CRITICAL VALUES FOR THE MOOD TEST OF EQUALITY OF DISPERSION

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Abstract. An exhaustive unconditional permutation distribution of a test statistic is necessary in the construction of exact tests of significance. Tables of exact critical values for the Mood test are scarce. In this paper, the exact permutation distribution of the Mood test statistic is generated empirically by actually obtaining all the distinct permutations of the variates in an experiment. The tables of exact critical values for the Mood test are produced.

1. Introduction. The risk in decision making cannot be totally eliminated, but it can be controlled if correct statistical procedures are employed. The unconditional permutation approach is a statistical procedure that ensures that the distribution of the test statistic is exact and that the resulting probability of a type I error is exactly α, see Agresti [1], Good [8] and Pesarin [13].

Scheffe [14] demonstrates that for a general class of problems, the permutation approach is the only possible method of constructing exact tests of significance. It is asymptotically as powerful as the best parametric test, see Hoeffding [10]. In this paper, consideration is given to the exhaustive permutation of the ranks of the observations in a two-sample experiment to arrive at the exact distribution of the Mood test statistic for equality of dispersion.

The idea of obtaining an exact test of significance through the permutation approach originated with Fisher [7]. The essential feature of the method is that all the distinct arrangements of the observations are considered, with the stipulation that all the permutations are equally likely under the null hypothesis. An exact test on the level of significance, α, is constructed by choosing a proportion, α, of the permutation as the critical region. The works of Siegel and Castellan [15], Conover [5], Headrick [9], Bagui and Bagui [2, 3], and Odiase and Ogbonmwan [11, 12] are contributions to the quest for exact critical values. The methodology for exact permutation tests presented in this paper is implemented to produce the exact distribution of the Mood test statistic for sample sizes $m, n \leq 15$.

2. Distribution-Free Mood Test. The Mood test is based on the sum of squared deviations of the ranks of one of the samples from the mean of the combined ranks of all the observations. The null hypothesis, H_0, is that there is no difference in spread while the alternative hypothesis,

H_1, is that there is some difference. Consider the layout of a two-sample experiment as

$$\begin{pmatrix} x_{11} & x_{21} \\ x_{12} & x_{22} \\ \vdots & \vdots \\ x_{1m} & x_{2n} \end{pmatrix},$$

where x_{ij} is the jth observation of the ith sample and $N = m + n$ is the total number of observations in the data set. Rank all the N observations from 1 (smallest x_{ij}) to N (largest x_{ij}). Let r_{1i} be the rank of x_{1i}; the Mood test statistic is

$$M = \sum \left(r_{1i} - \frac{m+n+1}{2} \right)^2$$

and the standardized version of M is

$$z = \frac{M - \frac{m(N^2-1)}{12}}{\sqrt{\frac{mn(N+1)(N^2-4)}{180}}}.$$

The large sample approximation of z is the standard normal distribution.

Permutation Test Procedure. Let $\pi_1, \pi_2, \ldots, \pi_n$ be a set of all distinct permutations of the ranks of the data set in the experiment. The permutation test procedure for the Mood test is as follows:

1. Rank the combined observed original data set of the experiment.
2. Compute the observed value of the Mood test statistic ($M_1 = t_0$).
3. Obtain a distinct permutation, π_i, of the ranks in Step 1.
4. Compute the Mood test statistic M_i for permutation π_i in Step 3, $M_i = M(\pi_i)$
5. Repeat Steps 3 and 4 for $i = 2(1)\eta$.
6. Construct an empirical cumulative distribution for M

$$p_0 = \hat{p}(M \le M_i) = \frac{1}{\eta} \sum_{i=1}^{\eta} \psi(t_0 - M_i),$$

   where $\psi(\cdot) = 1$, if $t_0 \ge M_i$, and $\psi(\cdot) = 0$ if $t_0 < M_i$.
7. Under the empirical distribution, if $p_0 \le \alpha$, reject the null hypothesis.

Under H_0, each distinct permutation of ranks is obtained, the value of M determined for each one, and the null distribution obtained by counting the number of times each value of M occurs.

A difficulty in using nonparametric tests is the nonavailability of exact critical values. This continues to be a problem as revealed by a survey of twenty (20) in-print general college statistics textbooks by Fahoome [6]; tables of critical values were not provided for the Mood test.

The purpose of this paper is to provide exact critical values for the Mood test that will ensure that the probability of a type I error is exactly α.

