Long-run availability of a two-unit standby system subjected to a priority rule

E. J. Vanderperre *

Abstract

We introduce a basic two-unit cold standby system subjected to a priority rule and attended by two different repairmen. In order to determine the invariant measure of the twin system, we employ a stochastic process endowed with probability measures satisfying general Hokstad-type differential equations. The solution procedure is based on advanced methods of complex analysis (sectionally holomorphic functions). Finally, we derive the long-run availability of the system.

1 Introduction

Standby redundancy provides a powerful tool to increase the reliability, availability and safety of operational plants, e.g. [1], [4], [12], [17 - 18].

However, redundant systems are often subjected to an appropriate priority rule. For instance, the *external* power supply station of a technical plant has usually overall priority in operation with regard to an *internal* (local) power generator in standby. The local generator is only used when the external unit is down.

Two-unit (cold or warm [1]) standby systems subjected to a priority rule and attended by a repair facility have received considerable attention in the current Literature [3], [6], [8 - 9], [13 - 16].

*Department of Quantitative Management University of South Africa
Received by the editors January 1999.
Communicated by M. Hallin.
1998 *Mathematics Subject Classification :* 60 K 10.
Key words and phrases : Twin system, priority rule, invariant measure, Hokstad-type differential equations, esctionally holomorphic function, long-run availability.

As a variant, we consider a twin system composed of a priority unit (the p-unit) and a non-priority unit (the n-unit) in cold standby. The p-unit has overall (break-in) priority in operation with regard to the n-unit, i.e. the n-unit is only used when the p-unit is down. In order to avoid undesirable delays in repairing failed units, we suppose that the entire system (henceforth called a **T**-system) is attended by *two* different repairmen.

The **T**-system satisfies the usual conditions, i.e. independent identically distributed random variables, instantaneous and perfect switch [1] and perfect repair [5].

Each repairman has his own particular task. Repairman $\mathcal{N}$ is skilled in repairing the n-unit, whereas repairman $\mathcal{P}$ is an expert in repairing the p-unit. Both repairmen are jointly busy if, and only if, both units (p-unit and n-unit) are down. In any other case, at least one repairman is idle.

In order to determine the invariant measure of the **T**-system, we introduce a stochastic process endowed with probability measures satisfying general Hokstad-type differential equations. The exact solution procedure is based on advanced methods of complex analysis, e.g. [2], [7]. Finally, we derive the long-run availability of the **T**-system.

2 Formulation

Consider a **T**-system satisfying the usual conditions. The p-unit has a constant failure rate $\lambda > 0$, [10] and a general repair time distribution $R(.)$, $R(0) = 0$ with mean ρ and variance σ^2.

The operative n-unit has a constant failure rate $\lambda_s > 0$, but a zero failure rate in standby (the so-called "cold" standby state) and a general repair time distribution $R_S(.)$, $R_S(0) = 0$ with mean ρ_S and variance σ_S^2.

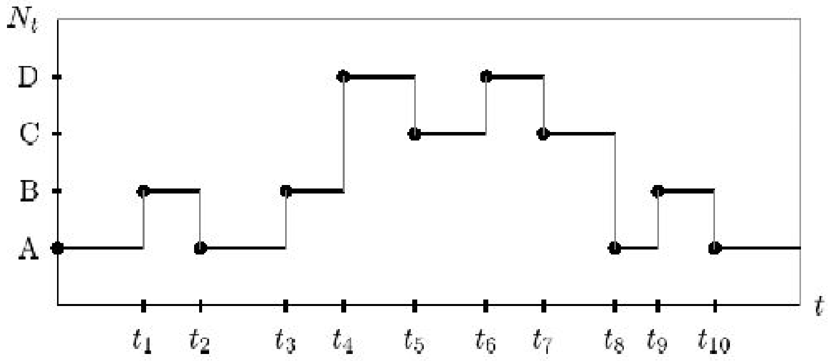

FIGURE 1. A sample path (trajectory) of $\{N_t, t \geq 0\}$.

