ON NANO RESOLVABLE SPACES

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Abstract. This paper introduces nano resolvable spaces and nano irresolvable spaces. Also, a new form of nano subspace topology is established. Several new characterizations of nano strongly irresolvable spaces are found and precise relationships are noted between nano strongly irresolvability and nano irresolvable space. Some weaker forms related to nano irresolvable are discussed. Also, comparisons between them are given.

1. Introduction

In topology, a subset A of a topological space X is said to be dense if the closure of A in X is given in [2]. A topological space is said to be resolvable if it is expressible as the union of two disjoint dense sets. A topological space that is not resolvable is termed as irresolvable. Ganster [5] discussed preopen sets and resolvable spaces. A space is hereditarily irresolvable if every subspace is irresolvable and a space is strongly irresolvable if every open subspace is irresolvable and the relationship among irresolvable and strongly irresolvable spaces are given in [11]. A new topology called a nano topology was introduced by Lellis Thivagar and Carmel Richard [12]. The topology introduced here is named nano topology because of its size, since it has at most five elements. Let $(\mathcal{U}, \tau_R(X))$ and $(\mathcal{V}, \tau_{R_1}(Y))$ be two nano topological spaces. Then a mapping $f : \mathcal{U} \longrightarrow \mathcal{V}$ is said to be nano open mapping if $f(G)$ is $\tau_{R_1}(Y)$-open for every $\tau_R(X)$ open set G. In this paper we focus on nano resolvable and nano irresolvable spaces. A brief study of nano strongly irresolvable space is also given. We present and study comparisons between nano irresolvable and nano strongly irresolvable spaces.

2. Preliminaries

In this section we recall some basic definitions and theorems.

Definition 2.1. [12] *Let $\mathcal{U}$ be a nonempty finite set of objects called the universe and R be an equivalence relation on $\mathcal{U}$ named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\mathcal{U},\ R)$ is said to be the approximation space. Let $X \subseteq \mathcal{U}$.*

(i) *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in\mathcal{U}}\{x : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.*
(ii) *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X) = \bigcup_{x\in\mathcal{U}}\{x : R(x) \cap X \neq \emptyset\}$.*
(iii) *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.*

Definition 2.2. [12] *Let $\mathcal{U}$ be an universe, R be an equivalence relation on $\mathcal{U}$ and $\tau_R(X) = \{\mathcal{U},\ \emptyset,\ L_R(X),\ U_R(X),\ B_R(X)\}$ where $X \subseteq \mathcal{U}$. $\tau_R(X)$ satisfies the following axioms:*

(i) *$\mathcal{U}$ and $\emptyset \in \tau_R(X)$.*
(ii) *The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*
(iii) *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*

That is, $\tau_R(X)$ forms a topology on $\mathcal{U}$ called the nano topology on $\mathcal{U}$ with respect to X. We say $(\mathcal{U},\ \tau_R(X))$ is the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets.

Proposition 2.3. [12] *Let $\mathcal{U}$ be a nonempty finite universe and $X \subset \mathcal{U}$, and $\mathcal{U}/R$ be an indiscernibility relation on $\mathcal{U}$. Then*

(i) ***Nano Type-1*** *($\mathcal{NT}_1$): If $L_R(X) = U_R(X) = X$, then the nano topology, $\tau_R(X) = \{\mathcal{U},\ \emptyset,\ L_R(X)\}$.*
(ii) ***Nano Type-2*** *($\mathcal{NT}_2$): If $L_R(X) = \emptyset$ and $U_R(X) \neq \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U},\ \emptyset,\ U_R(X)\}$.*
(iii) ***Nano Type-3*** *($\mathcal{NT}_3$): If $L_R(X) \neq \emptyset$ and $U_R(X) = \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U},\ \emptyset,\ L_R(X),\ B_R(X)\}$.*
(iv) ***Nano Type-4*** *($\mathcal{NT}_4$): If $L_R(X) = \emptyset$ and $U_R(X) = \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U},\ \emptyset\}$, is the indiscrete nano topology on U.*
(v) ***Nano Type-5*** *($\mathcal{NT}_5$): If $L_R(X) \neq U_R(X)$ where $L_R(X) \neq \emptyset$ and $U_R(X) \neq \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U},\ \emptyset,\ L_R(X),\ U_R(X),\ B_R(X)\}$.*

Theorem 2.4. [12] *Let $\mathcal{U}$ be a non-empty, finite universe and $X \subseteq \mathcal{U}$. Let $\tau_R(X)$ be the nano topology and $\mathcal{U}$ with respect to X. Then $[\tau_R(X)]^c$ whose elements are A^c for $A \in \tau_R(X)$, is a nano topology on $\mathcal{U}$.*

Remark 2.5. [12] *$[\tau_R(X)]^c$ is called the dual nano topology of $\tau_R(X)$. Elements of $[\tau_R(X)]^c$ are called nano closed sets. Thus, from the above theorem, we note that a subset A of $\mathcal{U}$ is nano closed in $\tau_R(X)$ if and only if $\mathcal{U} - A$ is nano open in $\tau_R(X)$.*

