LOCAL SEPARATION AXIOMS BETWEEN KOLMOGOROV AND FRÉCHET SPACES

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Abstract. Several separation axioms on topological spaces are described between Kolmogorov and Fréchet spaces as properties of the space at a particular point. After describing various equivalent descriptions, implications are established. Various examples are studied in order to show that the implications are strict.

1. Introduction

Studying separation axioms is one of the powerful ideas in Topology. See [5] for example. The separation axioms of topological spaces are usually denoted with the letter T which stands for the German word *Trennung*, meaning separation. For example, a topological space is called a T_1 space if and only if for any two distinct points, each point has an open neighborhood not containing the other. Similarly, a topological space is called a T_0 space if and only if for any two distinct points, at least one of these points has an open neighborhood not containing the other. In his study of locally connected spaces, Young [8] introduced the T_Y spaces that lie between T_0 and T_1.

Aull and Thron [3] studied separation axioms T_{DD}, T_D, T_{UD} between T_0 and T_1 which were later used in study of Zariski topology defined on the set of prime ideals of a commutative ring with identity [1]. For a detailed overview of separation axioms in recent research, see [7] and the bibliography therein.

Recall that a topological property is called a local property if it can be specified to a single point in the topological space. Although separation axioms are (local properties) associated with the points of a topological space, they have been studied under the assumption that the specific separation axiom holds for all points of the topological space. One of the present authors has studied various local compactness properties as a local property at a specific point in [4]. We will study separation axioms between T_0 and T_1 specific to a single point.

In this paper, we assume that X denotes any topological space, $\bar{A}$ denotes the closure of A (i.e., the smallest closed set containing A), A' denotes the complement of A in X, and $D(A) = \bar{A} \cap A'$ for all $A \subseteq X$. If $A = \{x\}$, then we write $D(x)$ for $D(A)$ and $\bar{x}$ for $\bar{A}$. Recall that any set that is both closed and open is called *clopen*.
Let us use the terminology *clorpen* for brevity. Any set that is either closed or open is called **clorpen**.

2. Local Separation Axioms

Recall that a topological space is a T_1-space (or Fréchet Space) if and only if each singleton is closed. Restricting a T_1-space axiom to a single point, we have some equivalent conditions. Let us first formally define a T_1-space at a particular point.

Definition 1. Let X be a topological space and $a \in X$. X is called T_1 at a if and only if for all $x \in X$ such that $x \neq a$, there exist open sets G and H such that $x \in G$, $a \notin G$, $a \in H$, and $x \notin H$.

Result 1. Let X be a topological space and $a \in X$. The following are equivalent.

1. X is T_1 at a.
2. For all $x \in X$, $x \neq a$, $a \notin \bar{x}$ and $x \notin \bar{a}$.
3. For all $x \in X$, $x \neq a$, $\bar{x} \nsubseteq \bar{a}$ and $\bar{a} \nsubseteq \bar{x}$.
4. $\{a\}$ is closed and it is an intersection of open sets.

Proof. Let X be T_1 at a, $x \in X$, and $x \neq a$. There exists an open set G such that $x \in G$ and $a \notin G$ meaning $x \notin \bar{a}$ and there exists an open set H, $a \in H$ and $x \notin H$, meaning $a \notin \bar{x}$. Thus, (1) $\Rightarrow$ (2).

Clearly (2) $\Rightarrow$ (3).

Assume (3). Let $x \in \bar{a}$. Since $\bar{x} \subseteq \bar{a}$, $x = a$. Thus, $\bar{a} = \{a\}$. Since $a \notin \bar{x}$ for all $x \neq a$, $\{a\} = \cap_{x \neq a} \bar{x}'$. Thus, (3) $\Rightarrow$ (4).

Assume (4). Let $x \in X$ and $x \neq a$. Clearly, $\{a\}$ is closed and $x \in \{a\}'$. Since x is not in the intersection of all open sets containing a, there exists an open set H of a not containing x. Thus, X is T_1 at a. □

Thus, X is T_1 at a if and only if it is T_C at a and T_N at a, according to the following definitions.

Definition 2. Let X be a topological space and $a \in X$.

1. X is called T_C at a if and only if for all $x \in X$ such that $x \neq a$, there exists an open set G such that $x \in G$, $a \notin G$ (i.e., $\{a\}$ is closed).

