Corrigenda to “Polarities of Symplectic Quadrangles”

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Lemma 1.2 of [1] claims that the elements of the group Γ_ℓ of all automorphisms of the symplectic quadrangle $\mathrm{W}(F)$ that fix a line ℓ pointwise are induced by block matrices of the form

$$\begin{pmatrix} c\mathbf{1} & 0 \\ X & c^{-1}\mathbf{1} \end{pmatrix}, \qquad \text{where } c \in F^\times, X \in F^{2\times 2} \text{ such that } (X\boldsymbol{j})' = X\boldsymbol{j}.$$

This is false if the ground field F contains non-squares. In fact, easy computations yield (contrary to what is stated in the paper) that Γ_ℓ is induced by the subgroup of GSp_4F consisting of all block matrices of the form

$$T(c,d,X) := \begin{pmatrix} c\mathbf{1} & 0 \\ X & d\mathbf{1} \end{pmatrix}, \text{ where } c,d \in F^\times, X \in F^{2\times 2} \text{ such that } (X\boldsymbol{j})' = X\boldsymbol{j}.$$

Scalar multiples do not affect the action on the quadrangle, and we obtain

Lemma. *The elements of Γ_ℓ are represented by the elements of the subgroup*

$$\left\{T(c,1,X) \,\middle|\, c \in F^\times, X \in F^{2\times 2}, (X\boldsymbol{j})' = X\boldsymbol{j}\right\} \le \mathrm{GSp}_4F\,.$$

For each $c \in F^\times$, the element $T(c,c^{-1},X)$ used above induces the same automorphism as $T(c^2,1,X)$. For any field with more than two elements, the subgroup $\Sigma \le \Gamma_\ell$ induced by $\left\{T(c^2,1,X) \mid c \in F^\times, X \in F^{2\times 2}, (X\boldsymbol{j})' = X\boldsymbol{j}\right\}$ consists of all squares of elements of Γ_ℓ. If every element of F is a square (in particular, in the case where F has two elements), the groups Γ_ℓ and Σ coincide.

Lemma 1.5.3 of [1] has to be corrected: For $|F| = 3$, the group $\mathrm{E}_\ell = \Sigma$ is the commutator subgroup of Γ_ℓ.

The vector space structure introduced in [1] 1.6 on E_ℓ only describes the action of scalars that are squares in F. Thus Lemma 1.7.3 gives $\dim_{F^\phi} \mathrm{E}_\ell$ rather than $\dim_F \mathrm{E}_\ell$. Nevertheless, the crucial observation that $\dim_F \mathrm{E}_\ell = \dim_F \mathrm{E}_p$ implies $F^\phi = F$ remains valid. After this observation, only perfect fields of characteristic 2 are considered, and the distinction between Σ and Γ_ℓ vanishes.

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References

[1] Stroppel, Markus: *Polarities of symplectic quadrangles*, Bull. Belg. Math. Soc. **10** (2003), 437–449.

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