Definition 2.6. [12] *If $(\mathcal{U},\ \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq \mathcal{U}$ and if $A \subseteq \mathcal{U}$, then the nano interior of A is defined as the union of all nano open subsets of A and it is denoted by $\mathcal{N}int(A)$. That is, $\mathcal{N}int(A)$ is the largest nano open subset contained in A. The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by $\mathcal{N}cl(A)$. That is, $\mathcal{N}cl(A)$ is the smallest nano closed set containing A.*

Definition 2.7. [12] *A subset A of a nano topological space $(\mathcal{U},\ \tau_R(X))$ is called nano dense if $\mathcal{N}cl(A) = \mathcal{U}$.*

Definition 2.8. [12] *Let $(\mathcal{U},\ \tau_R(X))$ be a nano topological space and $A \subseteq \mathcal{U}$. Then A is said to be*

(i) *nano semiopen if $A \subseteq \mathcal{N}cl(\mathcal{N}int(A))$.*
(ii) *nano pre-open if $A \subseteq \mathcal{N}int(\mathcal{N}cl(A))$.*
(iii) *nano regular open if $A = \mathcal{N}int(\mathcal{N}cl(A))$.*
(iv) *nano alpha-open if $A \subseteq \mathcal{N}int(\mathcal{N}cl(\mathcal{N}int(A)))$.*

NSO$(\mathcal{U}, X)$, NPO$(\mathcal{U}, X)$, NRO$(\mathcal{U}, X)$, NαO$(\mathcal{U}, X)$ or $\tau_R(\alpha)$, respectively, denote the families of all nano semiopen, nano preopen, nano regular open, nano alpha open subsets of $\mathcal{U}$.

Definition 2.9. [2] *Let $T = (S,\ \tau)$ be a topological space. Let $H \subseteq S$ be a non-empty subset of S. Define $\tau_H = \{U \cap H : U \in \tau \subseteq P(H)\}$. Then the topological space $T_H = (H,\ \tau_H)$ is called a (topological) subspace of T. The set τ_H is referred to as the subspace topology on H.*

Definition 2.10. [12] *Let $(\mathcal{U},\ \tau_R(X))$ and $(\mathcal{V},\ \tau_{R_1}(Y)$ be two nano topological spaces. Then a mapping $f : \mathcal{U} \longrightarrow \mathcal{V}$ is said to be nano continuous on $\mathcal{U}$ if the inverse image of every nano open set in $\mathcal{V}$ is nano open in $\mathcal{U}$.*

Definition 2.11. [12] *Let $(\mathcal{U},\ \tau_R(X))$ and $(\mathcal{V},\ \tau_{R_1}(Y)$ be two nano topological spaces. Then a mapping $f : \mathcal{U} \longrightarrow \mathcal{V}$ is said to be nano homeomorphism if*

(i) *f is bijective.*
(ii) *f is nano open and nano continuous.*

Definition 2.12. [12] *A property held by nano topological space* $(\mathcal{U},\ \tau_R(X))$ *is said to be nano topological property if each homeomorphic image of* $\mathcal{U}$ *also possesses that property. This property is also called nano topological invariant.*

Definition 2.13. [7] *A function* $f\colon (X,\ \tau) \longrightarrow (Y,\ \sigma)$ *is called somewhat open if for each nonempty open set* $U \in X$*,* $int f(U) \neq \emptyset$*.*

Definition 2.14. [4] *A function* $f\colon (X,\ \tau) \longrightarrow (Y,\ \sigma)$ *is said to be weakly somewhat open if for each dense set* $D \in (Y,\ \sigma)$ *with* $intD \neq \emptyset$*,* $f^{-1}(D)$ *is dense in* $(X,\ \tau)$*.*

3. Nano Relative Topology

In this section Nano relative topology is introduced and their properties are discussed.

Definition 3.1. *Let* $(\mathcal{U},\ \tau_R(X))$ *be a nano topological space on* $\mathcal{U}$ *with respect to* X *and a subset* $\mathcal{Y}$ *in* $\mathcal{U}$ *such that* $X \subseteq \mathcal{Y} \subseteq \mathcal{U}$ *and* $(\mathcal{Y}/R) \subseteq \mathcal{U}/R$*. Then the relative nano topology on* $\mathcal{Y}$ *is defined by* $\tau_R^*(X) = \{\mathcal{U},\ \emptyset,\ L_R^*(X),\ U_R^*(X),\ B_R^*(X)\}$*, where,* $L_R^*(X) = \{L_R(X) \cap \mathcal{Y}/L_R(X) \cap \mathcal{Y} \subseteq X,\ L_R(X) \in \tau_R(X)\}$*.*
$U_R^*(X) = \{\ U_R(X) \cap \mathcal{Y}/U_R(X) \cap \mathcal{Y} \neq \emptyset,\ U_R(X) \in \tau_R(X)\}$*.*
$B_R^*(X) = U_R^*(X)$ - $L_R^*(X)$*.*