2. X is called T_N at a if and only if for all $x \in X$ such that $x \neq a$, there exists an open set H such that $a \in H$ and $x \notin H$ (i.e., $\{a\}$ is an intersection of open sets in X).

   This lead to two more natural definitions.
3. X is called T_G at a if and only if $\{a\}$ is open (i.e., a is an isolated point of X).
4. X is called T_{CG} at a if and only if $\{a\}$ is clorpen.

The following example shows that, at a, T_G does not imply T_{CG} and T_N does not imply T_1.

Example 1. Let X be a set with at least two elements and $b \in X$ be a fixed element. We define a subset A of X to be open if and only if A contains b or A is empty.

- At b, X is T_G, T_N, not T_{CG}, not T_C. and not T_1.
- At all $x \neq b$, X is T_C and not T_N.
- X is not T_1 at any point $y \in X$.

The following example shows that, at a, T_1 does not imply T_{CG} and T_N does not imply T_G.

Example 2. Let X be an infinite set with cofinite topology i.e., a subset A of X is open if and only if A' is finite or X.

At any point $a \in X$, X is T_C, T_N (because $\{a\} = \cap_{x \neq a}\{x\}'$), T_1, and not T_G.

Now we define T_0 at a particular point and prove equivalent conditions.

Definition 3. Let X be a topological space and $a \in X$. X is called T_0 at a if and only if for all $x \in X$ such that $x \neq a$, there exists a clorpen set containing a but not x (i.e., either there exists an open set G such that $x \in G$ and $a \notin G$, or there exists an open set H such that $a \in H$ and $x \notin H$).

Result 2. Let X be a topological space and $a \in X$. The following are equivalent:

1. X is T_0 at a.
2. For all $x \in X$, $x \neq a$, $a \notin \bar{x}$ or $x \notin \bar{a}$.
3. For all $x \in X$, $x \neq a$, $\bar{x} \neq \bar{a}$.
4. For all $x \in D(a)$, $\bar{x} \subseteq D(a)$.
5. $D(a)$ is a union of closures of singletons (i.e., $D(a) = \cup_{x \in D(a)} \bar{x}$).
6. $D(a)$ is a union of closed sets.

Proof. Let X be T_0 at a, $x \in X$, and $x \neq a$. Either there exists an open set G such that $x \in G$ and $a \notin G$ meaning $x \notin \bar{a}$ or there exists an open set H, $a \in H$ and $x \notin H$, meaning $a \notin \bar{x}$. Thus, (1) $\Rightarrow$ (2).

Clearly (2) $\Rightarrow$ (3).

Assume (3). Let $x \in D(a)$. Since $x \neq a$, $\bar{x} \neq \bar{a}$. But $x \in \bar{a}$ so that $\bar{x} \subseteq \bar{a}$. Hence, $a \notin \bar{x}$. Hence, $\bar{x} \subseteq D(a)$. Thus, (3) $\Rightarrow$ (4).

Clearly (4) $\Rightarrow$ (5) $\Rightarrow$ (6).

Assume (6). Let $x \in X$ and $x \neq a$. If $x \in D(a)$, then $x \in C$ where C is closed and $C \subseteq D(a)$. Hence, $a \notin C$. Thus, C' is an open set containing a, but not x. If $x \notin D(a)$, then $x \notin \bar{a}$ so that $\bar{a}'$ is open set containing x but not a. Thus, X is T_0 at a. □

Note that, T_N at a implies T_0 at a. If X is T_N at a, then $\{a\}$ is an intersection of open sets $\{G_\alpha\}$; $\{a\}'$ is an union of closed sets $\{G'_\alpha\}$, hence, $D(a) = \bar{a} \cap \{a\}'$ is a union of closed sets $\{\bar{a} \cap G'_\alpha\}$, which means, X is T_0 at a.

Definition 4. Let X be a topological space and $a \in X$. X is called T_D at a if and only if $D(a)$ is closed.

Clearly, at a, $T_C \Rightarrow T_D$ (because $D(a)$ is empty), $T_G \Rightarrow T_D$ (because $D(a) = \bar{a} \cap \{a\}'$ is closed), and $T_D \Rightarrow T_0$.

