TRANSLATIONAL SURFACES

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Abstract. A translational surface is a rational surface generated from two rational space curves by translating one curve along the other curve. In this paper, we utilize matrices to represent translational surfaces, and give necessary and sufficient conditions for a real rational surface to be a translational surface.

1. Introduction

A translational surface is a rational surface generated from two rational space curves by translating either one of these curves parallel to itself in such a way that each of its points describes a curve that is a translation along the other curve. Since translational surfaces are generated from two space curves, translational surfaces have simple representations. The simplest and perhaps the most common representation of a translational surface is given by the rational parametric representation $\mathbf{h}^{\circ}(s;t) = \mathbf{f}^{\circ}(s) + \mathbf{g}^{\circ}(t)$, where $\mathbf{f}^{\circ}(s)$ and $\mathbf{g}^{\circ}(t)$ are two rational space curves. The goal of this paper is to utilize matrices to represent and identify translational surfaces.

Translational surfaces are being studied since they have uses within computer graphics and computer aided design. Naturally, finding ways to represent these surfaces along with identifying them using these representations is going to be an area of interest for study. Previous work has been done to find two space curves to generate a given translational surface which revolves around finding a parametrization of a curve that could help generate the translational surface and then finding the second curve needed, based on the first [2]. Other work has focused on taking the parametric representation of a translational surface and finding the implicit equation of the surface by utilizing a support function for the surface [1]. In this paper, we study translational surfaces via matrix representations. We utilize elementary matrix operations to provide necessary and sufficient conditions for a real rational surface to be a translational surface. As a by-product, we also obtain two generating space curves if the given rational surface is a translational surface.

This paper is structured in the following fashion. In Section 2, we introduce our notation, define translational surfaces, and provide several examples of translational surfaces. In Section 3, we give matrix representations for translational surfaces to prepare for the proof in the following section. In Section 4, we identify translational surfaces, that is to give necessary and sufficient conditions for a real rational surface to be a translational surface. In Section 5, we conclude the paper. We provide illustrative examples throughout the paper.

2. Translational Surfaces in Real 3-Space

Throughout this paper, let $\mathbb{R}[s]$ be the polynomial ring in one variable s, and $\mathbb{R}[s,t]$ the polynomial ring in two variables s and t. The following notation denotes the polynomial and rational forms of the same vector valued function $\mathbf{a}$:

$$\mathbf{a}(s) = [a_0(s), a_1(s), a_2(s), a_3(s)] \in \mathbb{R}^4[s], \qquad \text{polynomial form,}$$
$$\mathbf{a}^\circ(s) = \left(\frac{a_1(s)}{a_0(s)}, \frac{a_2(s)}{a_0(s)}, \frac{a_3(s)}{a_0(s)}\right) \in \mathbb{R}^3(s), \qquad \text{rational form.}$$

In real 3-space, a rational surface

$$\mathbf{h}^\circ(s;t) = \mathbf{f}^\circ(s) + \mathbf{g}^\circ(t), \tag{1}$$

is called the *translational surface* generated by the rational space curves $\mathbf{f}^\circ(s)$ and $\mathbf{g}^\circ(t)$

$$\mathbf{f}^\circ(s) = \left(\frac{f_1(s)}{f_0(s)}, \frac{f_2(s)}{f_0(s)}, \frac{f_3(s)}{f_0(s)}\right), \quad \mathbf{g}^\circ(t) = \left(\frac{g_1(t)}{g_0(t)}, \frac{g_2(t)}{g_0(t)}, \frac{g_3(t)}{g_0(t)}\right), \tag{2}$$

where $f_i \in \mathbb{R}[s]$ and $g_i \in \mathbb{R}[t]$ with $\max(\deg(f_i(s))) = m$, $\max(\deg(g_i(t))) = n$, and $\gcd(\mathbf{f}) = \gcd(f_0, f_1, f_2, f_3) = \gcd(\mathbf{g}) = \gcd(g_0, g_1, g_2, g_3) = 1$.