The corresponding repair times are denoted by r and r_S.

Characteristic functions (and their duals) are formulated in terms of a *complex* transform variable. For instance,

$$\mathbf{E}e^{i\omega r} = \int_0^\infty e^{i\omega x} dR(x), \ \text{Im}\ \omega \geq 0.$$

But note that

$$\mathbf{E}e^{-i\omega r} = \int_0^\infty e^{-i\omega x} dR(x) = \int_{-\infty}^0 e^{i\omega x} d(1 - R((-x)-)), \text{ Im } \omega \le 0.$$

The corresponding Fourier-Stieltjes transforms are called *dual* transforms. Without loss of generality (see our forthcoming remarks), we may assume that both repair time distributions have bounded densities (in the Radon-Nikodym sense) defined on $[0, \infty)$.

In order to analyse the random behaviour of the **T**-system, we introduce a stochastic process $\{N_t, t \ge 0\}$ with arbitrary discrete state space $\{A, B, C, D\}$, characterized by the following events:

$\{N_t = A\}$: The p-unit is operative and the n-unit is in cold standby at time t.
$\{N_t = B\}$: The n-unit is operative at time t.
$\{N_t = C\}$: The p-unit is operative and the n-unit is in repair at time t.
$\{N_t = D\}$: Both units are simultaneously down at time t.

Figure 1 displays a right-continuous sample path N_t, where $N_0 = A$ a.s.; $A = 1$, $B = 2$, $C = 3$, $D = 4$. An upwards (resp, downwards) jump corresponds to a failure (resp. repair) of a unit. Our priority rule entails that a transition from A to C is only possible via D.

A Markov characterization of the process $\{N_t, t \ge 0\}$ is piecewise and conditionally defined by:

$\{N_t\}$, if $N_t = A$ (i.e. if the event $\{N_t = A\}$ occurs).
$\{(N_t, X_t)\}$, if $N_t = B$, where X_t denotes the *remaining* repair time of the p-unit in progressive repair at time t.
$\{(N_t, Y_t)\}$, if $N_t = C$, where Y_t denotes the *remaining* repair time of the n-unit in progressive repair at time t.
$\{(N_t, X_t, Y_t)\}$, if $N_t = D$.

The state space of the underlying Markov process is given by

$$\{A\} \cup \{(B, x); x \ge 0\} \cup \{(C, y); y \ge 0\} \cup \{(D, x, y); x \ge 0, y \ge 0\}.$$

Next, we consider the **T**-system in stationary state (the so-called ergodic state) with *invariant* measure $\{p_K; K = A, B, C, D\}$, $\sum_K p_K = 1$, where

$$p_K := \mathbf{P}\{N = K\} := \lim_{t\to\infty} \mathbf{P}\{N_t = K | N_0 = A\}.$$

Finally, we introduce the measures

$$\begin{aligned}
\varphi_B(x)dx &:= \mathbf{P}\{N = B, X \in dx\} := \lim_{t\to\infty} \mathbf{P}\{N_t = B, X_t \in dx | N_0 = A\}, \\
\varphi_C(y)dy &:= \mathbf{P}\{N = C, Y \in dy\} := \lim_{t\to\infty} \mathbf{P}\{N_t = C, Y_t \in dy | N_0 = A\}, \\
\varphi_D(x,y)dxdy &:= \mathbf{P}\{N = D, X \in dx, Y \in dy\} := \lim_{t\to\infty} \mathbf{P}\{N_t = D, X_t \in dx, Y_t \in dy | N_0 = A\}.
\end{aligned}$$

Notations The indicator of an event $\mathcal{E}$ is denoted by $\mathbb{1}(\mathcal{E})$, i.e.

$$\mathbb{1}(\mathcal{E}) := \begin{cases} 1, & \text{if } \mathcal{E} \text{ occurs,} \\ 0, & \text{otherwise.} \end{cases}$$