The topological space considered in Example 1 is (T_C and hence) T_D, but not (T_N and hence not) T_G at any point other than b, which shows that T_D at a does not imply T_G at a.

Example 3. Let X be the set of all integers greater than 1 with the *Divisor Topology* whose base is $\{V_x : x \in X\}$ where V_x be the set of all divisors of x greater than 1. Note that for any $x \in X$ and $y \in X$,

$$V_x \cap V_y = V_{gcd(x,y)}$$

and

$$\bar{x} = \{kx : k \in \mathbb{N}\}.$$

$\{x\}$ is open (i.e., X is T_G (and hence T_D) at x) if and only if x is prime.

No two non-empty closed sets are disjoint because $\overline{xy} \subseteq \bar{x} \cap \bar{y}$.

T_0 at a does not imply T_D at a as seen in the following example.

Example 4. Let X be any infinite set and $b \in X$ be fixed. Declare any nonempty set G open in X if and only if $b \in G$ and G' is finite.

Note that $D(b) = \{b\}'$ is neither open nor closed. It is a union of closed sets $\{x\}$ where $x \neq b$.

At b, X is T_0, not T_D, and not T_C, because $\{b\}'$ is not open.

However, X is T_C at any $x \neq b$, because $\{x\}'$ is open.

Result 3. Let X be a topological space and $a \in X$. The following are equivalent:

1. X is T_D at a (i.e., $D(a)$ is closed).

2. There exists an open set G such that $a \in G$ and $G \cap \{a\}'$ is open.
3. There exist an open set G and a closed set C such that $\{a\} = G \cap C$.
4. There exist an open set G such that $\{a\} = G \cap \bar{a}$.

Proof. Let X be T_D at a. Take $G = D(a)'$. Then G is open and $G \cap \{a\}' = D(a)' \cap \{a\}' = \bar{a}'$ is open. Thus, (1) $\Rightarrow$ (2).

If (2) is true, then take $C = (G \cap \{a\}')'$ so that C is closed and $\{a\} = G \cap C$. Thus, (2) $\Rightarrow$ (3).

If (3) is true, then C is closed and $a \in C$ so that $\bar{a} \subseteq C$ and hence $\{a\} \subseteq G \cap \bar{a} \subseteq G \cap C = \{a\}$. Thus, (3) $\Rightarrow$ (4).

If (4) is true, then $\{a\} = G \cap \bar{a}$. Therefore, $G \cap D(a) = \phi$ and it follows that $D(a) \subseteq G'$. Hence, since $a \in G$, it follows that $D(a) = G' \cap D(a) = G' \cap \bar{a}$. Therefore, $D(a)$ is closed and so we have that (4) $\Rightarrow$ (1). □

Definition 5. Let X be a topological space and $a \in X$.

1. X is called T_E at a if and only if $D(x) \cap D(a) = \phi$ for all $x \in X$, $x \neq a$.
2. X is called T_{DD} at a if and only if it is T_D at a and T_E at a, i.e., $D(a)$ is closed and $D(x) \cap D(a) = \phi$ for all $x \in X$, $x \neq a$.
3. X is called T_{UD} at a if and only if $D(a)$ is a union of disjoint closed sets.

Clearly, at a, $T_C \Rightarrow T_E$ (because $D(a)$ is empty), and $T_C \Rightarrow T_{DD} \Rightarrow T_D \Rightarrow T_{UD} \Rightarrow T_0$.

The topological space considered in Example 1 is T_{DD} and T_E at b, but not T_C at b. The following example shows that T_D does not imply T_{DD}.

Example 5. Let X be a set and A be a proper subset of X. Let X be the topological space in which a subset G of X is open if and only if $G \subseteq A$ or $G = X$.

If $x \in A$, then $D(x) = A'$ is closed. Hence, X is T_D at all points of A.

If A has more than one point, X is not T_{DD} at any point of A.

If $x \in A'$, then $D(x) = A' \cap \{x\}'$.

If A' has more than one point and if $x \in A'$, then $D(x) = A' \cap \{x\}'$ is not closed. Hence, X is not T_D at any point of A'.

If $A' = \{a\}$, then $D(a) = \phi$ and X is T_{DD} at a.

