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Two-point homogeneous quandles with cardinality of prime power

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ABSTRACT. The main result of this paper classifies two-point homogeneous quandles with cardinality of prime power. More precisely, such quandles are isomorphic to Alexander quandles defined by primitive roots over finite fields. This result classifies all two-point homogeneous finite quandles, by combining with the recent result of Vendramin.

1. Introduction

Quandles were introduced to study knots by Joyce ([7]). Let X be a set, and assume that there exists a map $s : X \to \mathrm{Map}(X, X) : x \mapsto s_x$. Here $\mathrm{Map}(X, X)$ denotes the set of all maps from X to X. Then a pair (X, s) is called a *quandle* if s satisfies the conditions corresponding to Reidemeister moves of classical knots (see Definition 2.1). In knot theory, quandles provide a complete algebraic framework, and provide several invariants of knots (see [2, 4] and references therein). Among others, Carter, Jelsovsky, Kamada, Langford and Saito ([3]) gave strong invariants, called quandle cocycle invariants, defined by quandle cocycles. They apply it to prove the non-invertibility of the 2-twist spun trefoil by using a 3-cocycle of the dihedral quandle R_3 with cardinality 3. In [9], Mochizuki gave a systematic method for calculating some quandle cocycles of dihedral quandles. In addition, Nosaka ([10]) applied the method of Mochizuki, and provided quandle cocycles of some Alexander quandles. However, in general, calculation of quandle cocycles is difficult, even in the case of low cardinality. Therefore, it is of importance to study special classes of quandles, whose quandle structures are helpful to induce algebraic properties of quandle cohomologies.

From this point of view, we study two-point homogeneous quandles and quandles of cyclic type. A quandle (X, s) with $\#X \geq 3$ is said to be *two-point homogeneous* if for any $(x_1, x_2), (y_1, y_2) \in X \times X$ satisfying $x_1 \neq x_2$ and $y_1 \neq y_2$, there exists an inner automorphism f of (X, s) such that $(f(x_1), f(x_2)) = (y_1, y_2)$. On the other hand, a quandle (X, s) with finite

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cardinality $n \geq 3$ is said to be of *cyclic type* if s_x are cyclic permutations of order $n-1$ for any $x \in X$. These quandles have been studied in [5, 6, 8, 11, 12, 13]. In particular, all two-point homogeneous quandles with prime cardinality were classified in [12]. In addition, [12] proved that all quandles of cyclic type are two-point homogeneous, and gave the following conjecture. Note that this conjecture is true when the cardinalities are prime numbers ([12]).

CONJECTURE 1.1. *All two-point homogeneous quandles with finite cardinality are of cyclic type.*

Recently, Vendramin ([13]) proved that the cardinalities of two-point homogeneous quandles must be prime power.

In this paper, we classify two-point homogeneous quandles (X, s) with cardinality of prime power. A key of the proof is that (X, s) are simple crossed sets (Proposition 3.7). By using a classification of simple crossed sets with cardinality of prime power in [1], we have the following.

MAIN THEOREM (Theorem 4.3). Let q be a prime power and X be a quandle with cardinality q. Then the following conditions are mutually equivalent:
(1) (X, s) is two-point homogeneous,
(2) (X, s) is isomorphic to the Alexander quandle $(\mathbf{F}_q, \omega)$, where ω is a primitive root over the finite field $\mathbf{F}_q$,
(3) (X, s) is of cyclic type.

This result is an extension of the result of [12]. Moreover, by applying the result of Vendramin ([13]), we obtain a classification of all two-point homogeneous finite quandles (Corollary 4.5). In particular, Corollary 4.5 shows that Conjecture 1.1 is true.

This paper is organized as follows. In Section 2, we recall some notions of quandles. In Section 3, some properties of two-point homogeneous quandles and quandles of cyclic type are summarized. In Section 4, we prove the main result.

2. Preliminaries for quandles

In this section we recall some notions on quandles.