Example 3.2. *Let* $\mathcal{U} = \{a, b, c, d\}$ *with* $\mathcal{U}/R = \{\{a\},\ \{b, c\},\ \{d\}\}$ *and* $X = \{a,\ b\}$*. Then* $\tau_R(X) = \{\mathcal{U},\ \emptyset,\ \{a\},\ \{a, b, c\},\ \{b, c\}\}$*. Let* $\mathcal{Y} = \{a,\ b,\ c\}, \mathcal{Y}/R = \{\{a\},\ \{b, c\}\}$*, Now,* $\mathcal{U} \cap \mathcal{Y} = \mathcal{Y},\ \emptyset \cap \mathcal{Y} = \emptyset, \{a\} \cap \mathcal{Y} = \{a\}, \{b, c\} \cap \mathcal{Y} = \{b, c\}$*. Therefore,* $\tau_R^*(X) = \{\mathcal{Y},\ \emptyset,\ \{a\}, \{b, c\}\}$*.*

Definition 3.3. *The nano topological space* $(\mathcal{Y},\ \tau_R^*(X))$ *is called nano subspace of* $(\mathcal{U},\ \tau_R(X))$ *if* $X \subseteq \mathcal{Y} \subseteq \mathcal{U}$ *and* $(\mathcal{Y}/R) \subseteq \mathcal{U}/R$*, where* $\tau_R^*(X)$ *is a relative nano topology on a subset* $\mathcal{Y}$ *of* X*.*

Theorem 3.4. *Let* $(\mathcal{U},\ \tau_R(X))$ *is a nano topological space and* $X \subseteq \mathcal{Y} \subseteq \mathcal{U}$*,* $(\mathcal{Y}/R) \subseteq \mathcal{U}/R$ *and* $\tau_R^*(X) = \{\mathcal{U},\ \emptyset,\ L_R^*(X),\ U_R^*(X),\ B_R^*(X)\}$*, then* $\tau_R^*(X)$ *is a nano topology on* $\mathcal{Y}$*.*

Proof. The relative nano topology for $\mathcal{Y}$ with respect to X is given by $\tau_R^*(X) = \{\mathcal{U},\ \emptyset,\ L_R^*(X),\ U_R^*(X),\ B_R^*(X)\}$.

(i) Both $\mathcal{Y}$ and $\emptyset$ belong to $\tau_R^*(X)$. Now, $X \subseteq \mathcal{Y} \subseteq \mathcal{U}$ and $(\mathcal{Y}/R) \subseteq \mathcal{U}/R \Longrightarrow \mathcal{U} \cap \mathcal{Y} = \mathcal{Y}$. Therefore, this relation shows that $\mathcal{Y} \in \tau_R^*(X)$ by definition of $\tau_R^*(X)$, where $\mathcal{U}$ is $\tau_R(X)$-open. Also, $\emptyset \cap \mathcal{Y} = \emptyset$. Hence, $\emptyset$ belongs to $\tau_R^*(X)$.

(ii) $\tau_R^*(X)$ is closed for finite intersection. Now, $X \subseteq \mathcal{Y} \subseteq \mathcal{U}$ and $(\mathcal{Y}/R) \subseteq \mathcal{U}/R$ so $L_R(X) = L_R^*(X)$ and $U_R(X) = U_R^*(X)$, $B_R(X) =$

$B^*_R(X)$. Consider $(L_R(X)\cap\mathcal{Y})\cap(U_R(X)\cap\mathcal{Y}) = (L_R(X)\cap U_R(X))\cap \mathcal{Y} = (L_R(X)\cap\mathcal{Y}) \in \tau^*_R(X)$. Now, $(L_R(X)\cap\mathcal{Y})\cap(B_R(X)\cap\mathcal{Y}) = (L_R(X)\cap B_R(X))\cap\mathcal{Y} = (\emptyset\cap\mathcal{Y}) \in \tau^*_R(X)$. $(U_R(X)\cap\mathcal{Y})\cap(B_R(X)\cap \mathcal{Y}) = (U_R(X)\cap B_R(X))\cap\mathcal{Y} = (B_R(X)\cap\mathcal{Y}) \in \tau^*_R(X)$.

(iii) $\tau^*_R(X)$ is closed under arbitrary union. Now, $X \subseteq \mathcal{Y} \subseteq \mathcal{U}$ and $(\mathcal{Y}/R) \subseteq \mathcal{U}/R$ so $L_R(X) = L^*_R(X)$ and $U_R(X) = U^*_R(X)$, $B_R(X) = B^*_R(X)$. Consider $(L_R(X)\cap\mathcal{Y})\cup(U_R(X)\cap\mathcal{Y})\cup(B_R(X)\cap\mathcal{Y}) = L_R(X)\cup U_R(X)\cup B_R(X))\cap\mathcal{Y} = (\mathcal{U}\cap\mathcal{Y})$ belongs to $\tau^*_R(X)$. $(L_R(X)\cap \mathcal{Y})\cup(U_R(X)\cap\mathcal{Y}) = (L_R(X)\cup U_R(X))\cap\mathcal{Y} = U_R(X)\cap\mathcal{Y}$ belongs to $\tau^*_R(X)$, $(U_R(X)\cap\mathcal{Y})\cup(B_R(X)\cap\mathcal{Y}) = (U_R(X)\cup B_R(X))\cap\mathcal{Y} = (U_R(X)\cap\mathcal{Y})$ belongs to $\tau^*_R(X)$, $(L_R(X)\cup B_R(X))\cap\mathcal{Y} = (U_R(X)\cap\mathcal{Y})$ belongs to $\tau^*_R(X)$.