Result 4. Let X be a topological space and let $a \in X$ such that X is T_{DD} at a. Then $\bar{x} \cap \bar{a} = \phi$ or $\{x\}$ or $\{a\}$ for all $x \in X$, $x \neq a$.

Proof. Let $x \in X$ and $x \neq a$. $D(a)$ is closed and $D(x) \cap D(a) = \phi$. Note that $\bar{x} \cap \bar{a} = (D(x) \cap \{a\}) \cup (D(a) \cap \{x\})$. If $x \in D(a)$ and $a \in D(x)$, then $a \in D(x) \subseteq \bar{x} \subseteq D(a)$, which is a contradiction.

If $x \in D(a)$, then $a \notin D(x)$ and $\bar{x} \cap \bar{a} = D(a) \cap \{x\} = \{x\}$.

If $x \notin D(a)$, then $\bar{x} \cap \bar{a} = D(x) \cap \{a\} \subseteq \{a\}$. □

Definition 6. Let X be a topological space and $a \in X$.

1. X is called T_{YS} at a if and only if $\bar{x} \cap \bar{a} = \phi$ or $\{x\}$ or $\{a\}$ for all $x \in X$, $x \neq a$.
2. X is called T_Y at a if and only if $|\bar{x} \cap \bar{a}| \leq 1$ for all $x \in X$, $x \neq a$.
3. X is called T_S at a if and only if $\bar{x} = \{x\}$ for all $x \in D(a)$.

Clearly, at a, $T_{DD} \Rightarrow T_{YS} \Rightarrow T_Y$, $T_C \Rightarrow T_{YS}$ (because $\bar{a} = \{a\}$), and $T_S \Rightarrow T_0$.

The topological space, at b, considered in Example 4, is T_{UD} (since $D(b) = \{b\}'$ is the union of closed sets $\{x\}$ where $x \neq b$) and T_{YS} ($\bar{b} \cap \bar{x} = \{x\}$ where $x \neq b$), but not T_D (and hence not T_{DD}). This shows that, at a, T_{YS} does not imply T_{DD} and T_{UD} does not imply T_D.

The topological space X, considered in Example 5, is T_D and hence, T_{UD} at all points of A. However, X is not T_S at all points of A (since $D(x) = A'$ for any $x \in A$ and $\bar{y} = A'$ for any $y \in A'$). This shows T_{UD} does not imply T_S.

The following example shows that T_0 does not imply T_{UD}.

Example 6. Let X be the set of all real numbers in which a subset G of X is open if and only if G is ϕ, X, or (a, ∞) for some $a \in X$.

If $x \in X$, then $\bar{x} = (-\infty, x]$.

If $x < y$, then (x, ∞) is an open set containing y but not x. Thus, X is T_0 space at all points of X.

But, no two non-empty closed sets are disjoint. Indeed, $D(x) = (-\infty, x)$ is not a union of disjoint cloased sets. Thus, X is not T_{UD} at any point.

The following example shows that T_Y does not imply T_{YS}.

Example 7. Let X be a set. Let $\{A, B, C\}$ be a partition of X of nonempty proper subsets of X. Let X be the topological space in which a subset G of X is open if and only if G is ϕ, A, B, C' or X.

If $x \in A$, then $\bar{x} = B'$ and $D(x) = B' \cap \{x\}'$.

If $x \in B$, then $\bar{x} = A'$ and $D(x) = A' \cap \{x\}'$.

If $x \in C$, then $\bar{x} = C$ and $D(x) = C \cap \{x\}'$.

If each of the sets A, B, and C has more than one point, then $D(x)$ is not closed for all $x \in X$.

Assume $A = \{a\}$, $B = \{b\}$, and $C = \{c\}$. Then $\bar{x} \cap \bar{y} = \{c\}$ for all $x \neq y$. Thus, X is not T_{YS} at a or b. X is not T_{DD} at a or b. But X is T_Y at all points.

Note that, T_Y at a implies T_S at a. Indeed, assume X is T_Y at a. Let $x \in D(a)$. Since $x \in \bar{a} \cap \bar{x}$ and $|\bar{a} \cap \bar{x}| \leq 1$, we have $\{x\} = \bar{x} \cap \bar{a}$, which shows $\{x\}$ is closed. Thus, X is T_S at a.

The following example shows that T_S does not imply T_Y.