Translational surfaces are typical modeling surfaces in architecture and computer aided design. Among the non-degenerate quadratic surfaces, ellipsoids, elliptical (and circular) cones, and hyperboloids of one or two sheets cannot be constructed as translational surfaces, that is, these surfaces do not admit a rational parametrization given by equation (1). However, elliptical (and circular) cylinders, parabolic cylinders, hyperbolic cylinders, elliptical (and circular) paraboloids, and hyperbolical paraboloids can be constructed as translational surfaces. It is easy to picture that elliptical (and circular), parabolic, or hyperbolic cylinders are translational surfaces generated by translating an ellipse (and circle), parabola, or hyperbola along a straight line. Next, we provide examples of a circular paraboloid, a parabolic cylinder, and a hyperbolical paraboloid constructed as translational surfaces.

Example 2.1. *The circular paraboloid $z = x^2 + y^2 + 2$ is the translational surface given by the parametric representation:*

$$\mathbf{h}^\circ = \mathbf{f}^\circ + \mathbf{g}^\circ, \quad \mathbf{f}^\circ(s) = \frac{(2s, 0, 2s^2 + 2)}{2s^2}, \quad \mathbf{g}^\circ(t) = \frac{(0, 2t, 2t^2 + 2)}{2t^2},$$

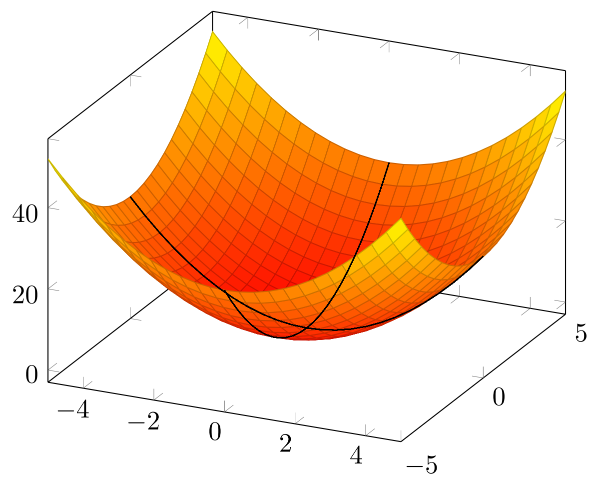

FIGURE 1. Circular paraboloid.

To verify the implicit equation of this parametric representation, we observe that

$$(x, y, z) = \frac{(2s, 0, 2s^2 + 2)}{2s^2} + \frac{(0, 2t, 2t^2 + 2)}{2t^2} = \left(\frac{1}{s}, \frac{1}{t}, 2 + \frac{1}{s^2} + \frac{1}{t^2}\right),$$

and therefore,

$$z = \frac{1}{s^2} + \frac{1}{t^2} + 2 = x^2 + y^2 + 2.$$

Hence, the circular paraboloid $z = x^2 + y^2 + 2$ is a translational surface.

Example 2.2. *The parabolic cylinder $z = \dfrac{-x^2}{4}$ is the translational surface given by the parametric representation:*

$$\mathbf{h}^\circ = \mathbf{f}^\circ + \mathbf{g}^\circ, \quad \mathbf{f}^\circ(s) = \frac{(4s^2, 0, -s^3)}{4s}, \quad \mathbf{g}^\circ(t) = \frac{(0, 3t^2, 0)}{3t}.$$

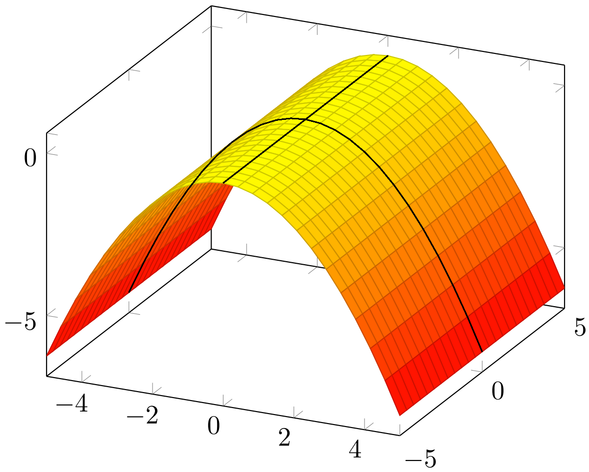

FIGURE 2. Parabolic cylinder.