□

Definition 3.5. *A property held by a nano topological space $(\mathcal{U},\ \tau_R(X))$ and also held by every nano subspace is called nano hereditary property.*

Theorem 3.6. *Let $(\mathcal{Y},\ \tau^*_R(X))$ be nano subspace of nano topological space $(\mathcal{U},\ \tau_R(X))$ and A is a nano subset of $\mathcal{Y}$ then $\tau_R(X) - int(A) = \tau^*_R(X) - int(A)$.*

Proof. Let x be a $\tau_R \in (X) - int(A)$ implies x belongs to largest nano open set contained in A. If $x \in \mathcal{N}int(A) = L_R(X)$ implies $x \in L_R(X)$. Since $L_R(X) = L^*_R(X)$, so $x \in L^*_R(X)$ implies $x \in \tau^*_R(X) - int(A)$. Now if $x \in \mathcal{N}int(A) = U_R(X)$ implies $x \in U_R(X)$. Since $U_R(X) = U^*_R(X)$, so $x \in U^*_R(X)$ implies $x \in \tau^*_R(X) - int(A)$. If $x \in \mathcal{N}int(A) = B_R(X)$ implies $x \in B_R(X)$. Since $B_R(X) = B^*_R(X)$, so $x \in B^*_R(X)$ implies $x \in \tau^*_R(X) - int(A)$. Hence, $\tau_R(X) - int(A) = \tau^*_R(X) - int(A)$. □

4. Nano Resolvable Spaces and Nano Irresolvable Spaces

In this section nano resolvable and nano irresolvable are introduced. Some of its properties are discussed.

Definition 4.1. *A nano topological space $(\mathcal{U},\ \tau_R(X))$ is called nano resolvable if there is a subset A of $(\mathcal{U},\ \tau_R(X))$ such that both A and its complement $(\mathcal{U} - A)$ are nano dense in $\mathcal{U}$.*

Example 4.2. *Let $\mathcal{U} = \{a,b,c,d\}$, $\mathcal{U}/R = \{\{a\},\ \{b,c\},\ \{d\}\}$. The family of equivalence classes of $\mathcal{U}$ by an equivalence relation R and $X = \{a,b,c\}$. Then $U_R(X) = \{a,b,c\}$, $L_R(X) = \{a,b,c\}$ and $B_R(X) = \emptyset$. Therefore, the nano topology, $\tau_R(X) = \{\mathcal{U},\ \emptyset,\ \{a,b,c\}\}$ and nano dense are $\{\mathcal{U},\{a\},\ \{b\},\ \{c\},\ \{a,b\},\ \{a,c\},\ \{a,d\},\ \{b,c\},\ \{b,d\},\ \{c,d\},\ \{a,b,c\},\ \{a,b,d\},\ \{b,c,d\},\ \{a,c,d\}\}$. Here $\{a\}$ and $\{b,c,d\}$ are both nano dense. Hence, $\mathcal{U}$ is Nano resolvable.*

Definition 4.3. *A nano topological space $(\mathcal{U},\ \tau_R(X))$ is called nano resolvable whenever it has two disjoint nano dense subsets and nano irresolvable otherwise.*

Definition 4.4. *A nano topological space $(\mathcal{U},\ \tau_R(X))$ is called nano k resolvable, where k is cardinal, if $\mathcal{U}$ contains k many disjoint nano dense subsets.*

Definition 4.5. *A space $\mathcal{U}$ is nano maximally resolvable if it is $\mathcal{N}\Delta(\mathcal{U})$ resolvable, where $\mathcal{N}\Delta(\mathcal{U}) = \min\{|G| : G \text{ is nonempty nano open set}\}$ and the cardinal $\mathcal{N}\Delta(\mathcal{U})$ is called nano dispersion character of $\mathcal{U}$.*

Example 4.6. *Let $\mathcal{U} = \{a,b,c,d\}$ and $\mathcal{U}/R = \{\{a,b\},\ \{c,d\}\}$, $X = \{a,b\}$. Then $\tau_R(X) = \{\mathcal{U},\ \emptyset,\ \{a,b\}\}$. Here $\mathcal{N}\Delta(\mathcal{U}) = \min\{2,4\} = 2$ and the nano dispersion character is 2.*

Definition 4.7. *A subset A of X is nano resolvable if the subspace $(A,\ \tau_R(X)/A)$ is nano resolvable.*

Example 4.8. *Let $\mathcal{U} = \{a,b,c,d,e\}$ with $\mathcal{U}/\mathcal{R} = \{\{a\},\{b\},\{c,d\},\{e\}\}$. Let $X = \{a,b,c\}$. Then $\tau_R(X) = \{\mathcal{U},\ \emptyset,\ \{a,b\},\ \{a,b,c,d\},\ \{c,d\}\}$. When $\mathcal{Y} = \{a,b,c,d\}$ and $\mathcal{Y}/R = \{\{a\},\ \{b\},\ \{c,d\}\}$, $\tau_R^*(X) = \{\mathcal{Y},\ \emptyset,\ \{a,b\},\ \{c,d\}\}$ then $\mathcal{Y}$ is nano resolvable.*