3. Unconditional Permutation Algorithm. Unconditional permutation involves allowing the column (treatment) and row totals of the layout of observations resulting from an experiment to vary with each rearrangement of the observations. This is unlike the conditional approach of fixing column and row totals (Agresti [1]). The first step in developing an unconditional permutation algorithm is to formulate an initial configuration of the ranks of the variates of an experiment, since a full enumeration of all the distinct permutations can be obtained from any configuration of the combined ranks.

Let the initial configuration of the ranks of the variate in a two-sample experiment be represented as

$$\begin{pmatrix} r_{11} & r_{21} \\ r_{12} & r_{22} \\ \vdots & \vdots \\ r_{1m} & r_{2n} \end{pmatrix}.$$

We expect to have $\frac{N!}{m!n!}$ distinct permutations for an exhaustive enumeration. Thus,

Stage 1.

$$\begin{pmatrix} r_{11} & r_{21} \\ r_{12} & r_{22} \\ \vdots & \vdots \\ r_{1m} & r_{2n} \end{pmatrix}.$$

The original arrangement of the data of the experiment yields $\binom{m}{0}\binom{n}{0} = 1$ permutation.

Stage 2.

$r_{11} \leftarrow r_{2i}$, $i = 1(1)n$ results in n permutations
$r_{12} \leftarrow r_{2i}$, $i = 1(1)n$ results in n permutations
...
$r_{1m} \leftarrow r_{2i}$, $i = 1(1)n$ results in n permutations

We have a total of $\binom{m}{1}\binom{n}{1}$ permutations (exchange of one rank from the first sample).

Stage 3.

$$\begin{pmatrix} r_{1s} \\ r_{1t} \end{pmatrix} \leftarrow \begin{pmatrix} r_{2i} \\ r_{2j} \end{pmatrix}; \; s \neq t, \; i \neq j,$$

yields $\binom{m}{2}\binom{n}{2}$ permutations (exchange of two ranks from the first sample).
...

Stage $\min(m,n)+1$.

$$\begin{pmatrix} r_{1s} \\ r_{1t} \\ \vdots \\ r_{1u} \end{pmatrix} \leftarrow \begin{pmatrix} r_{2i} \\ r_{2j} \\ \vdots \\ r_{2k} \end{pmatrix}; \; s \neq t \neq \ldots \neq u; \; i \neq j \neq \ldots \neq k$$

yields $\binom{m}{\min(m,n)}\binom{n}{\min(m,n)}$ permutations (exchange of all sample ranks). Therefore, the total number of distinct permutations in a complete enumeration is $\sum_{i=0}^{\min(m,n)} \binom{m}{i}\binom{n}{i}$, see Odiase and Ogbonmwan [11, 12].

The Mood test statistic is computed for each permutation in the complete enumeration of all the distinct permutations. Each value of the test statistic obtained from a complete enumeration occurs with probability $\left(\frac{N!}{m!n!}\right)^{-1}$. The distribution of the test statistic is obtained by tabulating the distinct values of the statistic against their probabilities of occurrence in the complete enumeration.

Tabulated exact critical values of a test statistic are usually provided for experiments with distinct observations, since it will be practically difficult to consider all possible occurrences of ties and create tables of exact critical values for each occurrence of ties for different sample sizes. This will result in several volumes of tables that will make the application of the Mood test unattractive. In order to arrive at the critical values provided in Table 1, the ranks of distinct observations (r_{ij}) were used as input in Algorithm (Mood) for various sample sizes. To provide exact critical values when ties occur, midranks are assigned as the ranks of tied observations, and Algorithm (Mood) is implemented with r_{ij} as input, composed of actual ranks containing ties.

A typical Intel Visual Fortran implementation of the methodology just described, that is, Stages 1 to $\min(m,n)+1$, for the exchange of five sample ranks requires the use of Algorithm (Mood).