Note that, for instance,

$$\mathbf{E}\{e^{i\omega X}e^{i\eta Y}\mathbb{1}(N=D)\} = \int_0^\infty\int_0^\infty e^{i\omega x}e^{i\eta y}\varphi_D(x,y)dxdy, \mathrm{Im}\ \omega\geq 0,\ \mathrm{Im}\ \eta\geq 0.$$

So that,

$$p_D = \int_0^\infty\int_0^\infty \varphi_D(x,y)dxdy.$$

The real line and the complex plane are denoted by $\mathbf{R}$ and $\mathbf{C}$, with obvious superscript notations, such as $\mathbf{C}^+, \mathbf{C}^-, \mathbf{C}^+\cup\mathbf{R}, \mathbf{C}^-\cup\mathbf{R}$. For instance, $\mathbf{C}^+ := \{\omega\in\mathbf{C} : \mathrm{Im}\ \omega > 0\}$.

3 Differential equations

In order to determine the φ-functions, we first construct a system of steady-state Hokstad-type differential equations based on a time independent version of Hokstad's supplementary variable technique (see e.g. Ref [13, p 526]). For $x>0, y>0$, we obtain

$$\begin{aligned}
\lambda p_A &= \varphi_B(0)+\varphi_C(0),\\
\left(\lambda_S - \frac{d}{dx}\right)\varphi_B(x) &= \varphi_D(x,0)+\lambda p_A\frac{d}{dx}R(x),\\
\left(\lambda - \frac{d}{dy}\right)\varphi_C(y) &= \varphi_D(0,y),\\
\left(-\frac{\partial}{\partial x}-\frac{\partial}{\partial y}\right)\varphi_D(x,y) &= \lambda_S\varphi_B(x)\frac{d}{dy}R_S(y)+\lambda\ \varphi_C(y)\frac{d}{dx}R(x).
\end{aligned}$$

4 Solution procedure

It should be noted that our equations are well adapted to an integral transformation. As a matter of fact, the integrability of the φ-functions and their corresponding derivatives implies that each φ-function vanishes at infinity irrespective of the asymptotic behaviour of the underlying repair time densities! Applying a routine Fourier transform technique to the equations and invoking the boundary condition $\lambda p_A = \varphi_B(0)+\varphi_C(0)$, reveals that

$$\begin{aligned}
&\lambda p_A(1-\mathbf{E}e^{i\omega r}) + (\lambda_S(1-\mathbf{E}e^{i\eta r_S})+i\omega)\mathbf{E}\{e^{i\omega X}\mathbb{1}(N=B)\}\ +\\
&(\lambda(1-\mathbf{E}e^{i\omega r})+i\eta)\mathbf{E}\{e^{i\eta Y}\mathbb{1}(N=C)\} + i(\omega+\eta)\mathbf{E}\{e^{i\omega X}e^{i\eta Y}\mathbb{1}(N=D)\} = 0. \qquad (1)
\end{aligned}$$

Observe that Eq (1) holds for any pair $(\omega,\eta)\in\mathbf{C}\times\mathbf{C} :\ \mathrm{Im}\ \omega\geq 0, \mathrm{Im}\ \eta\geq 0$. Therefore, substituting $\omega = t, \eta = -t$ $(t\in\mathbf{R})$ into Eq (1), yields the functional equation

$$\mathbf{E}\{e^{itX}\mathbb{1}(N=B)\}\psi^+(t) - \mathbf{E}\{e^{-itY}\mathbb{1}(N=C)\}\psi^-(t) = \psi(t), \qquad (2)$$

where for all $t\in\mathbf{R}$,

$$\psi^+(t) := \frac{p_A^{-1}}{1+\lambda\rho\varphi^+(t)}\ ,\quad \psi^-(t) := \frac{p_A^{-1}}{1+\lambda_S\rho_S\varphi^-(t)}\ ,$$

$$\varphi^+(t) := \frac{\mathbf{E}e^{itr} - 1}{it\rho}, \quad \varphi^+(0) := 1,$$

$$\varphi^-(t) := \frac{1 - \mathbf{E}e^{-itrs}}{it\rho_s}, \quad \varphi^-(0) := 1,$$

$$\psi(t) := \frac{\lambda\rho\varphi^+(t)}{(1+\lambda\rho\varphi^+(t))(1+\lambda_s\rho_S\varphi^-(t))}.$$