Remark 4.9. *Nano resolvable space is not nano hereditary.*

Example 4.10. *Let $\mathcal{U} = \{a,b,c,d\}$, $X = \{a,b\}$ with $\mathcal{U}/R = \{\{a,b\},\{c\},\{d\}\}$. Then $\tau_R(X) = \{\mathcal{U},\ \emptyset,\ \{a,b\}\}$. If $Y = \{b,c\}$, then $\tau_R^*(X) = \{\mathcal{Y},\ \emptyset,\{b\}\}$ is not nano resolvable. Therefore, nano resolvable is not nano hereditary.*

Theorem 4.11. *For a nano topological space the following statements are equivalent:*

(i) *$(\mathcal{U},\ \tau_R(X))$ is nano resolvable.*
(ii) *$(\mathcal{U},\ \tau_R(X))$ has a pair of nano dense sets A and B such that $A \subseteq \mathcal{U}\backslash B$.*

Proof. (i) $\implies$ (ii) Suppose that $(\mathcal{U},\ \tau_R(X))$ is nano resolvable. There exists a nano dense set A in $(\mathcal{U},\ \tau_R(X))$ such that $\mathcal{U}\backslash A$ is nano dense in $(\mathcal{U},\ \tau_R(X))$. Set $B = \mathcal{U}\backslash A$, then we have $A = \mathcal{U}\backslash B$.
(ii) $\implies$ (i) Suppose that the statement (ii) holds. Let $(\mathcal{U},\ \tau_R(X))$ be nano irresolvable. Then $\mathcal{U}\backslash B$ is not nano dense and $\mathcal{N}cl(A) \subseteq \mathcal{N}cl(\mathcal{U}\backslash B) \neq \mathcal{U}$. Hence, A is not nano dense. This contradicts the assumption. □

Theorem 4.12. *If $(\mathcal{U},\ \tau_R(X))$ is $\mathcal{N}\mathcal{T}_2$ topological space then $\mathcal{U}$ is nano resolvable.*

Proof. Let $\mathcal{U}$ be $\mathcal{N}\mathcal{T}_2$, since it has only $U_R(X)$ and $\mathcal{U}$ and $\mathcal{N}\Delta(\mathcal{U}) \geq 2$. Hence, it is clear that $\mathcal{U}$ is nano resolvable. □

Definition 4.13. *A nano topological space $(\mathcal{U},\ \tau_R(X))$ is called nano irresolvable if any two nano dense sets in $(\mathcal{U},\ \tau_R(X))$ intersects.*

Example 4.14. *Let $\mathcal{U} = \{a,b,c,d,e\}$, $\mathcal{U}/R = \{\{a\},\{b,c\},\{d,e\}\}$, the family of equivalence classes of $\mathcal{U}$ by an equivalence relation R and $X = \{a,e\}$. Then $U_R(X) = \{a,d,e\}$, $L_R(X) = \{a\}$ and $B_R(X) = \{d,e\}$. Therefore, $\tau_R(X) = \{\mathcal{U},\ \emptyset,\ \{a\},\ \{a,d,e\},\ \{d,e\}\}$ and nano dense are $\{\mathcal{U},\ \{a,e\},\ \{a,b,d\},\ \{a,b,e\},\ \{a,b,c,d\},\ \{a,b,c,e\},\ \{a,c,d\},\ \{a,c,e\},\ \{a,d,e\},\ \{a,c,d,e\},\ \{a,b,d,e\}\}$. Here the nano dense sets intersects. Therefore, $\mathcal{U}$ is Nano irresolvable.*

Theorem 4.15. *Nano irresolvability is a nano topological property.*

Proof. Let $(\mathcal{U},\ \tau_R(X))$ and $(\mathcal{V},\ \tau_{R_1}(X))$ be two nano topological spaces and $f\colon \mathcal{U} \longrightarrow \mathcal{V}$ be a nano continuous function. In case $\mathcal{U}$ is nano irresolvable, then we have to prove that $\mathcal{V}$ is nano irresolvable. Let $\mathcal{V}$ is nano resolvable implies that A and B be two disjoint nano dense sets in $\mathcal{V}$. But, f is nano continuous. Therefore, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nano dense in $\mathcal{U}$. Also, f is an onto mapping and A and B be disjoint nano dense sets in $\mathcal{V}$. Therefore, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nano dense in $\mathcal{U}$ as shown above. Hence, $(\mathcal{U},\ \tau_R(X))$ is nano resolvable. Thus, $(\mathcal{V},\ \tau_{R_1}(X))$ is nano resolvable implies $(\mathcal{U},\ \tau_R(X))$ is nano resolvable or $(\mathcal{U},\ \tau_R(X))$ is nano irresolvable implies $(\mathcal{V},\ \tau_{R_1}(X))$ is nano irresolvable. Thus, we conclude that nano irresolvability is a nano topological property. □

5. Characterization

In this section characterization of nano irresolvable and nano strongly irresolvable spaces are discussed.