Algorithm (Mood).

```
Generate ranks of observations (r_{i,j})
i ← 1
Rank1 ← r_{i,1}
for j ← i + 1, 2 do
    Rank2 ← r_{j,1}
    for m ← j + 1, 3 do
        Rank3 ← r_{m,1}
        for n ← m + 1, 4 do
            Rank4 ← r_{n,1}
            for o ← n + 1, 5 do
                Rank5 ← r_{o,1}
                l ← 2
                for i1 ← 1, 5 do
                    for l1 ← l, 2 do
                        if l ← l1 then
                            t ← i1 + 1
                        else
                            t ← 1
                        end if
                        for j1 ← t, 5 do
                            for l2 ← l1, 2 do
                                if l1 ← l2 then
                                    t1 ← j1 + 1
                                else
                                    t1 ← 1
                                end if
                                for j2 ← t1, 5 do
                                    for l3 ← l2, 2 do
                                        if l2 ← l3 then
                                            t2 ← j2 + 1
                                        else
                                            t2 ← 1
                                        end if
                                        for j3 ← t2, 5 do
                                            for l4 ← l3, 2 do
                                                if l3 ← l4 then
                                                    t3 ← j3 + 1
                                                else
                                                    t3 ← 1
                                                end if
                                                for j4 ← t3, 5 do
                                                    r_{i,1} ← r_{i1,l}; r_{i1,l} ← Rank1
                                                    r_{j,1} ← r_{j1,l1}; r_{j1,l1} ← Rank2
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$r_{m,1} \leftarrow r_{j2,l2};\ r_{j2,l2} \leftarrow Rank3$
$r_{n,1} \leftarrow r_{j3,l3};\ r_{j3,l3} \leftarrow Rank4$
$r_{o,1} \leftarrow r_{j4,l4};\ r_{j4,l4} \leftarrow Rank5$
Compute Mood statistic
end for
end for
end for
end for
end for
end for
end for
end for
end for
end for
end for
end for
end for
Construct frequency distribution for Mood Test Statistic
sort values of test statistic in ascending order
Construct cdf for Mood Test Statistic
Extract critical values

4. Critical Values for the Mood Test Statistic. Figure 1 clearly reveals that a group sample size of 5 or 7 is inadequate for the application of the large sample approximation of the Mood test. The wavy multimodal nature of the distribution for small m and n makes the application of the normal approximation to produce critical values unreliable. The distribution becomes more stable and closer to the normal distribution as the group sample size increases. For example, when $m = n = 15$, the exact and normal approximation of the Mood test statistic are very close.

