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Real hypersurfaces with harmonic Weyl tensor of a complex space form

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Abstract. We study real hypersurfaces of a complex space form $M_n(c)$. The purpose is to give another characterization of pseudo-Einstein hypersurfaces and then to prove that there are no real hypersurfaces with harmonic Weyl tensor of $M_n(c)$, $c \neq 0$, $n \geq 3$.

Introduction

A complex n-dimensional Kaehler manifold of constant holomorphic sectional curvature c is called a *complex space form*, which is denoted by $M_n(c)$. A complete and simply connected complex space form is a complex projective space P_nC, a complex Euclidean space C_n or a complex hyperbolic space H_nC, according as $c > 0$, $c = 0$ or $c < 0$. The induced almost contact metric structure of a real hypersurface of $M_n(c)$ is denoted by (ϕ, ξ, η, g).

Now, there exist many studies of real hypersurfaces of a complex space form. One of the first researches is the classification of homogeneous real hypersurfaces of a complex projective space P_nC by Takagi [12]. Some real hypersurfaces of a complex space form $M_n(c)$, $c \neq 0$, are characterized under the conditions for the shape operator (or principal curvatures) and one of the structure tensors. In particular, a real hypersurface M of $M_n(c)$, $c \neq 0$, is said to be *pseudo-Einstein* if the Ricci tensor S' satisfies

$$S' = ag + b\eta \otimes \eta\,,$$

where a and b are some functions on M. The structure of pseudo-Einstein hypersurfaces is investigated by Cecil and Ryan [2], Kon [6] and Montiel [9].

On the other hand, some studies about the non-existence for real hypersurfaces under natural linear conditions which can be imposed on S' or $\nabla S'$ have been made by Kimura [5], Kon [6] and Montiel [9]. It is seen in [6] and [9] that there are no Einstein real hypersurfaces of $M_n(c)$, $c \neq 0$, $n \geq 3$. In particular, it is proved by Kim [4] and Kwon and one of the authors [7] that there are no real hypersurfaces with harmonic curvature of $M_n(c)$, $c \neq 0$, $n \geq 3$, on which the structure vector ξ is principal. Recently, the first author [3]

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proves that there are no hypersurfaces with parallel Ricci tensor on $M_n(c)$, $c \neq 0$, $n \geq 3$.

The purpose of this paper is to give another characterization of pseudo-Einstein real hypersurfaces of $M_n(c)$, $c \neq 0$, under a tensorial condition given only by the Riemannian curvature tensor R and S'. In § 1 the theory of real hypersurfaces of a complex space form is recalled and in § 2 the main theorem (Theorem 2.1) is proved. As an application of properties obtained in § 2 it is in § 3 proved that there are no real hypersurfaces with harmonic Weyl tensor of $M_n(c)$, $c \neq 0$, $n \geq 3$.

1. Preliminaries

Let M be a real hypersurface of a complex n dimensional complex space form $M_n(c)$, $c \neq 0$, $n \geq 3$, and let C be a unit normal vector field on a neighborhood of a point x in M. We denote by J the almost complex structure of $M_n(c)$. For a local vector field X on a neighborhood of x in M, the transformations of X and C under J can be represented by

$$JX = \phi X + \eta(X)\xi\,, \qquad JC = -\xi\,,$$

where ϕ defines a skew-symmetric transformation on the tangent bundle TM of M, while η and ξ denote a 1-form and a vector field on a neighborhood of x in M, respectively. Moreover it is seen that $g(\xi, X) = \eta(X)$, where g denotes the induced Riemannian metric on M. By properties of the almost complex structure J, a set (ϕ, ξ, η, g) of tensors satisfies then

$$\phi^2 = -I + \eta \otimes \xi\,, \qquad \phi\xi = 0\,, \qquad \eta(\phi X) = 0\,, \qquad \eta(\xi) = 1\,,$$

where I denotes the identity transformation. Accordingly, the set is the almost contact metric structure. Furthermore the covariant derivatives of the structure tensors are given by

$$(1.1) \qquad \nabla_X\phi(Y) = \eta(Y)AX - g(AX, Y)\xi\,, \qquad \nabla_X\xi = \phi AX\,,$$

where ∇ is the Riemannian connection of g and A denotes the shape operator with respect to C on M.

