PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 (email: ccooper@cmsuvmb.cmsu.edu).

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk ($*$) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than September 30, 1992, although solutions received after that date will also be considered until the time when a solution is published.

41. *Proposed by Russell Euler, Northwest Missouri State University, Maryville, Missouri.*

Define the sequence $\{L_n\}_{n=1}^{\infty}$ by $L_1 = a$, $L_2 = b$ and $L_{n+2} = L_{n+1} + L_n$ for $n \geq 1$, where a and b are arbitrary integers. If $a = 1$ and $b = 2$, then $L_i = i$ for three consecutive integers i.

(i) Are there other values of a and b with this property?
(ii) Are there values of a and b such that $L_i = i$ for four consecutive values of i?
(iii)$*$ What happens if the 'consecutive' restriction is removed in (i) and (ii)?

42. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

Find the general solution to the differential equation

$$\sum_{k=0}^{1990} (1990)^{k+1} y^{(k)} = 0 \ ,$$

where $y^{(k)}$ represents the kth derivative of y.

43. *Proposed by Curtis Cooper and Robert E. Kennedy, Central Missouri State University, Warrensburg, Missouri.*

Let $n \geq 3$ be a positive integer and $m = \frac{n(n+1)}{2}$. Evaluate

$$\sum_{\substack{1 \leq a,b,c \leq n \\ a,b,c \text{ all distinct}}} \frac{abc}{m(m-a)(m-a-b)} .$$

44. *Proposed by Kandasamy Muthuvel, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.*

It is well known that the set of all real numbers R is a field under ordinary addition and multiplication, the set of all positive real numbers R^+ is a subgroup of the multiplicative group R^* (the set of all nonzero real numbers), $r^2 \in R^+$ for all $r \in R^*$, the characteristic of R is 0, and $R^+ - R^+ = R$.

Prove the generalized result, "Let G be a subgroup of the multiplicative group F^* of a field F such that $f^2 \in G$ for all f in F^*. If the characteristic of F is not equal to 2, 3, and 5, then $G - G = F$."

Remark. The above result is not true if the characteristic of F is equal to 2, 3, or 5.