PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to the problem editor, whose address appears on the inside back cover. An asterisk (∗) after a number indicates a problem submitted without a solution.

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than September 30, 1990, although solutions received after that date will also be considered until the time when a solution is published.

17. *Proposed by Stanley Rabinowitz, Westford, Massachusetts.*

Let $ABCD$ be an isosceles tetrahedron. Denote the dihedral angle at edge AB by $\angle AB$. Prove that

$$\frac{AB}{\sin\angle AB} = \frac{AC}{\sin\angle AC} = \frac{AD}{\sin\angle AD} \ .$$

18. *Proposed by Jayanthi Ganapathy, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.*

Let f be differentiable on an interval of the form (M,∞). Suppose

$$\lim_{x\to\infty}\big(f(x)+xf'(x)\big)=\alpha\ ,$$

where α is finite. Prove

$$\lim_{x\to\infty} f(x) \qquad \text{and} \qquad \lim_{x\to\infty} f'(x)$$

exist and evaluate these limits.

19. *Proposed by Russell Euler, Northwest Missouri State University, Maryville, Missouri.*

Prove that triangle ABC is isosceles if and only if

$$\tan(A-B) \;+\; \tan(B-C) \;=\; \tan(A-C)\ .$$

20. *Proposed by Troy Hicks, University of Missouri-Rolla, Rolla, Missouri.*

The useful lemma given below is part of the folklore of topology. A non-elementary proof was given in [1] and the lemma was used to prove the following theorem. A Tychonoff space X is separable and metrizable if and only if every compatible uniformity on X contains a compatible totally bounded uniformity. Then applications of this result were given to group actions, compactifications, and proximity classes of uniformities. It seems appropriate to have an elementary proof in the literature. Supply one.

Lemma. Let $(X,\mathcal{U})$ be a uniform space. Then for any closed subset A of X and any point $z \in X \setminus A$, there is a uniformly continuous function $f:(X,\mathcal{U}) \to [0,1]$ such that $f(z)=0$ and $f(A)=1$.

Reference

1. P. L. Sharma and T. L. Hicks, "Uniformities on Separable Metrizable Spaces," *Math. Japonica*, 25, No. 6 (1980), 677–680.