The complete algorithm was implemented in Intel Visual Fortran, and the exact critical values as obtained from the exhaustive unconditional permutation distribution of the Mood test statistic are presented in Table 1 (the two values in each cell represent the lower and upper critical values).

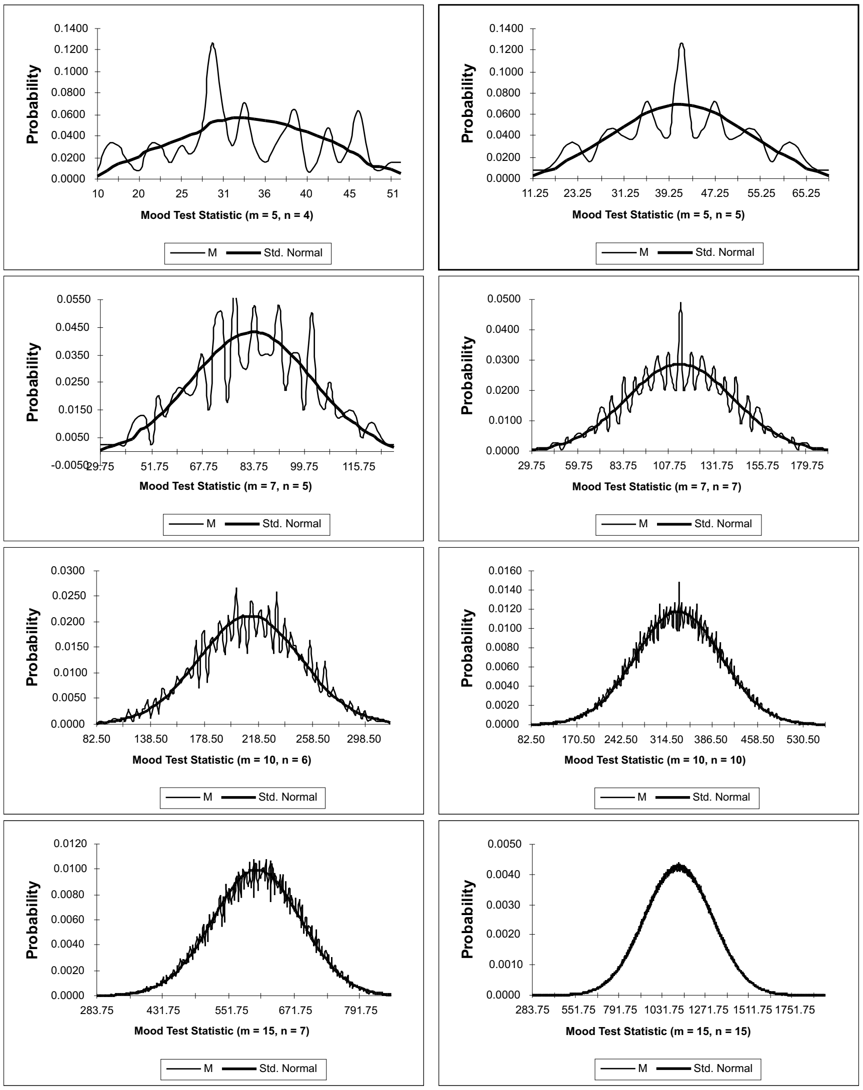

Figure 1: Probability distribution of Mood test statistic

Table 1: Exact Critical values for Mood test

Sample Size *m*	*n*	$M_{0.9000}$	$M_{0.9500}$	$M_{0.9750}$	$M_{0.9900}$	$M_{0.9950}$	$M_{0.9975}$	$M_{0.9990}$
3	2	2.00 9.00						
3	3	2.75 14.75						
4	2	9.00 15.00						
4	3	9.00 23.00	6.00 23.00					
4	4	11.00 31.00	9.00 33.00	9.00 33.00				
5	2	15.00 26.00	15.00 27.00					
5	3	17.25 35.25	15.25 37.25	11.25 39.25				
5	4	22.00 46.00	18.00 49.00	15.00 51.00	15.00 54.00			
5	5	25.25 57.25	21.25 61.25	17.25 65.25	15.25 67.25	11.25 71.25		
6	2	23.50 39.50	23.50 39.50					
6	3	28.00 51.00	26.00 55.00	24.00 55.00				
6	4	33.50 65.50	29.50 67.50	27.50 71.50	23.50 73.50	23.50 73.50		
6	5	40.00 80.00	34.00 85.00	31.00 90.00	26.00 95.00	24.00 95.00	19.00 95.00	
6	6	47.50 95.50	41.50 101.50	35.50 107.50	29.50 113.50	23.50 119.50	23.50 119.50	
7	2	35.00 56.00	35.00 59.00					
7	3	43.75 73.75	37.75 75.75	35.75 77.75	29.75 79.75			
7	4	51.00 89.00	44.00 95.00	40.00 96.00	35.00 101.00	35.00 104.00		
7	5	57.75 107.75	51.75 113.75	47.75 119.75	39.75 125.75	35.75 127.75	29.75 131.75	
7	6	67.00 127.00	59.00 136.00	52.00 143.00	44.00 148.00	40.00 154.00	35.00 156.00	35.00 163.00
7	7	77.75 149.75	67.75 159.75	59.75 167.75	49.75 177.75	43.75 183.75	41.75 185.75	35.75 191.75
8	2	50.00 80.00	50.00 80.00	50.00 80.00				
8	3	60.00 97.00	59.00 101.00	53.00 105.00	44.00 105.00			
8	4	72.00 120.00	64.00 124.00	60.00 128.00	50.00 134.00	50.00 134.00	50.00 134.00	
8	5	81.00 142.00	75.00 149.00	68.00 155.00	60.00 160.00	56.00 164.00	51.00 167.00	44.00 167.00
8	6	94.00 166.00	84.00 176.00	76.00 184.00	68.00 190.00	60.00 196.00	56.00 200.00	50.00 204.00
8	7	106.00 192.00	95.00 204.00	84.00 213.00	75.00 224.00	68.00 229.00	60.00 236.00	56.00 240.00
8	8	120.00 220.00	106.00 234.00	94.00 246.00	80.00 260.00	74.00 266.00	68.00 272.00	60.00 280.00

Table 1: Exact Critical values for Mood test (Contd.)

Sample Size *m*	*n*	$M_{0.9000}$	$M_{0.9500}$	$M_{0.9750}$	$M_{0.9900}$	$M_{0.9950}$	$M_{0.9975}$	$M_{0.9990}$
9	1	62.25						
		82.25						
9	2	76.00	69.00	69.00				
		105.00	108.00	109.00				
9	3	86.25	80.25	72.25	70.25	62.25		
		128.25	134.25	136.25	138.25	140.25		
9	4	96.00	89.00	81.00	76.00	69.00	69.00	
		153.00	161.00	167.00	168.00	173.00	176.00	
9	5	110.25	102.25	92.25	84.25	80.25	72.25	70.25
		180.25	190.25	196.25	204.25	206.25	210.25	212.25
9	6	125.00	114.00	104.00	93.00	85.00	80.00	76.00
		210.00	221.00	230.00	240.00	245.00	249.00	254.00
9	7	140.25	126.25	116.25	102.25	92.25	86.25	80.25
		242.25	254.25	266.25	278.25	284.25	290.25	296.25
9	8	157.00	141.00	128.00	113.00	104.00	96.00	85.00
		275.00	291.00	304.00	317.00	328.00	336.00	343.00