Note that

$$\psi(0) = \frac{\lambda\rho}{(1+\lambda\rho)(1+\lambda_s\rho_s)}.$$

Eq (2) constitutes a Plemelj boundary value problem on $\mathbf{R}$ which can be solved by the theory of sectionally holomorphic functions. As a matter of fact, a straightforward application of Rouché's theorem reveals that the function $1+\lambda\rho\varphi^+(\omega)$, $\mathrm{Im}\,\omega \geq 0$, has no zeros in $\mathbf{C}^+ \cup \mathbf{R}$, whereas $1+\lambda_S\rho_S\varphi^-(\omega)$, $\mathrm{Im}\,\omega \leq 0$, has no zeros in $\mathbf{C}^- \cup \mathbf{R}$. Consequently, the function

$$\mathbf{E}\{e^{i\omega X}\mathbb{1}(N=B)\}\psi^+(\omega), \ \mathrm{Im}\,\omega \geq 0,$$

is analytic in $\mathbf{C}^+$, bounded and continuous on $\mathbf{C}^+ \cup \mathbf{R}$ and

$$\lim_{\substack{|\omega| \to \infty \\ 0 \leq \arg\omega \leq \pi}} \mathbf{E}\{e^{i\omega X}\mathbb{1}(N=B)\}\psi^+(\omega) = 0.$$

On the other hand, the function

$$\mathbf{E}\{e^{-i\omega Y}\mathbb{1}(N=C)\}\psi^-(\omega), \ \mathrm{Im}\,\omega \leq 0,$$

is analytic in $\mathbf{C}^-$, bounded and continuous on $\mathbf{C}^- \cup \mathbf{R}$, whereas

$$\lim_{\substack{|\omega| \to \infty \\ \pi \leq \arg\omega \leq 2\pi}} \mathbf{E}\{e^{-i\omega Y}\mathbb{1}(N=C)\}\psi^-(\omega) = 0.$$

Moreover, ψ is (uniformly) Lipschitz continuous on $\mathbf{R}$. (Simply note that $|\psi'(t)|$ is bounded on $\mathbf{R}$. Therefore, our assertion follows from the mean value theorem.)

Finally, ψ is Hölder continuous at infinity, i.e. $|\psi(t)| = O(|t|^{-1})$, if $|t| \to \infty$.

Consequently, the Cauchy-type integral

$$\frac{1}{2\pi i}\int_\Gamma \psi(\tau)\frac{d\tau}{\tau-\omega}\ ,$$

(see Appendix) exists for all $\omega \in \mathbf{C}$ (real or complex), and defines a sectionally holomorphic function which vanishes at infinity.

An application of the Cauchy formulae for the regions $\mathbf{C}^+$ and $\mathbf{C}^-$, entails that

$$\mathbf{E}\{e^{i\omega X}\mathbb{1}(N=B)\} = \frac{1}{\psi^+(\omega)}\frac{1}{2\pi i}\int_\Gamma \psi(\tau)\frac{d\tau}{\tau-\omega}\ ,\ \omega \in \mathbf{C}^+\ ,$$

$$\mathbf{E}\{e^{-i\omega Y}\mathbb{1}(N=C)\} = \frac{1}{\psi^-(\omega)}\frac{1}{2\pi i}\int_\Gamma \psi(\tau)\frac{d\tau}{\tau-\omega}\ ,\ \omega \in \mathbf{C}^-.$$