DEFINITION 2.1. Let X be a set, and assume that there exists a map $s : X \to \mathrm{Map}(X, X) : x \mapsto s_x$. Then a pair (X, s) is called a *quandle* if s satisfies the following conditions:

(S1) $\forall x \in X,\ s_x(x) = x$,
(S2) $\forall x \in X,\ s_x$ is bijective, and
(S3) $\forall x, y \in X,\ s_x \circ s_y = s_{s_x(y)} \circ s_x$.

We denote by $\#X$ the cardinality of X.

Example 2.2. The following (X, s) are quandles:
(1) Let X be any set and $s_x := \mathrm{id}_X$ for every $x \in X$. Then the pair (X, s) is called the *trivial quandle*.
(2) Let $X := \{1, \dots, n\}$ and $s_i(j) := 2i - j \pmod{n}$ for any $i, j \in X$. Then the pair (X, s) is called the *dihedral quandle* with cardinality n.
(3) Let q be a prime power and $\mathbf{F}_q$ be the finite field of order q. If $\omega \in \mathbf{F}_q$ ($\omega \neq 0, 1$), then the pair $(\mathbf{F}_q, \omega)$ with the following operator is called the *Alexander quandle* of order q:

$$s_x(y) := \omega y + (1 - \omega)x. \tag{2.1}$$

Definition 2.3. Let (X, s^X) and (Y, s^Y) be quandles, and $f : X \to Y$ be a map.
(1) f is called a *homomorphism* if for every $x \in X$, $f \circ s_x^X = s_{f(x)}^Y \circ f$ holds.
(2) f is called an *isomorphism* if f is a bijective homomorphism.

An isomorphism from a quandle (X, s) onto itself is called an *automorphism*. The set of automorphisms of (X, s) forms a group, which is called the *automorphism group* and denoted by $\mathrm{Aut}(X, s)$.

Note that s_x $(x \in X)$ is an automorphism of (X, s). The subgroup of $\mathrm{Aut}(X, s)$ generated by $\{s_x \mid x \in X\}$ is called the *inner automorphism group* of (X, s) and denoted by $\mathrm{Inn}(X, s)$. A quandle (X, s) is said to be *connected* if $\mathrm{Inn}(X, s)$ acts on (X, s) transitively. Let $\mathrm{Inn}(X, s)_x$ be the stabilizer subgroup of $\mathrm{Inn}(X, s)$ at $x \in X$.

Definition 2.4. A quandle (X, s) is called a *crossed set* if $s_x(y) = y$ whenever $s_y(x) = x$.

Definition 2.5. Let (X, s^X) and (Y, s^Y) be finite quandles. A surjective homomorphism $f : X \to Y$ is called *trivial* if $\#Y$ is equal to either $\#X$ or 1.

Definition 2.6. A quandle (X, s) is *simple* if it is not a trivial quandle and any surjective homomorphism $f : X \to Y$ is trivial.

On classification of simple crossed sets with cardinality of prime power, Andruskiewitsch and Grana ([1]) give the following theorem.

Theorem 2.7 (Corollary 3.10 in [1]). *Let p be a prime number and $l \in \mathbf{N}$. If a quandle (X, s) with cardinality p^l is a simple crossed set, then (X, s) isomorphic to an Alexander quandle $(\mathbf{F}_{p^l}, \omega)$, where ω generates $\mathbf{F}_{p^l}$ over $\mathbf{F}_p$.*

Recall that $\omega \in \mathbf{F}_{p^l}$ is said to *generate* $\mathbf{F}_{p^l}$ if $\{1, \omega, \dots, \omega^{l-1}\}$ is a basis of $\mathbf{F}_{p^l}$ over $\mathbf{F}_p$.

3. Two-point homogeneous quandles and quandles of cyclic type

In this section, we recall the definitions and some properties of two-point homogeneous quandles and quandles of cyclic type, which are given in [12].

DEFINITION 3.1. A finite quandle (X, s) with $\#X = n \geq 3$ is said to be of *cyclic type* if for every $x \in X$, s_x acts on $X - \{x\}$ as a cyclic permutation of order $n - 1$.