To verify the implicit equation of this parametric representation, we observe that

$$(x, y, z) = \frac{(4s^2, 0, -s^3)}{4s} + \frac{(0, 3t^2, 0)}{3t} = \left(s, t, \frac{-s^2}{4}\right),$$

and therefore,

$$z = \frac{-s^2}{4} = \frac{-x^2}{4}.$$

Hence, the parabolic cylinder $z = \dfrac{-x^2}{4}$ *is a translational surface.*

Example 2.3. *The hyperbolical paraboloid* $z = y^2 - 3x^2 + 5$ *is the translational surface given by the parametric representation:*

$$\mathbf{h}^\circ = \mathbf{f}^\circ + \mathbf{g}^\circ, \quad \mathbf{f}^\circ(s) = \frac{(s^4, s^2, 1 + 3s^4)}{s^4}, \quad \mathbf{g}^\circ(t) = \frac{(t - t^2, 0, -3 + 2t^2)}{t^2}.$$

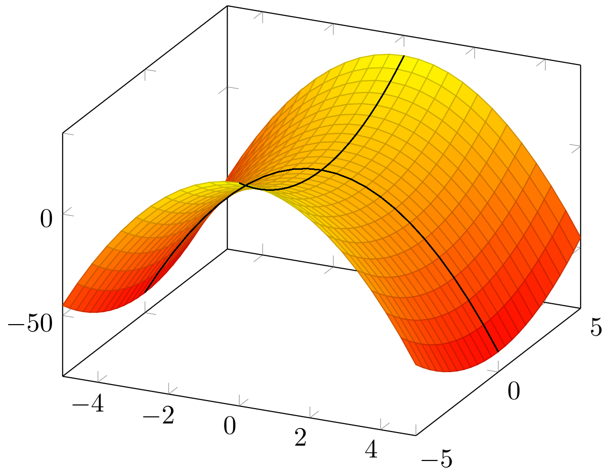

Figure 3. Hyperbolical paraboloid.

To verify the implicit equation of this parametric representation, we observe that

$$(x, y, z) = \frac{(s^4, s^2, 1+3s^4)}{s^4} + \frac{(t-t^2, 0, -3+2t^2)}{t^2} = \left(\frac{1}{t}, \frac{1}{s^2}, \frac{1}{s^4} - \frac{3}{t^2} + 5\right),$$

and therefore,

$$z = \frac{1}{s^4} - \frac{3}{t^2} + 5 = y^2 - 3x^2 + 5.$$

Hence, the hyperbolical paraboloid $z = y^2 - 3x^2 + 5$ is a translational surface.

3. Matrix Representations of Translational Surfaces

In this paper, we will study translational surfaces using matrices. To do so, we will first introduce a few notations.

Definition 3.1. Given any two vectors $\mathbf{a} = [a_0, a_1, a_2, a_3]$ and $\mathbf{b} = [b_0, b_1, b_2, b_3]$, we define the following operations:

$$\mathbf{a}^\bullet = [a_0, -a_1, -a_2, -a_3], \ \mathbf{a} * \mathbf{b} = [a_0 b_0,\ a_0 b_1 + b_0 a_1,\ a_0 b_2 + b_0 a_2,\ a_0 b_3 + b_0 a_3].$$

We note that there exists an identity element $\mathbf{e} = [1, 0, 0, 0]$ as $\mathbf{e} * \mathbf{a} = [a_0, a_1, a_2, a_3] = \mathbf{a}$. We also note that $\mathbf{a} * \mathbf{a}^\bullet = [a_0, a_1, a_2, a_3] * [a_0, -a_1, -a_2, -a_3] = [a_0^2, 0, 0, 0] = a_0^2 \mathbf{e}$.