9	9	174.25	156.25	140.25	122.25	112.25	104.25	92.25
		310.25	328.25	344.25	362.25	372.25	380.25	392.25
10	1	85.00						
		109.00						
10	2	100.50	92.50	92.50				
		136.50	140.50	140.50				
10	3	112.00	105.00	101.00	94.00	85.00		
		165.00	171.00	174.00	177.00	177.00		
10	4	128.50	120.50	110.50	104.50	100.50	92.50	92.50
		194.50	204.50	208.50	212.50	216.50	218.50	218.50
10	5	145.00	133.00	125.00	112.00	105.00	101.00	94.00
		227.00	238.00	246.00	253.00	258.00	262.00	265.00
10	6	162.50	148.50	136.50	124.50	116.50	110.50	105.50
		262.50	274.50	284.50	294.50	300.50	306.50	312.50
10	7	181.00	165.00	153.00	137.00	128.00	118.00	109.00
		298.00	313.00	325.00	340.00	348.00	353.00	364.00
10	8	202.50	182.50	166.50	150.50	138.50	128.50	118.50
		336.50	354.50	368.50	384.50	394.50	404.50	414.50
10	9	222.00	201.00	184.00	165.00	150.00	140.00	126.00
		378.00	398.00	415.00	435.00	447.00	458.00	469.00
10	10	244.50	220.50	200.50	178.50	164.50	150.50	136.50
		420.50	444.50	464.50	486.50	500.50	514.50	528.50
11	1	112.75						
		142.75						
11	2	130.00	121.00	121.00				
		177..00	178.00	181.00				
11	3	148.75	140.75	134.75	122.75	112.75		
		208.75	214.75	218.75	222.75	224.75		
11	4	166.00	154.00	145.00	134.00	130.00	121.00	121.00
		244.00	253.00	259.00	266.00	269.00	271.00	274.00
11	5	184.75	172.75	160.75	148.75	140.75	134.75	124.75
		280.75	292.75	300.75	310.75	316.75	318.75	322.75
11	6	207.00	191.00	178.00	163.00	154.00	145.00	134.00
		320.00	334.00	346.00	358.00	365.00	373.00	377.00
11	7	228.75	210.75	194.75	178.75	166.75	154.75	142.75
		362.75	378.75	392.75	408.75	418.75	426.75	434.75
11	8	253.00	232.00	214.00	193.00	181.00	169.00	155.00
		406.00	426.00	443.00	462.00	474.00	483.00	494.00
11	9	278.75	254.75	232.75	210.75	194.75	182.75	166.75
		452.75	476.75	496.75	518.75	530.75	542.75	558.75

Table 1: Exact Critical values for Mood test (Contd.)

Sample Size *m*	*n*	$M_{0.9000}$	$M_{0.9500}$	$M_{0.9750}$	$M_{0.9900}$	$M_{0.9950}$	$M_{0.9975}$	$M_{0.9990}$
11	10	304.00	277.00	254.00	229.00	211.00	197.00	180.00
		502.00	529.00	552.00	577.00	593.00	607.00	625.00
11	11	330.75	300.75	274.75	246.75	226.75	210.75	192.75
		554.75	584.75	610.75	638.75	658.75	674.75	692.75
12	1	146.00						
		181.00						
12	2	167.00	155.00	155.00				
		221.00	225.00	225.00				
12	3	190.00	179.00	170.00	159.00	157.00	146.00	
		259.00	266.00	270.00	275.00	275.00	275.00	
12	4	209.00	197.00	185.00	177.00	167.00	155.00	155.00
		299.00	309.00	317.00	323.00	327.00	331.00	331.00
12	5	233.00	218.00	205.00	191.00	181.00	170.00	166.00
		342.00	354.00	365.00	377.00	381.00	386.00	390.00
12	6	259.00	239.00	225.00	209.00	197.00	185.00	177.00
		387.00	403.00	415.00	431.00	437.00	443.00	451.00
12	7	284.00	263.00	246.00	226.00	212.00	202.00	186.00
		434.00	454.00	470.00	487.00	498.00	506.00	515.00
12	8	311.00	287.00	267.00	245.00	229.00	215.00	201.00
		485.00	507.00	527.00	547.00	561.00	571.00	585.00
12	9	341.00	314.00	290.00	265.00	248.00	233.00	215.00
		538.00	565.00	586.00	610.00	626.00	639.00	654.00