This notion is closely related to the notion of two-point homogeneous quandles.

DEFINITION 3.2. A quandle (X, s) with $\#X \geq 3$ is said to be *two-point homogeneous* if for any $(x_1, x_2), (y_1, y_2) \in X \times X$ satisfying $x_1 \neq x_2$ and $y_1 \neq y_2$, there exists $f \in \mathrm{Inn}(X, s)$ such that $(f(x_1), f(x_2)) = (y_1, y_2)$.

Note that quandles of cyclic type have finite cardinalities. On the other hand, two-point homogeneous quandles are not necessarily finite.

PROPOSITION 3.3 ([12]). *Every quandle of cyclic type is two-point homogeneous.*

The following are characterizations of quandles of cyclic type and two-point homogeneous quandles.

PROPOSITION 3.4 ([12]). *Let (X, s) be a finite quandle with $\#X \geq 3$. Then the following conditions are equivalent:*
(1) *(X, s) is of cyclic type,*
(2) *(X, s) is connected, and there exists $x \in X$ such that s_x acts on $X - \{x\}$ as a cyclic permutation of order $\#X - 1$.*

PROPOSITION 3.5 ([12]). *Let (X, s) be a quandle with $\#X \geq 3$. Then the following conditions are equivalent:*
(1) *(X, s) is two-point homogeneous,*
(2) *for every $x \in X$, the action of $\mathrm{Inn}(X, s)_x$ on $X - \{x\}$ is transitive,*
(3) *(X, s) is connected, and there exists $x \in X$ such that the action of $\mathrm{Inn}(X, s)_x$ on $X - \{x\}$ is transitive.*

The following lemma will be used to prove that every two-point homogeneous finite quandles is a simple crossed set.

LEMMA 3.6. *Let (X, s^X) and (Y, s^Y) be quandles, and $f : X \to Y$ be a homomorphism. Then for any $g \in \mathrm{Inn}(X, s^X)$, there exists $h \in \mathrm{Inn}(Y, s^Y)$*

satisfying

$$f \circ g = h \circ f. \tag{3.1}$$

Proof. The inner automorphism g can be written as

$$g = \prod_{i=1}^{k} (s_{x_i}^X)^{\varepsilon_i} \tag{3.2}$$

for some $x_i \in X$, $\varepsilon_i \in \mathbf{Z}$. By the assumption on f, we have

$$f \circ (s_x^X)^{\varepsilon} = (s_{f(x)}^Y)^{\varepsilon} \circ f \tag{3.3}$$

for any $\varepsilon \in \mathbf{Z}$. It follows that

$$h := \left(\prod_{i=1}^{k} (s_{f(x_i)}^Y)^{\varepsilon_i} \right) \in \mathrm{Inn}(Y, s^Y) \tag{3.4}$$

satisfies the required condition. □

This lemma gives the following proposition for two-point homogeneous quandles.

Proposition 3.7. *Every two-point homogeneous quandle with finite cardinality is a simple crossed set.*

Proof. Let (X, s) be a two-point homogeneous quandle with finite cardinality. We prove that (X, s) is crossed. Suppose that

$$s_x(y) = y \qquad (x, y \in X). \tag{3.5}$$

It is enough to consider the case $x \neq y$. By the definition of two-point homogeneous quandles, there exists $f \in \mathrm{Inn}(X, s)$ such that

$$f(x) = y, \qquad f(y) = x. \tag{3.6}$$

Hence one has

$$s_y(x) = s_{f(x)} \circ f(y) = f \circ s_x(y) = f(y) = x. \tag{3.7}$$

Next, we prove that (X, s) is simple. Take any quandle (Y, s') and any surjective homomorphism $f : X \to Y$. Assume that f is not injective, and we prove $\# Y = 1$. By the assumption, there exist $x, y \in X$ $(x \neq y)$ with $f(x) = f(y)$. Take any $z \in X - \{x, y\}$. Then there exists $g \in \mathrm{Inn}(X, s)$ satisfying

$$g(y) = z, \qquad g(x) = x. \tag{3.8}$$

Lemma 3.6 yields that there exists $h \in \mathrm{Inn}(Y, s')$ satisfying

$$f \circ g = h \circ f. \tag{3.9}$$

Hence we have

$$f(z) = f \circ g(y) = h \circ f(y) = h \circ f(x) = f \circ g(x) = f(x). \tag{3.10}$$