Since the ambient space is of constant holomorphic curvature c, the equations of Gauss and Codazzi are respectively given as follows:

$$(1.2) \qquad \begin{aligned} R(X, Y)Z = c\{&g(Y, Z)X - g(X, Z)Y \\ &+ g(\phi Y, Z)\phi X - g(\phi X, Z)\phi Y - 2g(\phi X, Y)\phi Z\}/4 \\ &+ g(AY, Z)AX - g(AX, Z)AY\,, \end{aligned}$$

$$(1.3) \qquad \nabla_X A(Y) - \nabla_Y A(X) = c\{\eta(X)\phi Y - \eta(Y)\phi X - 2g(\phi X, Y)\xi\}/4\,,$$

where R denotes the Riemannian curvature tensor of M and $\nabla_X A$ denotes the covariant derivative of the shape operator A with respect to X.

The Ricci tensor S' of M is a tensor ot type $(0, 2)$ given by $S'(X, Y) = \operatorname{tr}\{Z \to R(Z, X)Y\}$. But it may be also regarded as the tensor of type $(1, 1)$ and denoted by $S: TM \to TM$; it satisfies $S'(X, Y) = g(SX, Y)$. By the Gauss equation the Ricci tensor S is given by

$$S = c\{(2n+1)I - 3\eta \otimes \xi\}/4 + hA - A^2, \tag{1.4}$$

where h is the trace of the shape operator A. A real hypersurface M of $M_n(c)$ is said to be *pseudo-Einstein* if the Ricci tensor S satisfies

$$SX = aX + b\eta(X)\xi \tag{1.5}$$

for any vector field X tangent to M and some functions a and b on M.

2. Real cyclic Ryan hypersurfaces

This section is devoted to the investigation of real cyclic Ryan hypersurfaces of $M_n(c)$, $c \neq 0$, $n \geq 3$. Let N be a Riemannian manifold. For the Riemannian curvature tensor R_N, $R_N(X, Y)$ operates as a derivation on the algebra of tensor fields on N. For a tensor field F of type (r, s), $R_N(X, Y)\cdot F = \nabla_X\nabla_Y F - \nabla_Y\nabla_X F - \nabla_{[X,Y]}F$ is defined for any vector fields X and Y. The Riemannian manifold N is said to be *Ryan* (resp. *cyclic Ryan*), if it satisfies

$$(R_N(X, Y)\cdot S_N)(Z) = 0 \qquad (\text{resp. } \mathfrak{S}(R_N(X, Y)\cdot S_N)(Z) = 0) \tag{2.1}$$

for any vector fields, where S_N and $\mathfrak{S}$ denote the Ricci tensor and the cyclic sum with respect to X, Y and Z, respectively. The Ricci formula for the Ricci tensor gives rise to

$$(R_N(X, Y)\cdot S_N)(Z) = R_N(X, Y)(S_N Z) - S_N(R_N(X, Y)Z),$$

which implies that

$$\mathfrak{S}(R_N(X, Y)\cdot S_N)(Z) = \mathfrak{S}R_N(X, Y)(S_N Z), \tag{2.2}$$

because of the first Bianchi formula. By the definition of the Riemannian curvature tensor R_N, one gets

$$\begin{aligned} &\mathfrak{S}(R_N(X, Y)\cdot S_N)(Z) \\ &\quad = \mathfrak{S}\{\nabla_X(\nabla_Y S_N(Z) - \nabla_Z S_N(Y)) + \nabla_X S_N([Y, Z]) - \nabla_{[Y,Z]}S_N(X)\}. \end{aligned} \tag{2.3}$$

REMARK 2.1. It is seen in [2] and [9] that if a real hypersurface M of a complex space form $M_n(c)$, $c \neq 0$, is pseudo-Einstein, then the structure vector ξ is principal and M is cyclic Ryan, by means of (1.1), (1.3) and (2.3).

Now, let M be a real cyclic Ryan hypersurface of $M_n(c)$, $c \neq 0$, $n \geq 3$. By taking account of (1.2) and (1.4), the equation (2.2) is equivalent to

$$\mathfrak{S}[g(\phi X, HZ)\phi Y - g(\phi Y, HZ)\phi X + 2g(\phi X, Y)\phi HZ - 3\eta(Z)\{\eta(AX)AY - \eta(AY)AX\}] = 0\,, \tag{2.4}$$

because of $c \neq 0$, where we put $H = hA - A^2$. In order to give some formulas in the conventional form, given a linear transformation T a function T_m for an integer $m(\geq 1)$ is introduced as $T_m = \eta(T^m\xi)$. If we set $X = \xi$ in (2.4), then we obtain