Remark 3.2. *Note that the operation $*$ is commutative and associative.*

Proof. To verify this, we check

$$\begin{aligned}\mathbf{a}*\mathbf{b} &= [a_0b_0,\, a_0b_1+b_0a_1,\, a_0b_2+b_0a_2,\, a_0b_3+b_0a_3]\\ &= [b_0a_0,\, b_0a_1+a_0b_1,\, b_0a_2+a_0b_2,\, b_0a_3+a_0b_3]\\ &= \mathbf{b}*\mathbf{a}.\end{aligned}$$

Moreover, let $\mathbf{c}=[c_0,c_1,c_2,c_3]$. Then

$$\begin{aligned}\mathbf{a}*(\mathbf{b}*\mathbf{c}) &= \mathbf{a}*[b_0c_0,\, b_0c_1+c_0b_1,\, b_0c_2+c_0b_2,\, b_0c_3+c_0b_3]\\ &= [a_0b_0c_0,\, a_0b_0c_1+a_0c_0b_1+a_1b_0c_0,\\ &\qquad a_0b_0c_2+a_0c_0b_2+a_2b_0c_0,\, a_0b_0c_3+a_0c_0b_3+a_3b_0c_0]\\ &= [(a_0b_0)c_0,\, (a_0b_0)c_1+(a_0b_1+a_1b_0)c_0,\\ &\qquad (a_0b_0)c_2+(a_0b_2+a_2b_0)c_0,\, (a_0b_0)c_3+(a_0b_3+a_3b_0)c_0]\\ &= [a_0b_0,\, a_0b_1+b_0a_1,\, a_0b_2+b_0a_2,\, a_0b_3+b_0a_3]*\mathbf{c}\\ &= (\mathbf{a}*\mathbf{b})*\mathbf{c}.\end{aligned}$$

□

To represent the translational surfaces via matrices, it is necessary to consider a homogeneous representation of the translational surface $\mathbf{h}(s,t)$ generated by the curves $\mathbf{f}(s)$ and $\mathbf{g}(t)$, where

$$\begin{aligned}\mathbf{f}(s) &= [f_0(s), f_1(s), f_2(s), f_3(s)] \in \mathbb{R}^4[s],\\ \mathbf{g}(t) &= [g_0(t), g_1(t), g_2(t), g_3(t)] \in \mathbb{R}^4[t],\\ \mathbf{h}(s;t) &= [h_0(s;t),\, h_1(s;t),\, h_2(s;t),\, h_3(s;t)]\\ &= [f_0(s)g_0(t),\, f_0(s)g_1(t)+f_1(s)g_0(t),\\ &\qquad f_0(s)g_2(t)+f_2(s)g_0(t),\, f_0(s)g_3(t)+f_3(s)g_0(t)]\\ &= \mathbf{f}(s)*\mathbf{g}(t) \quad \text{by Definition 3.1.}\end{aligned}$$

Observe that the parametrization $\mathbf{h}(s;t)$ has matrix representations:

$$\begin{aligned}\mathbf{h} &= \begin{bmatrix} f_0g_0 & f_0g_1+f_1g_0 & f_0g_2+f_2g_0 & f_0g_2+f_3g_0\end{bmatrix} = \mathbf{f}*\mathbf{g}\\ &= \begin{bmatrix} f_0\\ f_1\\ f_2\\ f_3\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} g_0 & g_1 & g_2 & g_3\\ 0 & g_0 & 0 & 0\\ 0 & 0 & g_0 & 0\\ 0 & 0 & 0 & g_0\end{bmatrix} = \mathbf{f}M_{\mathbf{g}}, \text{ where } M_{\mathbf{g}} = \begin{bmatrix} g_0 & g_1 & g_2 & g_3\\ 0 & g_0 & 0 & 0\\ 0 & 0 & g_0 & 0\\ 0 & 0 & 0 & g_0\end{bmatrix},\end{aligned}$$

or

$$= \begin{bmatrix} g_0\\ g_1\\ g_2\\ g_3\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} f_0 & f_1 & f_2 & f_3\\ 0 & f_0 & 0 & 0\\ 0 & 0 & f_0 & 0\\ 0 & 0 & 0 & f_0\end{bmatrix} = \mathbf{g}M_{\mathbf{f}}, \text{ where } M_{\mathbf{f}} = \begin{bmatrix} f_0 & f_1 & f_2 & f_3\\ 0 & f_0 & 0 & 0\\ 0 & 0 & f_0 & 0\\ 0 & 0 & 0 & f_0\end{bmatrix}.$$