12	10	371.00	341.00	315.00	287.00	267.00	251.00	231.00
		595.00	623.00	649.00	677.00	695.00	709.00	729.00
12	11	403.00	369.00	341.00	308.00	287.00	268.00	246.00
		652.00	686.00	714.00	746.00	767.00	786.00	807.00
12	12	437.00	399.00	367.00	331.00	307.00	287.00	263.00
		713.00	751.00	783.00	819.00	843.00	863.00	887.00
13	1	185.25						
		227.25						
13	2	215.00	206.00	195.00	195.00			
		271.00	275.00	278.00	279.00			
13	3	235.25	221.25	211.25	199.25	197.25	185.25	
		317.25	325.25	331.25	333.25	335.25	337.25	
13	4	260.00	245.00	234.00	221.00	210.00	206.00	195.00
		362.00	374.00	382.00	390.00	394.00	397.00	399.00
13	5	287.25	271.25	255.25	239.25	227.25	221.25	211.25
		409.25	425.25	437.25	449.25	455.25	461.25	463.25
13	6	317.00	296.00	279.00	259.00	246.00	234.00	222.00
		462.00	480.00	495.00	511.00	519.00	527.00	535.00
13	7	347.25	323.25	303.25	281.25	265.25	251.25	237.25
		515.25	537.25	555.25	575.25	587.25	595.25	607.25
13	8	379.00	352.00	330.00	303.00	286.00	271.00	254.00
		573.00	598.00	619.00	642.00	656.00	669.00	682.00
13	9	413.25	383.25	355.25	327.25	307.25	291.25	271.25
		633.25	661.25	685.25	713.25	729.25	745.25	761.25
13	10	448.00	414.00	385.00	351.00	330.00	311.00	289.00
		695.00	728.00	756.00	787.00	807.00	824.00	844.00
13	11	485.25	447.25	413.25	377.25	353.25	333.25	307.25
		761.25	797.25	829.25	863.25	887.25	907.25	931.25
13	12	523.00	481.00	445.00	405.00	378.00	355.00	327.00
		829.00	871.00	906.00	945.00	971.00	994.00	1021.00
13	13	563.25	515.25	477.25	433.25	403.25	377.25	347.25
		899.25	947.25	985.25	1029.25	1059.25	1085.25	1115.25
14	1	231.00						
		279.00						

Table 1: Exact Critical values for Mood test (Contd.)

Sample Size m	n	$M_{0.9000}$	$M_{0.9500}$	$M_{0.9750}$	$M_{0.9900}$	$M_{0.9950}$	$M_{0.9975}$	$M_{0.9990}$
14	2	263.50	253.50	241.50	241.50			
		331.50	335.50	337.50	337.50			
14	3	291.00	276.00	264.00	255.00	244.00	231.00	
		382.00	391.00	395.00	402.00	403.00	403.00	
14	4	319.50	301.50	287.50	271.50	263.50	253.50	241.50
		433.50	445.50	455.50	463.50	469.50	473.50	475.50
14	5	350.00	331.00	315.00	294.00	283.00	271.00	259.00
		488.00	504.00	518.00	531.00	539.00	543.00	550.00
14	6	383.50	361.50	341.50	319.50	305.50	291.50	277.50
		545.50	565.50	581.50	599.50	609.50	617.50	627.50
14	7	419.00	392.00	369.00	343.00	327.00	311.00	294.00
		607.00	631.00	650.00	671.00	684.00	695.00	707.00
14	8	455.50	425.50	399.50	369.50	351.50	333.50	313.50
		669.50	697.50	721.50	745.50	763.50	775.50	791.50
14	9	494.00	459.00	431.00	398.00	376.00	356.00	334.00
		737.00	769.00	796.00	825.00	844.00	860.00	879.00
14	10	533.50	495.50	463.50	427.50	403.50	381.50	355.50
		807.50	843.50	873.50	907.50	929.50	947.50	971.50
14	11	575.00	533.00	497.00	456.00	430.00	406.00	378.00
		880.00	921.00	955.00	994.00	1018.00	1040.00	1066.00
14	12	619.50	571.50	533.50	487.50	459.50	431.50	401.50
		955.50	1001.50	1039.50	1083.50	1111.50	1135.50	1165.50
14	13	664.00	613.00	570.00	520.00	488.00	459.00	426.00
		1035.00	1085.00	1128.00	1176.00	1207.00	1236.00	1269.00