This shows $\# f(X) = 1$. Since f is surjective, we have $\# Y = 1$. □

4. Main theorem

Let $q \geq 3$ be a prime power. In this section, we classify two-point homogeneous quandles with cardinality q. We also show that all two-point homogeneous quandles with cardinality q are of cyclic type. Note that two-point homogeneous quandles with prime cardinality are already classified in [12]. The main theorem is an extension of the result of [12].

4.1. The inner automorphism group of $(\mathbf{F}_q, \omega)$**.** In this subsection, we determine the inner automorphism group of the Alexander quandle $(\mathbf{F}_q, \omega)$ of order q with $\omega \in \mathbf{F}_q - \{0, 1\}$. Recall that the map $s : \mathbf{F}_q \to \mathrm{Map}(\mathbf{F}_q, \mathbf{F}_q)$ is given by assigning

$$s_x : \mathbf{F}_q \to \mathbf{F}_q : y \mapsto \omega y + (1 - \omega)x$$

to each $x \in \mathbf{F}_q$.

For each $x \in \mathbf{F}_q$, we define a map ψ_x as follows:

$$\psi_x : \mathbf{F}_q \to \mathbf{F}_q : y \mapsto y + x. \tag{4.1}$$

PROPOSITION 4.1. *The inner automorphism group of* $(\mathbf{F}_q, \omega)$ *satisfies*

$$\mathrm{Inn}(\mathbf{F}_q, \omega) = \{(s_x)^k \mid x \in \mathbf{F}_q, k \in \mathbf{Z}\} \cup \{\psi_y \mid y \in \mathbf{F}_q\}. \tag{4.2}$$

PROOF. Let us denote by G the right side of (4.2). First of all, we prove $G \subset \mathrm{Inn}(\mathbf{F}_q, \omega)$. It is clear that $(s_x)^k \in \mathrm{Inn}(\mathbf{F}_q, \omega)$ for any $k \in \mathbf{Z}$ and $x \in \mathbf{F}_q$. Hence we have only to prove $\psi_x \in \mathrm{Inn}(\mathbf{F}_q, \omega)$ for each $x \in \mathbf{F}_q$. Note that there exists the inverse $(1 - \omega)^{-1}$. For $x, y \in \mathbf{F}_q$, one has

$$\begin{aligned} (s_{x(1-\omega)^{-1}})(s_0)^{q-2}(y) &= s_{x(1-\omega)^{-1}}(\omega^{q-2} y) \\ &= \omega^{q-1} y + x(1-\omega)(1-\omega)^{-1} \\ &= y + x \\ &= \psi_x(y). \end{aligned} \tag{4.3}$$

This shows $\psi_x = (s_{x(1-\omega)^{-1}})(s_0)^{q-2} \in \mathrm{Inn}(\mathbf{F}_q, \omega)$, and hence $G \subset \mathrm{Inn}(\mathbf{F}_q, \omega)$.

Next, we prove $G \supset \mathrm{Inn}(\mathbf{F}_q, \omega)$. Since G contains generators $\{s_x \mid x \in \mathbf{F}_q\}$ of $\mathrm{Inn}(\mathbf{F}_q, \omega)$, it is enough to prove that G is a group. For any $a, b \in \mathbf{Z}$, we show

$$(s_x)^a (s_y)^b, (s_x)^a \psi_y, \psi_y (s_x)^a \in G. \tag{4.4}$$

Case (1): We show $(s_x)^a (s_y)^b \in G$. For any $z \in \mathbf{F}_q$, we have

$$\begin{aligned}(s_x)^a (s_y)^b (z) &= (s_x)^a (\omega^b z + (1-\omega^b) y) \\ &= \omega^a (\omega^b z + (1-\omega^b) y) + (1-\omega^a) x \\ &= \omega^{(a+b)} z + (1-\omega^b)\omega^a y + (1-\omega^a) x. \end{aligned} \tag{4.5}$$

Let $\alpha := (1-\omega^b)\omega^a y + (1-\omega^a) x$.