Let

$$N_{\mathbf{g}} = \begin{bmatrix} g_0 & -g_1 & -g_2 & -g_3 \\ 0 & g_0 & 0 & 0 \\ 0 & 0 & g_0 & 0 \\ 0 & 0 & 0 & g_0 \end{bmatrix} \qquad \text{and} \qquad N_{\mathbf{f}} = \begin{bmatrix} f_0 & -f_1 & -f_2 & -f_3 \\ 0 & f_0 & 0 & 0 \\ 0 & 0 & f_0 & 0 \\ 0 & 0 & 0 & f_0 \end{bmatrix}.$$

Then $M_{\mathbf{g}}N_{\mathbf{g}} = N_{\mathbf{g}}M_{\mathbf{g}} = g_0^2\,\mathbf{I}$ and $M_{\mathbf{f}}N_{\mathbf{f}} = N_{\mathbf{f}}M_{\mathbf{f}} = f_0^2\,\mathbf{I}$, where $\mathbf{I}$ is the identity matrix. In the next section, we will determine whether a given rational parameterized surface is a translational surface via matrices.

4. When is a Rational Surface a Translational Surface?

In this section, we shall give necessary and sufficient conditions for a real rational surface to be a translational surface and follow up with some examples.

Theorem 4.1. *Given a parametric representation of a rational surface:*

$$\mathbf{H}(s;t) = [H_0(s;t), H_1(s;t), H_2(s;t), H_3(s;t)] \in \mathbb{R}^4[s;t].$$

Suppose $H_0(s;t) \neq 0$, choose any s_0 for which $H_0(s_0;t) \neq 0$ and $\gcd(\mathbf{H}(s_0;t)) = 1$. *Set*

$$\hat{\mathbf{H}}(s;t) = \frac{\mathbf{H}(s;t) * \mathbf{H}^{\bullet}(s_0;t)}{H_0(s_0;t)^2}.$$

Then $\mathbf{H}(s;t)$ is a translational surface if and only if $\hat{\mathbf{H}}(s;t) \in \mathbb{R}^4[s]$ and $\gcd(\hat{\mathbf{H}}(s;t)) = 1$.

Proof. ($\Rightarrow$) If $\mathbf{H}(s;t)$ is a translational surface, then there exist two rational space curves $\mathbf{f}(s)$ and $\mathbf{g}(t)$ such that $\mathbf{H}(s;t) = \mathbf{f}(s) * \mathbf{g}(t)$, and $\gcd(\mathbf{f}(s)) = \gcd(\mathbf{g}(t)) = 1$. Therefore, $H_0(s_0;t) = f_0(s_0)g_0(t) \neq 0$ implies that $f_0(s_0) \neq 0$, $g_0(t) \neq 0$, and

$$\begin{aligned} \hat{\mathbf{H}}(s;t) &= \frac{\mathbf{H}(s;t) * \mathbf{H}^{\bullet}(s_0;t)}{H_0(s_0;t)^2} = \frac{\mathbf{H}(s;t) * \mathbf{H}^{\bullet}(s_0;t)}{(f_0(s_0)g_0(t))^2} \\ &= \frac{\mathbf{f}(s) * \mathbf{g}(t) * \mathbf{H}^{\bullet}(s_0;t)}{(f_0(s_0)g_0(t))^2}. \end{aligned}$$

Note that

$$\begin{aligned}
&\mathbf{g}(t) * \mathbf{H}^{\bullet}(s_0;t)\\
&= \begin{bmatrix} g_0(t) \\ g_1(t) \\ g_2(t) \\ g_3(t) \end{bmatrix} * \begin{bmatrix} H_0(s_0;t) \\ -H_1(s_0;t) \\ -H_2(s_0;t) \\ -H_3(s_0;t) \end{bmatrix} = \begin{bmatrix} g_0(t)H_0(s_0;t) \\ -g_0(t)H_1(s_0;t) + g_1(t)H_0(s_0;t) \\ -g_0(t)H_2(s_0;t) + g_2(t)H_0(s_0;t) \\ -g_0(t)H_3(s_0;t) + g_3(t)H_0(s_0;t) \end{bmatrix}\\
&= \begin{bmatrix} f_0(s_0)(g_0(t))^2 \\ -f_0(s_0)g_0(t)g_1(t) - f_1(s_0)(g_0(t))^2 + f_0(s_0)g_0(t)g_1(t) \\ -f_0(s_0)g_0(t)g_2(t) - f_2(s_0)(g_0(t))^2 + f_0(s_0)g_0(t)g_2(t) \\ -f_0(s_0)g_0(t)g_3(t) - f_3(s_0)(g_0(t))^2 + f_0(s_0)g_0(t)g_3(t) \end{bmatrix}\\
&= \begin{bmatrix} f_0(s_0)(g_0(t))^2 \\ -f_1(s_0)(g_0(t))^2 \\ -f_2(s_0)(g_0(t))^2 \\ -f_3(s_0)(g_0(t))^2 \end{bmatrix} = (g_0(t))^2\mathbf{f}^{\bullet}(s_0).
\end{aligned}$$