Definition 5.1. *A nano topological space $(\mathcal{U},\ \tau_R(X))$ is said to be nano quasi maximal space if for every nano dense set D in $(\mathcal{U},\ \tau_R(X))$ with $\mathcal{N}int(D) \neq \emptyset$, $\mathcal{N}int(D)$ is also nano dense in $(\mathcal{U},\ \tau_R(X))$.*

Example 5.2. *Let $\mathcal{U} = \{a,b,c\}$, $X = \{a,b\}$ and $\mathcal{U}/\mathcal{R} = \{\{a\},\ \{b,c\}\}$. Then, $\tau_R(X) = \{\mathcal{U},\ \emptyset,\ \{a\}\}$, Nano dense are $\{\mathcal{U},\ \{a\},\ \{a,b\},\ \{a,c\}\}$. Hence, $\mathcal{U}$ is nano quasi maximal.*

Theorem 5.3. *If $\tau_R(X) = \tau_R^{\alpha}(X)$, then $(\mathcal{U},\ \tau_R(X))$ is either nano irresolvable or nano resolvable or not a nano quasi maximal space.*

Proof. Case(i). If $\tau_R(X) = \tau_R^{\alpha}(X)$ and $\mathcal{N}\Delta(\mathcal{U}) = 1$. Then it is obvious that $\mathcal{U}$ is nano irresolvable by definition.
Case(ii). Suppose if $(\mathcal{U},\ \tau_R(X))$ is nano indiscrete and $\tau_R(X) = \tau_R^{\alpha}(X)$. Therefore, the complement of a nano dense set is also nano dense and hence the nano topological space is nano resolvable but not nano quasimaximal space. □

Definition 5.4. *A nano subset A of a space $(\mathcal{U},\ \tau_R(X))$ is nano faintly open if either $A=\emptyset$ or $\mathcal{N}int(A)\neq\emptyset$. The collection of all nano faint open sets are denoted by NFOS.*

Example 5.5. *Let $\mathcal{U}=\{a,b,c,d\}$, $\mathcal{U}/R=\{\{a\},\ \{b,c,d\}\}$ and $X=\{a,b\}$. Then, $\tau_R(X)=\{\mathcal{U},\ \emptyset,\ \{a\},\ \{b,c,d\}\}$. Then $NFOS=\{\mathcal{U},\ \emptyset,\ \{a\},\ \{a,b\},$ $\{a,c\},\ \{a,d\},\ \{a,b,c\},\ \{a,b,d\},\ \{b,c,d\},\ \{a,c,d\}\}$.*

Proposition 5.6. *A nano topological space $(\mathcal{U},\ \tau_R(X))$ is nano irresolvable if and only if every nano dense set is nano faintly open.*

Proof. A space is nano irresolvable if and only if no nano dense set is nano codense set. □

Definition 5.7. *A nano subset D is NPO-dense if every nonempty $A\in NPO(\mathcal{U},\ \tau_R(X))$ has $A\cap D\neq\emptyset$.*

Lemma 5.8. *A nano subset D is NPO-dense if and only if $D\cup\mathcal{N}cl(\mathcal{N}int(D))=\mathcal{U}$.*

Proof. It is known that the union of all nano preopen subsets of $\mathcal{U}-D$ is $(\mathcal{U}-D)\cap\mathcal{N}int(\mathcal{N}cl(\mathcal{U}-D))$. If D is NPO dense, $(\mathcal{U}-D)\cap\mathcal{N}int(\mathcal{N}cl(\mathcal{U}-D))=\emptyset$ so that $\mathcal{U}=D\cup\mathcal{N}cl(\mathcal{N}int(D))$. □

Proposition 5.9. *For any nano subset of a space $(\mathcal{U},\ \tau_R(X))$ the following are equivalent:*

(i) *$A\in NPO(\mathcal{U},\ \tau_R(X))$.*
(ii) *There is nano regular open set $B\subseteq\mathcal{U}$ such that $A\subseteq B$ and $\mathcal{N}cl(A)\subseteq\mathcal{N}cl(B)$.*
(iii) *A is the intersection of a nano regular open set and nano dense set.*
(iv) *A is the intersection of an nano open set and a nano dense set.*

Proof. (i) $\Longrightarrow$ (ii) Let $A\in NPO(\mathcal{U},\ \tau_R(X))$, that is $A\subseteq\mathcal{N}int(\mathcal{N}cl(A))$. Let $B=\mathcal{N}int(\mathcal{N}cl(A))$. Then B is nano regular open with $A\subseteq B$ and $\mathcal{N}cl(A)=\mathcal{N}cl(B)$.

(ii) $\Longrightarrow$ (iii) Suppose (ii) holds. Let $D=A\cup(\mathcal{U}\backslash A)$. Then D is nano dense and $A=B\cap D$.

(iii) $\Longrightarrow$ (iv) This is trivial.

