REVIEWS
Edited by Joseph B. Dence

Reviews should be sent to Joseph B. Dence, Department of Chemistry, University of Missouri, 8001 Natural Bridge Rd., St. Louis, MO, 63121. Books on any area of undergraduate mathematics, mathematics education, or computer science are appropriate for consideration in this column. Reviews may be typed or neatly printed, and should be about two pages in length. The editor may undertake minor editing of a review, but only in connection with matters unrelated to the essential content or opinion of the review.

Sherman Stein. *Archimedes: What Did He Do Besides Cry Eureka?* The Mathematical Association of America, Washington, D.C., 1999.

While Archimedes is commonly hailed as one of the greatest mathematicians of all time, most of us would be at a loss to explain why. It is easy to be unimpressed by the area of a circle, volume and surface area of a sphere, or π estimated to two decimal place accuracy. His (dubious) "burning mirrors", catapults, and other mechanical inventions seem more likely catalysts for future fame. This slim volume captures the brilliance and innovation of Archimedes' arguments in the context of the mathematics known to his contemporaries and the reader is left full of wonder at the beauty of his achievements.

The language of mathematics in the third century B.C. was geometry (length, area, volume, lines, curves), and proofs, while accompanied by useful drawings, were narrative because algebraic notation had not been developed yet. Euclid's *Elements* and works of Aristaeus and Apollonius were Archimedes' sources for plane geometry, properties of conic sections, and of conoids. The Greek standard of proof was rigorous, and Archimedes' manuscripts typically presented a large number of (at times seemingly unrelated) lemmas, followed by major results whose proofs depend on the lemmas and results from other sources. Numbers were not abstract; each number had a context (e.g., length, area, weight, number of units), and, for example, length and area could not be compared. However, ratios could be compared. Archimedes developed his "law of levers" as $\frac{w}{w'} = \frac{d'}{d}$ (we would write $wd = w'd'$) which allowed him to apply it to lengths, areas, or volumes, as well as weights. The Greeks did not have algebraic formulas, but through very ingenious methods could determine things like the sum of a geometric series, or the sums of the squares of integers.

To get a glimpse of how Archimedes did mathematics, it is valuable to look at a method of exploration which he developed, which led him to many of his results. In a letter to Eratosthenes, Archimedes writes:

> Some things, which first became clear to me by the mechanical method, were afterwards proved geometrically, because their investigation by that method does not furnish an actual demonstration. It is easier to supply the proof when we have previously acquired, by the method, some knowledge of the question than it is to find it without any previous knowledge (p. 33).

Results found using this method are accurate, but do not fit the Greek notion of "proof," a situation somewhat reminiscent of both our Calculus students' "proof by calculator" and the "computer proof" of the Four Color Theorem. An example, from his *Quadrature of the Parabola,* will illustrate the process.

A parabola is cut by a chord AB (see Figure 1), C is the midpoint of the chord, segment AD is tangent to the parabola, and segments BD and CE are

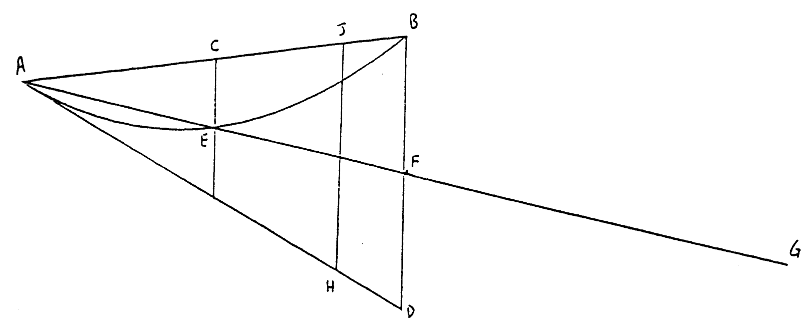

Figure 1: Proof by mechanical method

parallel to the axis of the parabola. Segment AE is extended so that AF and FG are the same length. He then shows that for any of the parallel segments cutting $\triangle ABD$ (e.g., HJ), the part of the segment within the parabola, placed at G, balances the whole segment where it is. He concludes, then, that the whole parabolic section, placed at G balances $\triangle ABD$ where it is. As the center of gravity of the triangle is one third of the way from F to A, then the area of the triangle must be three times the area of the parabolic section. Finally, he shows the area of $\triangle ABD$ is four times that of $\triangle ABE$; thus, the ratio of the area of the section of the parabola cut off by chord AB to the area of the inscribed $\triangle ABE$ is four-thirds.

The formal geometric proof follows a different line of reasoning. Starting with the parabola cut by segment AB (see Figure 2), with midpoint C, and segment EC parallel to the axis of the parabola, Archimedes inscribes $\triangle ABE$.

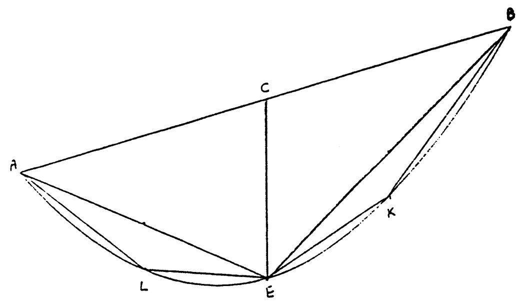

Figure 2: Geometric proof

Since both AE and EB cut off sections of the parabola, then $\triangle AEL$ and $\triangle EBK$ can be inscribed in a similar way. He shows from properties of the parabola that each of the smaller triangles has one-eighth the area of $\triangle ABE$. If the process is continued, the ratio of the sum of the areas of all the inscribed triangles to $\triangle ABE$ is a partial sum of a geometric series (ratio $\frac{1}{4}$) which approaches $\frac{4}{3}$ as the process continues. As this area also approaches the area of the parabolic section, then he concludes that the ratio of the area of this section cut from the parabola by segment AB to the area of $\triangle ABE$ is four-thirds.

The author's express purpose for this book is to make "Archimedes' most mathematically significant [in the author's opinion] discoveries accessible to the busy people of the mathematical community, ... anyone who recognizes the equation of a parabola (p. ix)." In this worthy goal I believe he succeeds admirably. The book is quite readable, interesting, and self-contained. By laying out the foundational mathematics that Archimedes would have known or developed, then proceeding through proofs of several major results, the author shows the diversity and ingenuity of Archimedes' work. By contrast with the classic English translations of Archimedes' work ([H, D]), he dispenses with all but the main argument of each proof, describing them using modern notation. In this way the interested reader could jump in at any point without being completely lost. The chosen results span classic planar area, volume, surface area, but also include amazing floating body problems and an estimate for π which uses inscribed and circumscribed 96-gons. There seem to be just enough examples to show the variety of style and argument without getting repetitive. Of course, some of the necessary mathematics is beyond what may be known to a wide audience; in such cases, the author deftly teaches the material first, using examples and exercises, then shows how Archimedes applied it.

As a quick historical reference, this book should be in each mathematics teacher's library; and it should be in every undergraduate library (along with [H] and [D]) in the hope that students will stumble across it often. I would highly recommend it as a companion for a History of Math course. I will end with a warning: a reader might wish to only dabble in this small book, but the writer's efficient and comfortable style, and ability to bring ancient mathematics to life will make it difficult to put down.

References

1. E. J. Dijksterhuis, *Archimedes,* Princeton University Press, Princeton, 1987.
2. T. L. Heath, *The Works of Archimedes,* Dover Publications, New York, 1912.

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