In particular, we obtain

$$p_B = p_A(1+\lambda\rho)\Phi^+(0) \ , \tag{3}$$

$$p_C = p_A(1+\lambda_S\rho_S)\Phi^-(0) \ , \tag{4}$$

where

$$\Phi^+(0) := \lim_{\substack{\omega \to 0 \\ \omega \in \mathbf{C}^+}} \frac{1}{2\pi i}\int_\Gamma \psi(\tau)\frac{d\tau}{\tau-\omega} \ , \qquad \Phi^-(0) := \lim_{\substack{\omega \to 0 \\ \omega \in \mathbf{C}^-}} \frac{1}{2\pi i}\int_\Gamma \psi(\tau)\frac{d\tau}{\tau-\omega} \ .$$

Applying the Sokhotskii-Plemelj formulae yields

$$\Phi^+(0) = \frac{1}{2}\psi(0) + \frac{1}{2\pi i}\int_\Gamma \psi(\tau)\frac{d\tau}{\tau} \ , \qquad \Phi^-(0) = -\frac{1}{2}\psi(0) + \frac{1}{2\pi i}\int_\Gamma \psi(\tau)\frac{d\tau}{\tau} \ .$$

Subtracting both equations reveals that

$$\Phi^+(0) - \Phi^-(0) = \frac{\lambda\rho}{(1+\lambda\rho)(1+\lambda_S\rho_S)} \ . \tag{5}$$

On the other hand, we have

$$p_D = \lim_{\substack{\omega \to 0 \\ \omega \in \mathbf{C}^+}} \lim_{\substack{\eta \to 0 \\ \eta \in \mathbf{C}^+}} \mathbf{E}\{e^{i\eta Y}e^{i\omega X}\mathbb{1}(N=D)\}.$$

Applying the limit procedure to Eq (1), and invoking the condition $p_A + p_B + p_C + p_D = 1$, yields the additional relation

$$p_A + p_B(1+\lambda_S\rho_S) = 1 \ . \tag{6}$$

Substituting Eq (3) into Eq (6) yields

$$p_A = \frac{1}{1+(1+\lambda\rho)(1+\lambda_S\rho_S)\Phi^+(0)} \ .$$

Finally, eliminating $\Phi^+(0)$ and $\Phi^-(0)$ by means of the equations (3), (4) and (5), entails that

$$p_A + p_C = \frac{1}{1+\lambda\rho} \ , \qquad p_B + p_D = \frac{\lambda\rho}{1+\lambda\rho} \ .$$

Note that we have completely determined the invariant measure simply and solely depending upon $\Phi^+(0)$.

5 Long-run availability

We recall that the **T**-system is only available (operative) in states A, B or C. Therefore, the long-run availability of the **T**-system, denoted by $\mathcal{A}$, is given by $\mathcal{A} = 1 - p_D$. We summarize explicitly as follows.
Result

$$\mathcal{A} = \frac{1}{1+\lambda\rho} + \frac{(1+\lambda\rho)\Phi^+(0)}{1+(1+\lambda\rho)(1+\lambda_S\rho_S)\Phi^+(0)} \ ,$$

where

$$\Phi^+(0) = \lim_{\substack{\omega \to 0 \\ \omega \in \mathbf{C}^+}} \frac{1}{2\pi i}\int_\Gamma \psi(\tau)\frac{d\tau}{\tau-\omega} \ .$$

Remarks It should be noted that the kernel $\psi(t),\ t \in \mathbf{R}$, preserves all the relevant properties to ensure the existence of the Cauchy integral for *arbitrary* repair time distributions with finite mean and variance. First of all, the order relation $|\psi(t)| = O(|t|^{-1}),\ |t| \to \infty$, also holds for arbitrary characteristic functions. Moreover, the H-continuity of ψ on $\mathbf{R}$ does not depend on the canonical structure (decomposition) of R or R_S. For instance, the Hölder inequality

$$|\mathbf{E}e^{it_2 r} - \mathbf{E}e^{it_1 r}| \le \rho|t_2 - t_1|,\ (t_2,\ t_1 \in \mathbf{R}),$$

always holds for *any* r with mean ρ.