Subcase (1)-i: We consider the case of $1 - \omega^{a+b} = 0$. By (4.5), we have

$$(s_x)^a (s_y)^b (z) = z + \alpha = \psi_\alpha (z) \tag{4.6}$$

for any $z \in \mathbf{F}_q$. This yields that $(s_x)^a (s_y)^b = \psi_\alpha \in G$.

Subcase (1)-ii: We deal with the case of $1 - \omega^{a+b} \neq 0$. In this case, there exists the inverse $(1-\omega^{a+b})^{-1}$. Therefore (4.5) yields

$$\begin{aligned}(s_x)^a (s_y)^b (z) &= \omega^{(a+b)} z + (1-\omega^{a+b})(1-\omega^{a+b})^{-1} \alpha \\ &= (s_{(1-\omega^{a+b})^{-1}\alpha})^{(a+b)} (z). \end{aligned} \tag{4.7}$$

This yields $(s_x)^a (s_y)^b = (s_{(1-\omega^{a+b})^{-1}\alpha})^{(a+b)} \in G$.

Case (2): We show $(s_x)^a \psi_y \in G$. Let $z \in \mathbf{F}_q$.

Subcase (2)-i: If $1 - \omega^a = 0$, then one has

$$(s_x)^a (z) = \omega^a z + (1-\omega^a) x = z. \tag{4.8}$$

This yields

$$(s_x)^a \psi_y = \psi_y \in G. \tag{4.9}$$

Subcase (2)-ii: Suppose that $1 - \omega^a \neq 0$. Note that there exists the inverse $(1-\omega^a)^{-1}$. For any $z \in \mathbf{F}_q$, we have

$$\begin{aligned}(s_x)^a \psi_y (z) &= \omega^a (z + y) + (1-\omega^a) x \\ &= \omega^a z + (1-\omega^a)(1-\omega^a)^{-1} \omega^a y + (1-\omega^a) x \\ &= (s_{(1-\omega^a)^{-1}\omega^a y + x})^a (z). \end{aligned} \tag{4.10}$$

This yields $(s_x)^a \psi_y = (s_{(1-\omega^a)^{-1}\omega^a y + x})^a \in G$.

Case (3): We show $\psi_y(s_x)^a \in G$. For any $z \in \mathbf{F}_q$, we have

$$\psi_y(s_x)^a(z) = \omega^a z + (1-\omega^a)x + y. \tag{4.11}$$

By considering two cases as in Case (2), we have $\psi_y(s_x)^a \in G$. □

Let us concern the stabilizer subgroup of $\mathrm{Inn}(\mathbf{F}_q, \omega)$ at 0,

$$\mathrm{Inn}(\mathbf{F}_q, \omega)_0 := \{f \in \mathrm{Inn}(\mathbf{F}_q, \omega) \mid f(0) = 0\}. \tag{4.12}$$

COROLLARY 4.2. *The group* $\mathrm{Inn}(\mathbf{F}_q, \omega)_0$ *is generated by* s_0.

PROOF. Since $s_0 \in \mathrm{Inn}(\mathbf{F}_q, \omega)_0$, we have

$$\mathrm{Inn}(\mathbf{F}_q, \omega)_0 \supset \langle s_0 \rangle. \tag{4.13}$$

Hence, we have only to prove

$$\mathrm{Inn}(\mathbf{F}_q, \omega)_0 \subset \langle s_0 \rangle. \tag{4.14}$$

Take any $g \in \mathrm{Inn}(\mathbf{F}_q, \omega)_0$. In view of Proposition 4.1, we have only to consider the following two cases.