$$\begin{aligned}\eta(H\phi Y)\phi Z - \eta(H\phi Z)\phi Y + 2g(\phi Y, Z)\phi H\xi &= 3[\eta(Z)\{A_1AY - \eta(AY)A\xi\} + \{\eta(AY)AZ - \eta(AZ)AY\} \\ &\quad + \eta(Y)\{\eta(AZ)A\xi - A_1AZ\}]\,,\end{aligned} \tag{2.5}$$

because the transformation H is symmetric. For any point x on M we can choose an orthonormal basis $\{E_1, \ldots, E_{2n-1}\}$ for the tangent space T_xM such that $E_{2a} = \phi E_a$, $a = 1, \ldots, n-1$ and $E_{2n-1} = \xi$. Putting $Z = E_i$ in the last equation, taking the inner product of this with ϕE_i and summing up with respect to i, we have

$$3AU = (2n-1)\phi H\xi\,, \qquad \eta(AU) = 0\,, \tag{2.6}$$

where we set $U = \nabla_\xi\xi$. We notice here that

$$\phi U = -A\xi + A_1\xi\,, \tag{2.7}$$

because of (1.1) and the property of the almost contact metric structure. This implies that the structure vector ξ is principal if and only if $\phi U = 0$, namely, the vector field U vanishes identically. By making use of (2.6) and (2.7), the relationship (2.5) is reduced to

$$\begin{aligned}(2n-1)[g(\phi Y, U)AZ - g(\phi Z, U)AY + \{\eta(Y)\eta(AZ) - \eta(Z)\eta(AY)\}A\xi] \\ = -g(U, AY)\phi Z + g(U, AZ)\phi Y + 2g(\phi Y, Z)AU\,.\end{aligned} \tag{2.8}$$

When we take the inner product of this equation and U, it follows from (2.6) that we get

$$(n-1)\{g(U, AZ)g(\phi Y, U) - g(AY, U)g(\phi Z, U)\} = g(AU, U)g(\phi Y, Z)\,,$$

and hence it turns out that

$$(n-1)\{g(AY, U)\phi U + g(\phi Y, U)AU\} = g(AU, U)\phi Y\,, \tag{2.9}$$

$$(n-2)g(AU, U)\phi U = 0\,. \tag{2.10}$$

Since it is easily seen that $\eta(U) = 0$ and $\eta(AU) = 0$ by the definition, (2.8) yields

$$(2.11) \qquad (2n-3)g(\phi Y, U)AU + g(U, AY)\phi U = g(AU, U)\phi Y,$$

by putting $Z = U$ in (2.8).

For a tube of radius r over a submanifold of a complex space form $M_n(c)$, cf. Cecil and Ryan [2] and Montiel [9]. A Montiel tube of a complex hyperbolic space is only defined here. Let H_1^{2n+1} be a $(2n+1)$-dimensional anti-de Sitter space in C^{n+1}, which is a Lorentz manifold of constant curvature $c/4$ (<0). Given the real hypersurface M of a complex hyperbolic space H_nC, one can construct a Lorentz hypersurface N of H_1^{2n+1} which is a principal S^1-bundle over M with time-like totally geodesic fibers and projection π: $N \to M$ in such a way that the diagram

$$\begin{tikzcd} N \arrow[r, "i'"] \arrow[d, "\pi"'] & H_1^{2n+1} \arrow[d, "\pi"] \\ M \arrow[r, "i"'] & H_nC \end{tikzcd}$$

is commutative (i, i' being the isometric immersions). In particular, let $N(t)$ be the Lorentz hypersurface of H_1^{2n+1} in C^{n+1} given by

$$-|z_0|^2 + \textstyle\sum_{j=1}^n |z_j|^2 = 1, \qquad |z_0 - z_1|^2 = t.$$

It is seen in [9] that $M_n^*(t) = \pi(N(t))$ is a pseudo-Einstein real hypersurface of H_nC. Then $M_n^* = M_n^*(1)$ is called a *Montiel-tube*.

THEOREM 2.1. *Let M be a complete real cyclic Ryan hypersurface of $M_n(c)$, $c \neq 0$, $n \geq 3$. Then M is congruent to one of the following spaces*:

(1) *In case $M_n(c) = P_nC$,*
  (a) *a geodesic hypersphere,*
  (b) *a tube of radius r over a totally geodesic P_kC $(1 \leq k \leq n-2)$, where $0 < r < \pi/2$ and $\cot^2 r = k/(n-k-1)$,*
  (c) *a tube over a complex quadric Q_{n-1}.*
(2) *In case $M_n(c) = H_nC$,*
  (a) *a geodesic hypersphere,*
  (b) *a tube of arbitrary radius over a complex hyperplane,*
  (c) *a Montiel-tube M_n^*.*

In order to prove Theorem 2.1, we begin with the following lemma.

