

**REVIEWS****Edited by Joseph B. Dence**

Reviews should be sent to Joseph B. Dence, Department of Chemistry, University of Missouri, 8001 Natural Bridge Rd., St. Louis, MO, 63121. Books on any area of undergraduate mathematics, mathematics education, or computer science are appropriate for consideration in this column. Reviews may be typed or neatly printed, and should be about two pages in length. The editor may undertake minor editing of a review, but only in connection with matters unrelated to the essential content or opinion of the review.

J. B. Dence and T. P. Dence, *Elements of the Theory of Numbers*. Academic Press, San Diego, 1999, pp. 517.

Number theory is such an enjoyable subject that many are inspired to write number theory texts. As I scan my private library, I find that the book under review is the twenty-third elementary number theory book in my collection. This means that while writing such books may be great fun for the authors, it may be difficult to decide whether there are any real differences amongst the books.

So let me begin by noting that I really like this book. First of all, it covers the main topics that are essential in a basic course, and it does so with flair. Each chapter ends with a collection of “Research Problems.” Of these, the authors assert: “These are intended as a source of further stimulation for the more motivated students.”

For example, at the end of Chapter 1 (on primes), the first research problem reads:

“It is plausible that the density of twin primes might decrease as one moves farther out on the positive real axis. We already know that this behavior is true of the primes themselves. Investigate the behavior of the ratio of the density of twin primes to the density of the primes.”

This is a great problem. The authors point the students toward an investigation which could plausibly lead to thinking about sieves and Brun’s Theorem.

It is possible when covering basic number theory to stick so closely to the theorem-proof format that some of the initial excitement ebbs. The authors have worked hard to avoid this pitfall. In the chapter on polynomial congruences they include a wonderfully elementary proof of the Lucas-Lehmer test (due to J. W. Bruce). A short introduction to cryptography and the RSA method are included in the congruences chapter.

There are many, many exercises concluding most sections of each chapter. Here the authors have been careful to include a wide range of problems extending from

very easy to very hard. As they say in the Preface: “A great many are proof-oriented; several others require the writing of computer programs to carry out computations. Problems appear after every section in the book: we recommend that lots of them be assigned as homework, especially since, for many students, this course may be their first in-depth experience with the mechanics of formal proof.”

Amen!

Now we come to Part 2 of the book: “Special Topics.” In this part, the authors provide four topics: Representation problems (centering around Fermat’s last theorem), number fields, partitions and recurrence relations. In my admittedly biased opinion, they could not have done better. The pace is just right, and the topics are wonderfully appealing. Given that my favorite topic in mathematics is the theory of partitions, I was especially ready to be critical of Chapter 10, the one on partitions. To my delight, I can only say that it is very well done. It starts slowly and carefully, with just the right mixture of motivation and proof. It concludes with Fabian Franklin’s magnificent elementary proof of Euler’s pentagonal number theorem.

In conclusion, let me warmly recommend this book. It is an excellent text for a first course on number theory.

REVIEWED BY

George E. Andrews  
Department of Mathematics  
The Pennsylvania State University  
University Park, PA 16802