Example 8. Let $X = \{a, x, y\}$ be a set. Let $A = \{x\}$ and $B = \{a, x\}$. Let X be the topological space in which a subset G of X is open if and only if G is ϕ, A, B, or X.

Now, $\bar{a} = \{a, y\}$ and $\bar{x} = X$, so $\bar{a} \cap \bar{x} = \{a, y\}$, and hence, X is not T_Y at a. Since $\bar{y} = \{y\}$, X is T_S at a.

T_{YS} is equivalent to T_S and T_E, as shown below.

Result 5. Let X be a topological space and $a \in X$.

X is T_{YS} at a if and only if X is T_S at a and T_E at a (i.e., $D(a) \cap D(x) = \phi$ for all $x \neq a$).

Proof. Let X be T_{YS} at a. If $x \in D(a)$, then $x \neq a$ and $\bar{x} \cap \bar{a} = \{x\}$ which means $\{x\}$ is closed. Thus, X is T_S at a.

Let $x \in X$ and $x \neq a$. If $\bar{x} \cap \bar{a} = \phi$, clearly $D(x) \cap D(a) = \phi$. Similarly, $\bar{x} \cap \bar{a} = \{x\}$ or $\{a\}$ clearly implies that $D(x) \cap D(a) = \phi$.

Conversely, assume that X is T_S at a and $D(x) \cap D(a) = \phi$ for all $x \neq a$. Let $x \neq a$. Now, $\bar{x} \cap \bar{a} = (D(x) \cap \{a\}) \cup (D(a) \cap \{x\})$.

If $x \in D(a)$, then $\bar{x} \cap \bar{a} \subseteq \bar{x} = \{x\}$.

If $x \notin D(a)$, then $\bar{x} \cap \bar{a} = D(x) \cap \{a\} \subseteq \{a\}$.

Thus, X is T_{YS} at a. □

Example 8 (together with the fact $T_{YS} \implies T_Y$) shows that T_S does not imply T_{YS}. The following example shows that T_E does not imply T_{YS}.

Example 9. Let $X = I\!N \times \{0, 1\}$ be the topological space in which a subset G of X is open if and only if there is a subset A of $I\!N$ such that $G = A \times \{0, 1\}$.

If $n \in I\!N$, then $\overline{(n, 0)} = \{n\} \times \{0, 1\}$ and $\overline{(n, 1)} = \{n\} \times \{0, 1\}$, Thus, X is T_E at all points of X and it is not T_{YS} at any point of X.

If $n \in I\!N$, then every open set containing $(n, 0)$ or $(n, 1)$ must contain the open set $\{n\} \times \{0, 1\}$, thus, X is not T_0 at any point of X.

If $n \in I\!N$, then every clorpen set containing $(n, 0)$ or $(n, 1)$ must contain the clorpen set $\{n\} \times \{0, 1\}$, thus, X is not T_{CG} at any point of X.

So far we have considered properties of topological spaces that distinguishes two points. We will consider properties at a single point with regard to finite sets.

Definition 7. Let X be a topological space and $a \in X$.

1. X is called T_F at a if and only if for every finite subset B of X not containing a, there exists a clorpen set G containing a and disjoint from B.
2. X is called T_{FF} at a if and only if for any two disjoint finite subsets A and B of X such that $a \in A$, there exists a clorpen set G containing A and disjoint from B.

3. X is called T_{SS} at a if and only if for all elements $u \in \{a\}'$ and x, y in $\bar{u} \cap \bar{a}$ such that $\{u, y\} \cap \{a, x\} = \phi$, there exists a clorpen set G such that $\{u, y\} \subseteq G$ and $\{a, x\} \subseteq G'$.

Clearly, at any point, T_G or T_C implies T_F.

Result 6. Let X be a topological space and $a \in X$. If X is T_Y at a, X is also T_F at a.

Proof. Assume X is T_Y at a. Let B be a finite set not containing a.

If there is $x \in B$ such that $a \in \bar{x}$, then $\{a\} = \bar{x} \cap \bar{a}$ and hence, $\{a\}$ is a closed set containing a, disjoint from B.

So assume that $a \notin \bar{x}$ for all $x \in B$. Define $K = \cup\{\bar{x} : x \in B\}$. K is closed and hence, K' is an open set containing a disjoint from B.