LEMMA 2.2. *Let M be a real cyclic Ryan hypersurface of $M_n(c)$, $c \neq 0$, $n \geq 3$. Then the structure vector ξ is principal.*

PROOF. Let M_0 be a set consisting of points x at which the function $(A_2 - A_1{}^2)(x) \neq 0$. Then the structure vector at any point x in M_0 is not principal and therefore the vector U is not zero at x.

Suppose that M_0 is not empty. It suffices to show that a contradiction is drawn on the open set. By (2.9) and (2.10) we have

$$g(AU, U) = 0\,, \qquad g(AY, U)\phi U = -g(\phi Y, U)AU \quad \text{on} \quad M_0\,,$$

from which (2.11) is reduced to $(n-2)g(AU, Y)\phi U = 0$ for any vector field Y tangent to M_0 and hence we get $AU = 0$ on M_0, because of $\phi U \neq 0$ on M_0 and $n \geq 3$. The above condition and (2.6) enable us to get the property $\phi H\xi = 0$ for the shape operator A and hence it turns out that

$$A^2\xi = hA\xi + (A_2 - hA_1)\xi \quad \text{on} \quad M_0\,. \tag{2.12}$$

Combining the equation stated above together with (1.4), one finds

$$S\xi = S_1\xi\,, \qquad S_1 = (n-1)c/2 - A_2 + hA_1\,.$$

On the other hand, it is seen in [3] that under the cyclic Ryan condition, the following equation holds:

$$\begin{aligned} 2(n-1)\{SX - \eta(SX)\xi - \eta(X)S\xi\} - (r - S_1)X \\ + \{r + (2n-3)S_1\}\eta(X)\xi = 0\,, \end{aligned} \tag{2.13}$$

which means that M_0 is pseudo-Einstein, that is, $S = aI + b\eta \otimes \xi$ on M_0, where the coefficients a and b are given by $a = (r - S_1)/2(n-1)$ and $b = \{-r + (2n-1)S_1\}/2(n-1)$, and hence we have

$$4(A^2 - hA) = \{(2n+1)c - 4a\}I - (3c + 4b)\eta \otimes \xi\,. \tag{2.14}$$

Transforming the last equation by A, we obtain

$$4(A^3 - hA^2) = \{(2n+1)c - 4a\}A - (3c + 4b)\eta \otimes A\xi\,,$$

from which, by taking the skew-symmetric part, it is seen that

$$(3c + 4b)\{\eta(AX)\xi - \eta(X)A\xi\} = 0 \quad \text{on} \quad M_0\,.$$

So it turns out that $b = -3c/4$ and hence (2.14) is reduced to $4H = \{4a - (2n+1)c\}I$, from which together with (2.4) it follows that

$$\begin{aligned} \eta(Z)\{\eta(AX)AY - \eta(AY)AX\} + \eta(X)\{\eta(AY)AZ - \eta(AZ)AY\} \\ + \eta(Y)\{\eta(AZ)AX - \eta(AX)AZ\} = 0\,. \end{aligned}$$

For the orthonormal basis $\{E_j\}$ we put $X = \xi$ and $Z = E_j$ in the left hand side of the above equation, whose vector is denoted by $F(Y, E_j)$. By noting

$\eta(X) = g(\xi, X)$ and by taking account of $4A^3\xi = \{4h^2 - 4a + (2n+1)c\}A\xi - h\{4a - (2n+1)c\}\xi$, it follows from a straightforward calculation that

$$\begin{aligned} 4\sum g(F(Y, E_j), AE_j) &= [\eta(AY)\{4(-h^2 + h_2 - A_2 + hA_1 + a) - (2n+1)c\} \\ &\qquad + 2A_1\eta(Y)\{(2n+1)c - 4a + 2(h^2 - h_2)\}] \\ &= 0\,, \end{aligned}$$

where h_2 denotes the square of the norm of the second fundamental form. On the other hand, by the definition of the linear transformation H we have

$$\begin{aligned} 4(A_2 - hA_1) &= -4a + (2n+1)c\,, \\ 4(h_2 - h^2) &= -(2n-1)\{4a - (2n+1)c\}\,, \end{aligned}$$

and hence it turns out that

$$\{4a - (2n+1)c\}(A\xi - A_1\xi) = 0 \quad \text{on} \quad M_0\,,$$

which implies $4a = (2n+1)c$ and $S = c\{(2n+1)I - 3\eta \otimes \xi\}/4$, and hence we have $A^2 - hA = 0$.