14	14	709.50	655.50	607.50	553.50	519.50	487.50	451.50
		1117.50	1171.50	1219.50	1273.50	1307.50	1339.50	1375.50
15	1	283.75						
		339.75						
15	2	319.00	308.00	295.00	295.00			
		398.00	403.00	404.00	407.00			
15	3	349.75	335.75	325.75	313.75	299.75	297.75	
		455.75	463.75	469.75	475.75	477.75	479.75	
15	4	385.00	367.00	351.00	335.00	323.00	310.00	295.00
		513.00	528.00	540.00	548.00	553.00	556.00	561.00
15	5	421.75	397.75	379.75	357.75	343.75	331.75	313.75
		573.75	593.75	607.75	623.75	629.75	635.75	643.75
15	6	459.00	433.00	411.00	386.00	370.00	355.00	340.00
		639.00	662.00	680.00	699.00	711.00	719.00	730.00
15	7	499.75	469.75	443.75	415.75	395.75	379.75	357.75
		707.75	733.75	755.75	777.75	793.75	805.75	817.75
15	8	540.00	507.00	478.00	445.00	424.00	404.00	382.00
		778.00	809.00	834.00	862.00	880.00	895.00	911.00
15	9	583.75	545.75	513.75	477.75	453.75	431.75	405.75
		851.75	887.75	915.75	949.75	969.75	987.75	1007.75
15	10	629.00	587.00	551.00	511.00	484.00	460.00	431.00
		930.00	970.00	1003.00	1039.00	1063.00	1084.00	1108.00
15	11	675.75	629.75	589.75	545.75	515.75	487.75	457.75
		1009.75	1055.75	1091.75	1133.75	1161.75	1185.75	1213.75
15	12	725.00	674.00	631.00	581.00	548.00	519.00	484.00
		1094.00	1144.00	1186.00	1232.00	1264.00	1291.00	1323.00
15	13	775.75	719.75	671.75	617.75	583.75	549.75	511.75
		1181.75	1235.75	1281.75	1335.75	1369.75	1399.75	1435.75
15	14	828.00	767.00	716.00	657.00	618.00	583.00	542.00
		1271.00	1332.00	1383.00	1441.00	1479.00	1513.00	1554.00
15	15	883.75	815.75	759.75	697.75	655.75	617.75	571.75
		1363.75	1431.75	1487.75	1549.75	1591.75	1629.75	1675.75

5. Conclusion. Results obtained from asymptotic procedures and resampling techniques are commonly adopted in several nonparametric tests as alternatives to tabulated exact critical values. Fahoome [6] conducted a Monte Carlo study and recommends the asymptotic approximation of the Mood test when group sample sizes exceed 4, based on Bradley's [4] conservative estimates of $0.045 <$ Type I error rate < 0.055 for $\alpha = 0.05$. To ensure that the probability of a type I error is exactly α when applying the large sample approximation, the group sample size should well exceed 5 as revealed in the distributions in Figure 1. This paper provides exact critical values for the group sample size up to $\max(m, n) = 15$.

It is clear that for $m, n \leq 7$, the normal distribution will poorly approximate the exact distribution of the Mood test statistic. As the group sample size increases, the shape of the distribution of the Mood test begins to look more like the normal, as seen in Figure 1 when $m, n \geq 10$. The critical values of the Mood test statistic in Table 1 are obtained from the enumeration of all the distinct permutations of the ranks of the variates in an experiment. These critical values are exact and therefore, ensure that the probability of a type I error in decisions arising from the use of the Mood test is exactly α.

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