Thus, X is T_F at a. □

In Example 5, if $|A| \geq 2$ and $|A'| \geq 2$, then $\bar{x} \cap \bar{y}$ contains $\{x, y\}$ for all $\{x, y\} \subseteq A$, and hence, the topological space is not T_Y at any point. But it is T_F at all points, because it is T_G at all points of A and T_C at all points of A'. Thus, T_F does not imply T_Y.

Result 7. Let X be a topological space and $a \in X$. Then X is T_Y at a if and only if X is T_{SS} at a.

Consequently, if X is T_{FF} at a, then it is also T_Y at a

Proof. Suppose X is not T_Y at a. Then there exists $u \neq a$ such that $|\bar{a} \cap \bar{u}| > 1$.

Case 1. Assume $a \in \bar{u}$. Then, $\bar{a} \cap \bar{u} = \bar{a}$. Choose $y \in \bar{a}$ and $y \neq a$. Then, if G is any open set containing a, then $u \in G$ (because $a \in \bar{u}$). On the other hand, if G is any open set containing y, then $a \in G$ (because $y \in \bar{a}$). Thus, $\{u, y\} \cap \{a\} = \phi$ and there is no clorpen set G containing one of these sets and disjoint from the other set.

Case 2. Assume $a \notin \bar{u}$. Choose $x \in \bar{a} \cap \bar{u}$ such that $x \neq u$. Choose $y \in \bar{a} \cap \bar{u}$ such that $y \neq x$. If G is any open set containing x, then $u \in G$ (because $x \in \bar{u}$). On the other hand, if G is any open set containing y, then $a \in G$ (because $y \in \bar{a}$). Thus, $\{u, y\} \cap \{a, x\} = \phi$ and there is no clorpen set G containing one of these sets and disjoint from the other set.

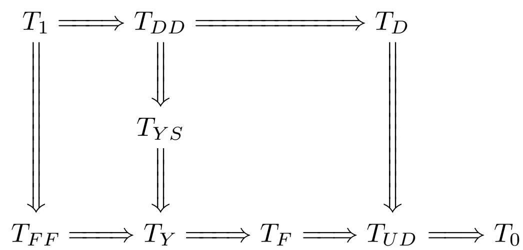
Thus, X is not T_{SS} at a.

Conversely, if X is not T_{SS} at a, then clearly there exists $u \neq a$ such that $\bar{u} \cap \bar{a}$ has at least two elements and hence, X is not T_Y at a. □

In Example 7, the topological space X is not T_{FF} at a because X is the only clorpen set containing a. Thus, T_Y (and hence, T_F) does not imply T_{FF}.

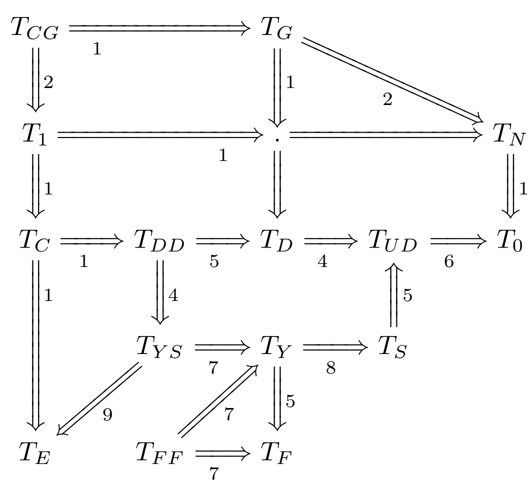
If the separation axiom is valid at every point, then T_1, T_C, and T_N are equivalent; T_G and T_{CG} are equivalent to the topology being discrete.

It has been known [3] that the following implications hold when the axioms are true for all points.



3. Conclusion

The following diagram summarizes the implications of separation axioms where the number under an implication arrow represents the example number given above which shows the implication is strict.



It would be interesting to explore local conditions on the topological spaces that assures equivalence of these local separation axioms between Kolmogorov and Fréchet Spaces. However, that will lead us in different direction which could be a topic of another study.

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MSC2010: 08A25, 08A60, 08A30, 08C05, 17A30, 18A40, 18B99, 18D15, 54A05

Key words and phrases: Separation axioms, topological spaces, Kolmogorov space, Fréchet spaces

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