This implies that

$$\begin{aligned}
\frac{\mathbf{f}(s) * \mathbf{g}(t) * \mathbf{H}^{\bullet}(s_0;t)}{(f_0(s_0)g_0(t))^2} = \frac{\mathbf{f}(s) * (g_0(t))^2\mathbf{f}^{\bullet}(s_0)}{(f_0(s_0)g_0(t))^2} = \frac{\mathbf{f}(s) * \mathbf{f}^{\bullet}(s_0)}{(f_0(s_0))^2}\\
= \hat{\mathbf{H}}(s) \in \mathbb{R}^4[s].
\end{aligned}$$

Therefore,

$$\hat{\mathbf{H}}(s;t) = \frac{\mathbf{H}(s;t) * \mathbf{H}^{\bullet}(s_0;t)}{H_0(s_0;t)^2} = \frac{\mathbf{f}(s) * \mathbf{f}^{\bullet}(s_0)}{(f_0(s_0))^2} = \hat{\mathbf{H}}(s) \in \mathbb{R}^4[s].$$

Moreover, since

$$\mathbf{f}(s) * \mathbf{f}^{\bullet}(s_0) = \begin{bmatrix} f_0(s)f_0(s_0) \\ f_1(s)f_0(s_0) - f_0(s)f_1(s_0) \\ f_2(s)f_0(s_0) - f_0(s)f_2(s_0) \\ f_3(s)f_0(s_0) - f_0(s)f_3(s_0) \end{bmatrix},$$

the entries of $\mathbf{f}(s) * \mathbf{f}^{\bullet}(s_0)$ are linear combinations of the entries of $\mathbf{f}(s)$. We must have that

$$\gcd(\hat{\mathbf{H}}(s)) = \gcd(\mathbf{f}(s) * \mathbf{f}^{\bullet}(s_0)) = \gcd(\mathbf{f}(s)) = 1.$$

Thus, we have shown that if $\mathbf{H}(s;t)$ is a translational surface, then $\hat{\mathbf{H}}(s;t) \in \mathbb{R}^4[s]$ and $\gcd(\hat{\mathbf{H}}(s;t)) = 1$.

($\Leftarrow$) If $\hat{\mathbf{H}}(s;t) = \dfrac{\mathbf{H}(s;t) * \mathbf{H}^{\bullet}(s_0;t)}{H_0(s_0;t)^2} \in \mathbb{R}^4[s]$, then let $\mathbf{f}(s) = \hat{\mathbf{H}}(s)$, and $\mathbf{g}(t) = \mathbf{H}(s_0;t)$. By assumption, $\gcd(\mathbf{f}(s)) = \gcd(\hat{\mathbf{H}}(s)) = 1$, and

$\gcd(\mathbf{g}(t)) = \gcd(\mathbf{H}(s_0;t)) = 1$. Furthermore,

$$\begin{aligned}\mathbf{f}(s) * \mathbf{g}(t) &= \hat{\mathbf{H}}(s;t) * \mathbf{H}(s_0;t) = \frac{\mathbf{H}(s;t) * \mathbf{H}^{\bullet}(s_0;t)}{H_0(s_0;t)^2} * \mathbf{H}(s_0;t)\\ &= \frac{\mathbf{H}(s;t) * (\mathbf{H}^{\bullet}(s_0;t) * \mathbf{H}(s_0;t))}{H_0(s_0;t)^2}\\ &= \frac{\mathbf{H}(s;t)H_0(s_0;t)^2}{H_0(s_0;t)^2} \text{ (by Definition 3.1)}\\ &= \mathbf{H}(s;t).\end{aligned}$$

Thus, we have shown that if $\hat{\mathbf{H}}(s;t) \in \mathbb{R}^4[s]$ and $\gcd(\hat{\mathbf{H}}(s;t)) = 1$, then the rational surface $\mathbf{H}(s;t)$ is a translational rational surface. □

We will provide two examples to illustrate our theorem.