Case (1): We deal with the case $g = (s_x)^a$ for $x \in \mathbf{F}_q$ and $a \in \mathbf{Z}$. Since $(s_x)^a(0) = 0$, one has

$$0 = (s_x)^a(0) = (1-\omega^a)x. \tag{4.15}$$

Since $\mathbf{F}_q$ is a field, one has $1-\omega^a = 0$ or $x = 0$. If $1-\omega^a = 0$, then $(s_x)^a(y) = (1-\omega^a)x + \omega^a y$ yields

$$g = (s_x)^a = \mathrm{id} \in \langle s_0 \rangle. \tag{4.16}$$

If $x = 0$, then we have

$$g = (s_0)^a \in \langle s_0 \rangle. \tag{4.17}$$

Case (2): We deal with the case $g = \psi_x$. Since $\psi_x(0) = 0$, we clear have $x = 0$ and $\psi_x = \mathrm{id} \in \mathrm{Inn}(\mathbf{F}_q, \omega)_0$. Therefore (4.14) holds. □

4.2. Proof of the main theorem. In this subsection, we prove the main theorem. Let q be a prime power and $\mathbf{F}_q$ be a finite field of order q. Recall that $\omega \in \mathbf{F}_q$ is called a *primitive root modulo* q if $\langle \omega \rangle := \{1, \omega, \ldots, \omega^{q-2}\} = \mathbf{F}_q - \{0\}$.

THEOREM 4.3. *Let q be a prime power and X be a quandle with cardinality q. Then the following conditions are mutually equivalent:*

(1) *(X, s) is two-point homogeneous,*

(2) *(X, s) is isomorphic to the Alexander quandle $(\mathbf{F}_q, \omega)$, where ω is a primitive root over the finite field $\mathbf{F}_q$,*

(3) *(X, s) is of cyclic type.*

Proof. First of all, we deal with (1) $\Rightarrow$ (2). By Proposition 3.7, X is simple and crossed. Thus, by Theorem 2.7, there exists $\omega \in \mathbf{F}_q$ satisfying $(X, s) \cong (\mathbf{F}_q, \omega)$. It is enough to show that ω is a primitive root modulo q. Note that $(\mathbf{F}_q, \omega)$ is two-point homogeneous. Hence, by Proposition 3.5, $\mathrm{Inn}(\mathbf{F}_q, \omega)_0$ acts on $\mathbf{F}_q - \{0\}$ transitively. Thus Corollary 4.2 yields

$$\mathbf{F}_q - \{0\} = \mathrm{Inn}(\mathbf{F}_q, \omega)_0 \cdot \{1\} = \langle s_0 \rangle \cdot \{1\} = \langle \omega \rangle. \tag{4.18}$$

Hence ω is a primitive root modulo q.

Next, we prove (2) $\Rightarrow$ (3). Note that $(\mathbf{F}_q, \omega)$ is connected from Proposition 4.1. Hence it is enough to prove that s_0 is a cyclic permutation of order $q-1$ from Proposition 3.4. One knows $(s_0)^t(x) = \omega^t x$ for any $x \in \mathbf{F}_q - \{0\}$ and $t \in \mathbf{Z}$. Since ω is a primitive root, we have

$$\langle s_0 \rangle \cdot \{1\} = \{1, \omega, \omega^2, \ldots, \omega^{q-2}\} = \mathbf{F}_q - \{0\}. \tag{4.19}$$

This means that s_0 is a cyclic permutation of order $q-1$.

The implication (3) $\Rightarrow$ (1) follows from Proposition 3.3. □

This theorem yields the existence of two-point homogeneous quandles with cardinality of prime power.

Corollary 4.4. *For any prime power $q \geq 3$, there exists a quandle of cyclic type, and hence a two-point homogeneous quandle, with cardinality q.*

In addition, by combining the result of [13] with Theorem 4.3, we obtain the following corollary.

Corollary 4.5. *All two-point homogeneous quandles with finite cardinality are isomorphic to Alexander quandles defined by primitive roots over finite fields. In particular, they are of cyclic type.*

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