(iv) $\Longrightarrow$ (i) Suppose $A=B\cap D$ where B is nano open and D is nano dense. Then $\mathcal{N}cl(A)=\mathcal{N}cl(B)$. Hence, $A\subseteq B\subseteq\mathcal{N}cl(B)=\mathcal{N}cl(A)$. □

Definition 5.10. *A space $(\mathcal{U},\ \tau_R(X))$ is nano strongly irresolvable if each nano open subspace is nano irresolvable.*

Example 5.11. *Let $\mathcal{U} = \{a, b, c\}$ with $\mathcal{U}/R = \{\{a\}, \{b, c\}\}$ and let $X = \{a\}$. Then $\tau_R(X) = \{\mathcal{U},\ \emptyset,\ \{a\}\}$. Here each nano open subspace is nano irresolvable.*

Theorem 5.12. *The following are equivalent for a space $(\mathcal{U},\ \tau_R(X))$.*

(i) $\mathcal{U}$ is nano strongly irresolvable.
(ii) $NPO(\mathcal{U},\ \tau_R(X)) \subseteq NSO(\mathcal{U},\ \tau_R(X))$.
(iii) $NPO(\mathcal{U},\ \tau_R(X)) \subseteq NFO(\mathcal{U},\ \tau_R(X))$.
(iv) Every nano dense set is NPO-dense.

Proof. (i) $\Longrightarrow$ (ii) If $(\mathcal{U},\ \tau_R(X))$ is nano strongly irresolvable and $A = B \cap D \in NPO(\mathcal{U},\ \tau_R(X))$ for some $B \in \tau_R(X)$ and nano dense D, then $\mathcal{N}int(D)$ is nano dense and $\mathcal{N}cl(\mathcal{U} \cap \mathcal{N}int(D)) = \mathcal{N}cl(B)$ so that $B \cap D \subseteq B \subseteq \mathcal{N}cl(\mathcal{N}int(B \cap \mathcal{N}int(D))) \subseteq \mathcal{N}cl(\mathcal{N}int(B \cap D))$. Thus, $A \in NSO(\mathcal{U},\ \tau_R(X))$.

(ii)$\Longrightarrow$(iii) This is clear since $NSO(\mathcal{U},\ \tau_R(X)) \subseteq NFO(\mathcal{U},\ \tau_R(X))$.

(iii)$\Longrightarrow$(iv) This is clear since every nonempty nano faintly open set has nonempty nano interior which must then intersect every nano dense set.

(iv)$\Longrightarrow$(i) If D is nano dense in $(\mathcal{U},\ \tau_R(X))$ then D is NPO-dense and by Lemma 5.8 $\mathcal{U} = D \cup \mathcal{N}cl(\mathcal{N}int(D))$. Then if $\mathcal{N}int(D)$ is not nano dense, $B = \mathcal{U} - \mathcal{N}cl(\mathcal{N}int(D)) \in \tau_R(X)$ is nonempty and $\mathcal{U} \subseteq \mathcal{N}int(D) - \mathcal{N}int(D) = \emptyset$. This contradiction shows that $\mathcal{N}int(D)$ is nano dense and hence, $(\mathcal{U},\ \tau_R(X))$ is nano strongly irresolvable. □

Theorem 5.13. *Every nano strongly irresolvable space is nano irresolvable.*

Proof. Let $A \subseteq \mathcal{U}$ be nano open since $\mathcal{U}$ is nano strongly irresolvable where $X \subseteq A$ and $X \subseteq \mathcal{U}$. Also, there exist a pair of nano dense sets intersects, it is true for each nano open sets. Hence, $\mathcal{U}$ is nano irresolvable. □

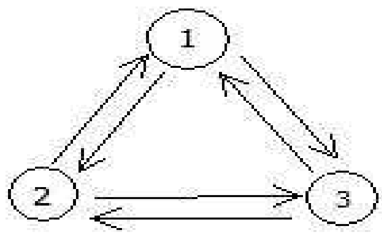
Remark 5.14. *Converse of the above, Theorem 5.13 is not true as shown in the following example.*

Example 5.15. *Let $\mathcal{U} = \{a, b, c\}$, $X = \{a, b\}$ and $\mathcal{U}/R = \{\{a\}, \{b, c\}\}$. Then $\tau_R(X) = \{\mathcal{U},\ \emptyset,\ \{a\},\ \{b, c\}\}$. Here $\mathcal{N}\Delta(\mathcal{U}) = 1$ and hence, it is nano irresolvable and since nano open subspace is not nano irresolvable the space is not nano strongly irresolvable.*

Theorem 5.16. *If $(\mathcal{U},\ \tau_R(X))$ is an $\mathcal{N}\mathcal{T}_1$ topological space and $\mathcal{N}\Delta(\mathcal{U}) = 1$, then $\mathcal{U}$ is nano irresolvable if and only if $\mathcal{U}$ is nano strongly irresolvable.*

Proof. If $\mathcal{U}$ is $\mathcal{NT}_1$, here $L_R(X) = U_R(X) = X$ since the nano dispersion character is 1 then $\mathcal{U}$ is nano irresolvable so $L_R(X)$ is nano irresolvable since complement of $L_R(X)$ is not nano dense, hence, $\mathcal{U}$ is nano strongly irresolvable. Conversely, $\mathcal{U}$ is nano strongly irresolvable that is every nano open subspace is nano irresolvable so each nano open set is a superset of X this is not possible so the only case is $\mathcal{NT}_1$. Since the nano dispersion character is 1 hence, $\mathcal{U}$ is nano irresolvable. □

Remark 5.17. *In general, nano irresolvable does not imply nano strongly irresolvable but if $(\mathcal{U},\ \tau_R(X))$ is $\mathcal{NT}_1$ topological space and $\mathcal{N}\Delta(\mathcal{U}) = 1$, then the following implication holds.*



1)Nano strongly irresolvable 2)Nano irresolvable 3)Nano quasi maximal.