Example 4.2. *Consider the rational surface*

$$\mathbf{H}(s;t) = [s^2t^3, s^2t + t^3s, st^3 + t^3 + s^2t^2, s^2 + t^3].$$

Since $\mathbf{H}(1;t) = (t^3, t + t^3, 2t^3 + t^2, 1 + t^3)$, $H_0(1;t) = t^3$, $\gcd(\mathbf{H}(1;t)) = 1$,

$$\hat{\mathbf{H}}(s;t) = \frac{\mathbf{H}(s;t) * \mathbf{H}^{\bullet}(s_0;t)}{H_0(s_0;t)^2} = (s^2, -s^2 + s, -2s^2 + s + 1, -s^2 + 1) \in \mathbb{R}^4[s],$$

and $\gcd(\hat{\mathbf{H}}(s;t)) = 1$*, this rational surface* $\mathbf{H}$ *is a translational surface given by*

$$\begin{aligned}\mathbf{H}(s;t) &= \mathbf{f}(s) * \mathbf{g}(t), \quad \textit{where}\\ \mathbf{f}(s) &= \hat{\mathbf{H}}(s;t) = [s^2, -s^2 + s, -2s^2 + s + 1, -s^2 + 1],\\ \mathbf{g}(t) &= \mathbf{H}(1;t) = [t^3, t + t^3, 2t^3 + t^2, 1 + t^3].\end{aligned}$$

Remark 4.3. *As shown in the above example, if a rational surface is a translational surface, then* $\hat{\mathbf{H}}(s;t) \in \mathbb{R}^4[s]$ *and* $\mathbf{H}(s_0;t) \in \mathbb{R}^4[t]$ *are the generating curves of this translational surface.*

Example 4.4. *Consider the rational surface*

$$\begin{aligned}\mathbf{H}(s;t) = [&s^3t^3 - s^2t^2 - st - 1,\ s^2t^3 + s^3t^2 - t + s,\\ &st^3 + t^2 + s^3t - s^2,\ t^3 - st^2 + s^2t + s^3].\end{aligned}$$

Since $\mathbf{H}(0;t) = (-1, -t, t^2, t^3)$, $H_0(1;t) = -1$, $\gcd(\hat{\mathbf{H}}(0;t)) = 1$, *and*

$$\begin{aligned}
\hat{\mathbf{H}}(s;t) &= \frac{\mathbf{H}(s;t) * \mathbf{H}^{\bullet}(s_0;t)}{H_0(s_0;t)^2} = \mathbf{H}(s;t) * \mathbf{H}^{\bullet}(s_0;t) \\
&= \begin{bmatrix} s^3t^3 - s^2t^2 - st - 1 \\ s^2t^3 + s^3t^2 - t + s \\ st^3 + t^2 + s^3t - s^2 \\ t^3 - st^2 + s^2t + s^3 \end{bmatrix} * \begin{bmatrix} -1 \\ -t \\ t^2 \\ t^3 \end{bmatrix} \\
&= \begin{bmatrix} -s^3t^3 + s^2t^2 + st + 1 \\ s^3t^4 - s^3t^2 - 2s^2t^3 - st^2 - s \\ -s^3t^5 + s^2t^4 - s^3t + s^2 \\ -s^3t^6 + s^2t^5 + st^4 - s^3 - s^2t + st^2 \end{bmatrix} \notin \mathbb{R}^4[s],
\end{aligned}$$

this rational surface $\mathbf{H}$ *is not a translational surface.*

5. Conclusion

In this paper, we have given a matrix representation for a translational surface. Utilizing this representation and simple matrix operations, we provided necessary and sufficient conditions to identify translational surfaces. For future study, we would want to investigate if the set of translational surfaces form some sort of algebraic structure with this matrix representation. Also, while we represent the surfaces as matrices, we can also look to identify operations which will preserve the property of being a translational surface.

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