|u||\zeta_{tt}|=\int_{Q_T}|u|\zeta^{\frac1q}|\zeta_{tt}|\zeta^{-\frac1q}.$$

Using Hölder's inequality, we obtain

$$\int_{Q_T}|u||\zeta_{tt}|\le\Big(\int_{Q_T}|u|^q\zeta\Big)^{\frac1q}\Big(\int_{Q_T}|\zeta_{tt}|^{\frac{q}{q-1}}\zeta^{-\frac{1}{q-1}}\Big)^{\frac{q-1}{q}}.$$

Arguing as above we have

$$\int_{Q_T}|u||\Delta\zeta|\le\Big(\int_{Q_T}|u|^q\zeta\Big)^{\frac1q}\Big(\int_{Q_T}|\Delta\zeta|^{\frac{q}{q-1}}\zeta^{-\frac{1}{q-1}}\Big)^{\frac{q-1}{q}},$$

$$\int_{Q_T}|u|g(t)|\zeta_t|\le\Big(\int_{Q_T}|u|^q\zeta\Big)^{\frac1q}\Big(\int_{Q_T}g^{\frac{q}{q-1}}|\zeta_t|^{\frac{q}{q-1}}\zeta^{-\frac{1}{q-1}}\Big)^{\frac{q-1}{q}}$$

and

$$\int_{Q_T}|u|g'(t)\zeta\le\Big(\int_{Q_T}|u|^q\zeta\Big)^{\frac1q}\Big(\int_{Q_T}g'^{\frac{q}{q-1}}\zeta\Big)^{\frac{q-1}{q}}.$$

Finally, we obtain

$$\int_{Q_T}|v|^p\zeta\le\Big(\int_{Q_T}|u|^q\zeta\Big)^{\frac1q}A_q, \tag{21}$$

where

$$\begin{aligned}A_q=\Big(\int_{Q_T}|\zeta_{tt}|^{\frac{q}{q-1}}\zeta^{-\frac{1}{q-1}}\Big)^{\frac{q-1}{q}}+\Big(\int_{Q_T}|\Delta\zeta|^{\frac{q}{q-1}}\zeta^{-\frac{1}{q-1}}\Big)^{\frac{q-1}{q}}\\ +\Big(\int_{Q_T}g^{\frac{q}{q-1}}|\zeta_t|^{\frac{q}{q-1}}\zeta^{-\frac{1}{q-1}}\Big)^{\frac{q-1}{q}}+\Big(\int_{Q_T}g'^{\frac{q}{q-1}}\zeta\Big)^{\frac{q-1}{q}}.\end{aligned}$$

Also, we have

$$\int_{Q_T} |u|^q \zeta \le \left(\int_{Q_T} |v|^p \zeta\right)^{\frac{1}{p}} A_p, \tag{22}$$

where

$$A_p = \left(\int_{Q_T} |\zeta_{tt}|^{\frac{p}{p-1}} \zeta^{-\frac{1}{p-1}}\right)^{\frac{p-1}{p}} + \left(\int_{Q_T} |\Delta\zeta|^{\frac{p}{p-1}} \zeta^{-\frac{1}{p-1}}\right)^{\frac{p-1}{p}}$$

$$+\left(\int_{Q_T} f^{\frac{p}{p-1}} |\zeta_t|^{\frac{p}{p-1}} \zeta^{-\frac{1}{p-1}}\right)^{\frac{p-1}{p}} + \left(\int_{Q_T} f'^{\frac{p}{p-1}} \zeta\right)^{\frac{p-1}{p}}.$$

Using the later inequality into the former one, we obtain

$$\left(\int_{Q_T} |v|^p \zeta\right)^{\frac{pq-1}{pq}} \le A_q \ .A_p^{\frac{1}{q}}. \tag{23}$$

Next we consider the scale of variables

$$t = R^2\ \tau, \quad x = R\ y$$

then

$$\left(\int_{Q_T} |v|^p \zeta\right)^{\frac{pq-1}{pq}} \le C\Big[R^{s_1} + R^{s_2} + R^{s_3} + R^{s_4}\Big] \times \Big[R^{s_5} + R^{s_6} + R^{s_7} + R^{s_8}\Big]^{\frac{1}{q}} \tag{24}$$

where

$$s_1 = -4 + (2+n)\frac{q-1}{q}, \quad s_2 = -2 + (2+n)\frac{q-1}{q}, \quad s_3 = 2\beta - 2 + (2+n)\frac{q-1}{q},$$

$$s_4 = 2\beta - 2 + (2+n)\frac{q-1}{q}, \quad s_5 = -4 + (2+n)\frac{p-1}{p}, \quad s_6 = -2 + (2+n)\frac{p-1}{p},$$

$$s_7 = 2\alpha - 2 + (2+n)\frac{p-1}{p}, \quad s_8 = 2\alpha - 2 + (2+n)\frac{p-1}{p}.$$

We deduce

$$\left(\int_{Q_T} |v|^p \zeta\right)^{\frac{pq-1}{pq}} \le C\ R^{s_4 + \frac{s_8}{q}}. \tag{25}$$

If $s_4 + \dfrac{s_8}{q} < 0$, the right hand side of (25) goes to 0, as R goes to infinity, while the left hand side of (25) goes to

$$\left(\int_{R^+ \times R^n} |v|^p \zeta\right)^{\frac{pq-1}{pq}}.$$

This implies that $v = 0$ and hence $u = 0$.
If $s_4 + \dfrac{s_8}{q} = 0$, we get

$$\int_{R^+ \times R^n} |v|^p\ dxdt < \infty.$$

Using again Hölder's inequality we infer

$$\int_{Q_T} |u|^q \zeta \leq \left(\int_{\{R^2 \leq t^2 + |x|^4 \leq 2R^2\}} |v|^p \zeta \right)^{\frac{1}{p}} A_p.$$

Since

$$\int_{R^+ \times R^n} |v|^p \, dxdt < \infty$$

we get

$$\lim_{R \to +\infty} \int_{\{R^2 \leq t^2 + |x|^4 \leq 2R^2\}} |v|^p \zeta = 0.$$

The later inequality implies

$$\int_{R^+ \times R^n} |u|^q \, dxdt = 0$$

which ends the proof.

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