PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 (email: ccooper@cmsuvmb.cmsu.edu).

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk (*) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than October 15, 1993, although solutions received after that date will also be considered until the time when a solution is published.

53. *Proposed by Russell Euler, Northwest Missouri State University, Maryville, Missouri.*

Prove analytically that

$$\sqrt[3]{19+9\sqrt{6}}+\sqrt[3]{19-9\sqrt{6}}$$

is an integer.

54. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

Let n be any positive integer greater than one, and let

$$x_n - x_{n-1} = \sum_{k=1}^{n}(-1)^{k+1}\binom{n}{k}\sum_{m=1}^{k}\frac{1}{m}, \qquad x_1 = 1.$$

Prove that

$$\sum_{k=1}^{n} k\left(kx_k^{-1}\right)^k < (n+1)\,!\ .$$

55. *Proposed by Stanley Rabinowitz, Westford, Massachusetts.*

Let F_n and L_n denote the nth Fibonacci and Lucas numbers, respectively. Find a polynomial $f(x, y)$ with constant coefficients such that $f(F_n, L_n)$ is identically zero for all positive integers n or prove that no such polynomial exists.

56. *Proposed by Curtis Cooper and Robert E. Kennedy, Central Missouri State University, Warrensburg, Missouri.*

Let

$$A_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and

$$A_{n+1} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & 1 & 0 & \cdots & 0 \\ 1 & 0 & & & & & \\ 0 & 1 & & & & & \\ 0 & 0 & & & A_n & & \\ \vdots & \vdots & & & & & \\ 0 & 0 & & & & & \end{pmatrix}$$

for $n \geq 1$. Find $\det A_{1993}$.