6. Application

Here in this section we applied nano irresolvable spaces in nano weakly somewhat open function which gives an equivalent statement.

Definition 6.1. *A function $f\colon (\mathcal{U},\ \tau_R(X)) \longrightarrow (\mathcal{V},\ \tau_{R_1}(Y))$ is called nano somewhat open if for each nonempty nano open set $A \in \tau_R(X)$, where $X \subseteq \mathcal{U}$, $\mathcal{N}int(f(A)) \neq \emptyset$.*

Definition 6.2. *A function $f\colon (\mathcal{U},\ \tau_R(X)) \longrightarrow (\mathcal{V},\ \tau_{R_1}(Y))$ is said to be nano weakly somewhat open if for each nano dense $D \in (\mathcal{V},\ \tau_{R_1}(Y))$ with $\mathcal{N}intD \neq \emptyset, f^{-1}(D)$ is nano dense in $(\mathcal{U},\ \tau_R(X))$.*

Example 6.3. *Let $\mathcal{U} = \{a,b,c,d\}$, $X = \{a,b\}$, and $\mathcal{U}/R = \{\{a\},\{c\},\{b,d\}\}$. Then $\tau_R(X) = \{\mathcal{U},\ \emptyset,\ \{a\},\ \{a,b,d\},\ \{b,d\}\}$ and the following are nano dense: $\{\mathcal{U},\ \{a,b\},\{a,d\},\ \{a,b,c\},\ \{a,b,d\}\}$. Now $\mathcal{V} = \{a,b,c,d,e\}$, $Y = \{a,e\}$, $\mathcal{V}/R_1 = \{\{a\},\{d,e\},\{b,c\}\}, \sigma_{R_1}(Y) = \{\mathcal{V},\ \emptyset,\ \{a\},\{a,d,e\},\{d,e\}\}$. Nano dense $\mathcal{V}$ are $\{V,\{a,d\},\ \{a,e\},\ \{a,b,d\},\ \{a,b,e\},\ \{a,b,c,d\},\ \{a,b,c,e\},$ $\{a,c,d\},\ \{a,c,e\},\ \{a,d,e\},\ \{a,c,d,e\},\ \{a,b,d,e\}\}$. Define a function $f\colon \mathcal{U} \longrightarrow \mathcal{V}$ where $f(a) = a$, $f(b) = a$, $f(c) = c$, $f(d) = d$. Therefore, this function satisfies the condition of nano weakly somewhat open.*

Theorem 6.4. *The following statements are equivalent for a space $(\mathcal{V},\ \tau_{R_1}(Y))$:*

(i) $(\mathcal{V},\ \tau_{R_1}(Y))$ *is nano irresolvable.*

(ii) *For every nano topological space* $(\mathcal{U},\ \tau_R(X))$*, every nano weakly somewhat open function* $f\colon (\mathcal{U},\ \tau_R(X)) \longrightarrow (\mathcal{V},\ \tau_{R_1}(Y))$ *is nano somewhat open.*

Proof. (i) $\Longrightarrow$ (ii) It follows from the definition and it is obvious.
(ii) $\Longrightarrow$ (i) Let $D_1, D_2 \in \mathcal{V}$ since f is nano weakly somewhat open $\mathcal{N}intD_1 \neq \emptyset$ and $\mathcal{N}intD_2 \neq \emptyset$, $f^{-1}(D_1) \in D$ and $f^{-1}(D_2) \in D$ where D is in U. Therefore $f^{-1}(D_1) \cap f^{-1}(D_2) \in D$. $f^{-1}(D_1 \cap D_2) \in D$. Since f is nano somewhat open then $\mathcal{N}intf(f^{-1}(D_1 \cap D_2)) \neq \emptyset$. Now $\mathcal{N}intf(f^{-1}(D_1 \cap D_2)) \subseteq \mathcal{N}int(D_1 \cap D_2) \neq \emptyset$. Therefore, $\mathcal{N}int(D_1 \cap D_2) \neq \emptyset$. Hence, $D_1 \cap D_2 \neq \emptyset$. □

Conclusion

We conclude this paper by giving the table for hereditary property and topological property in nano resolvable and nano irresolvable space.

Table 1

Spaces	*Hereditary property*	*Topological property*
NanoResolvable	0	1
NanoIrresolvable	0	1

Here in the table 0 represents false and 1 represents true. Also we focused on topological results by combining nano irresolvable and nano strongly irresolvable and we hope the results given in this paper will further enrich nano topological space.

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