The requirement of finite variances is extremely mild. As a matter of fact, the current probability distributions employed to model repair times [1], even have moments of all orders!

Consequently, our initial assumptions concerning the existence of repair time densities are totally superfluous to ensure the existence of an invariant measure.

6 Evaluation

Next, we propose a (brief) outline to evaluate the C-type integral for an arbitrary R_S (or R). Let, for instance,

$$\mathbf{E}e^{itr} = \frac{Q_p(t)}{Q_q(t)},\ 0 \le p < q,$$

where $Q_k(t);\ k = p,\ q$ is a polynomial of degree k. D.R. Cox has shown that this family is surprisingly large. Furthermore, let R_S be *arbitrary.* Evaluating

$$\frac{\lambda\rho\varphi^+(t)}{1+\lambda\rho\varphi^+(t)} \ ,$$

in terms of $Q_k(t)$, entails that

$$\Phi^+(0) = \lim_{\substack{\omega \to 0 \\ \omega \in \mathbf{C}^+}} \frac{1}{2\pi i}\int_\Gamma \frac{1}{1+\lambda_S\rho_S\varphi^-(t)}\frac{\lambda(Q_p(t)-Q_q(t))}{itQ_q(t)+\lambda(Q_p(t)-Q_q(t))}\frac{dt}{t-\omega} \ .$$

Note that

$$0 < \rho = \frac{Q_p'(0) - Q_q'(0)}{iQ_q(0)} < \infty,$$

so that $t = 0$ is a removable singularity of the integrand.

The equation

$$izQ_q(z) + \lambda(Q_p(z) - Q_q(z)) = 0,$$

has (apart from the removable pole $z = 0$) q roots z_j; $j = 1, \dots, q$ located in the lower half plane $\mathbf{C}^-$. (We recall that the function $1 + \lambda_S \rho_S \varphi^-(z)$ has no zeros in $\mathbf{C}^- \cup \mathbf{R}$). Evaluating the integral in terms of the so-called *operating characteristics* (Cf [17], for further details), yields

$$\Phi^+(0) = -\sum_j \mathrm{Res}_{z=z_j} \left\{ \frac{1}{1 + \lambda_S \rho_S \varphi^-(z)} \frac{\lambda(Q_p(z) - Q_q(z))}{izQ_q(z) + \lambda(Q_p(z) - Q_q(z))} \frac{1}{z} \right\}.$$

Observe that the minus sign appears due to the *clockwise* integration along a suitable semi-circle enclosing the poles z_j.

The simplest case occurs when $R(x) = 1 - e^{-\rho^{-1}x}$, i.e. let

$$\mathbf{E}e^{itr} = \frac{1}{1 - i\rho t} .$$

Some algebra reveals that

$$\Phi^+(0) = \frac{\lambda}{\lambda + \rho^{-1} + \lambda_S(1 - \mathbf{E}e^{-(\lambda + \rho^{-1})r})}.$$

Finally, we remark that a repair time distribution with a complicated FST (such as the Weibull of Log-normal distribution) could be substituted by an accurate *finite* mixture of Coxian distributions (see e.g. [17, Ref 15]). Consequently, our proposed technique allows to compute $\Phi^+(0)$ for an *arbitrary* R_S (or R) by exact methods based on approximations.

7 Conclusion

A concatenation of Hokstad's supplementary variable technique and the theory of sectionally holomorphic functions provides a powerful tool to analyse our proposed **T**-system.

The invariant measure holds for *arbitrary* repair time distributions with finite mean and variance. Moreover, the evaluation of the Cauchy-type integral

$$\frac{1}{2\pi i} \int_\Gamma \psi(t) \frac{dt}{t - \omega} ,$$

does not actually impose an insuperable computational problem. As a matter of fact, there exists a huge Literature to evaluate the integral by exact or numerical computer aided routines.

Appendix

Let $f(t),\ t \in \mathbf{R}$ be a bounded and continuous function. f is called Γ-integrable, if

$$\lim_{\substack{T \to \infty \\ \varepsilon \downarrow 0}} \int_{\Gamma_{T,\varepsilon}} f(t) \frac{dt}{t-u}\ ,\ u \in \mathbf{R},$$

exists, where $\Gamma_{T,\varepsilon} := (-T, u-\varepsilon] \cup [u+\varepsilon, T)$.

The corresponding integral, denoted by

$$\frac{1}{2\pi i} \int_{\Gamma} f(t) \frac{dt}{t-u},$$

is called a Cauchy principal value in double sense.

References

[1] Birolini, A. (1994), *Quality and Reliability of Technical Systems*, Springer-Verlag, Berlin.

[2] Bliev, N. (1997), *Generalized Analytic Functions in Fractional Spaces*, Addison-Wesley Longman Inc., Harlow.

[3] Buzacott, J.A. (1971), Availability of priority redundant systems, *IEEE Trans. Reliab.* **R-20**, 60-63.

[4] Dhillon, B.S. and Yang, N. (1997), Formulae for analyzing a redundant robot configuration with a built-in safety system, *Microelectron. Reliab.* **37**, 557-563.

[5] Gertsbakh, I.B. (1989), *Statistical Reliability Theory*, Marcel Dekker, New York.

[6] Li, W. and Shi, D.H. (1993), Reliability analysis of a two-unit parallel system with a preemptive priority rule, *Microelectron. Reliab.* **33**, 1447-1453.

[7] Muskhelishvili, N.I. (1977), *Singular Integral Equations*, Noordhof, Leiden.

[8] Nakagawa, T. and Osaki, S. (1975), Stochastic behaviour of a two-unit priority standby redundant system with repair, *Microelectron. Reliab.* **14**, 309-313.

[9] Osaki, S. (1970), Reliability analysis of a two-unit standby redundant system with priority, *Canadian J. Oper. Res.* **8**, 60-62.

[10] Shaked, M. and Shantikumar, J.G. (1990), *Reliability and Maintainability*, In: Handbook in Operations Research and Management Science **2**, Heyman, D.P. and Sobel, M.J. (Eds.), North-Holland, Amsterdam.

[11] Shao, J. and Lamberson, L.R. (1991), Reliability model of cold standby systems with built-in-test, *Microelectron. Reliab.* **31**, 443-451.

[12] Shao, J. and Lamberson, L.R. (1988), Impact of BIT design parameters on systems RAM, *Reliab. Eng. Syst. Safety* **23**, 219-246.

[13] Shi, D.H. and Liu, L. (1996) Availability analysis of a two-unit series system with a priority shut-off rule, *Naval Res. Log.* **43**, 1009-1024.

[14] Subramanian, R. and Ravichandran, N. (1980), A two-unit priority redundant system with preemptive resume repair, *IEEE Trans. Reliab.* R-**29**, 183-184.

[15] Val'Des Kastro, K. (1986), Optimal policies in a priority standby model, *Moscow Univ. Math. Bull.* **41**, 10-15.

[16] Vanderperre, E.J. (1989), Reliability analysis of a two-unit cold standby system subjected to a priority rule, *CCERO, Oper. Res.* **31**, 159-165.

[17] Vanderperre, E.J. (1998), On the reliability of Gaver's parallel system sustained by a cold standby unit and attended by two repairmen, *J. Oper. Res. Soc. Japan*, **41**, 171-180.

[18] Vanderperre, E.J., Makhanov, S.S. and Suchatvejapoom, S. (1997), Long-run availability of a repairable parallel system, *Microelectron. Reliab.* **37**, 525-527.

Correspondence Address:
Dr. E.J. Vanderperre
Residentie Neptunus
Ruzettelaan 183
Bus 158
8370 Blankenberge
Belgium
e-mail : evanderperre@hotmail.com