The rank of A at a point x in M is called a *type number* and is denoted by $t(x)$. The above equation means that the type number $t(x)$ of any point x in M_0 is at most 1. It is however seen (cf. Yano and Kon [13]) that any point in M_0 is geodesic under the type number. So it is a contradiction to the fact that $(A_2 - A_1{}^2)(x) \neq 0$ at x in M_0. This implies that M_0 is empty, which yields that the structure vector ξ is principal. This completes the proof.

By taking account of the classification theorem of pseudo-Einstein real hypersurfaces of $M_n(c)$ by Cecil and Ryan [2] and Montiel [9], in order to prove Theorem 2.1 it suffices to show that M is pseudo-Einstein.

LEMMA 2.3. *Let M be a real cyclic Ryan hypersurface of $M_n(c)$, $c \neq 0$, $n \geq 3$. Then M is pseudo-Einstein.*

PROOF. Under the assumption of this lemma, Lemma 2.2 gives rise to $S\xi = S_1\xi$, $S_1 = (n-1)c/2 - A_1{}^2 + hA_1$, because of (1.4), while it is seen that (2.13) holds true on the whole on M. This implies that the Ricci tensor S satisfies $S = aI + b\eta \otimes \xi$, where the coefficients a and b are functions.

REMARK 2.2. By the classification theorem [2] and [9] of real pseudo-Einstein hypersurfaces of P_nC or H_nC, $n \geq 3$, it is seen that the coefficients a and b are constant, and then there are no real Einstein hypersurfaces of $M_n(c)$, $c \neq 0$, $n \geq 3$.

3. Harmonic Weyl tensor

In this section we are concerned with real hypersurfaces with harmonic Weyl tensor of $M_n(c)$, $c \neq 0$. Let (N, g_N) be an n-dimensional Riemannian manifold. The Ricci tensor S'_N can be regarded as a 1-form with values in the cotangent bundle T^*N. Then N is said to have *harmonic curvature* or *harmonic Weyl tensor*, if S'_N or $S'_N - r_N g_N/2(n-1)$ for the scalar curvature r_N is a Codazzi tensor, that is, it satisfies

$$dS'_N = 0 \quad \text{or} \quad d\{S'_N - r_N g_N/2(n-1)\} = 0\,,$$

where d denotes the exterior differential. For the harmonic Weyl tensor, it is seen that in the case of $n \geq 4$ the Weyl curvature tensor W which is regarded as a 2-form with values in the bundle $\Lambda^2 T^*N$ is closed and coclosed, namely, it is harmonic. In the case of $n = 3$ the Riemannian manifold N is conformally flat. For details, see Besse [1].

REMARK 3.1. By means of (2.3) it is easily seen that a Riemannian manifold with harmonic curvature is cyclic Ryan.

Now, let M be a real hypersurface with harmonic Weyl tensor of a complex space form $M_n(c)$, $c \neq 0$. First of all, the following lemma is proved.

LEMMA 3.1. *Let M be a real hypersurface with harmonic Weyl tensor of a complex space form $M_n(c)$, $c \neq 0$, $n \geq 3$. Then M is cyclic Ryan.*

PROOF. The Ricci tensor S' and the scalar curvature r satisfy by definition $d\{S' - rg/4(n-1)\} = 0$, that is,

$$\nabla_X S(Y) - \nabla_Y S(X) = \{dr(X)Y - dr(Y)X\}/4(n-1) \tag{3.1}$$

for any vector fields X and Y tangent to M. By substituting (3.1) into (2.3) and by calculating directly, the conclusion is given.

THEOREM 3.2. *There are no real hypersurfaces with harmonic Weyl tensor of $M_n(c)$, $c \neq 0$, $n \geq 3$.*

PROOF. For the real hypersurface with harmonic Weyl tensor of $M_n(c)$, Lemmas 2.2 and 2.3 show that M is pseudo-Einstein and the structure vector ξ is principal. In particular, by Remark 2.2, the coefficients a and b of the Ricci tensor are both constant and hence the scalar curvature r becomes constant. This means that M has harmonic curvature.

Differentiating the Ricci tensor S covariantly, we find

$$\nabla_X S(Y) = b\{\eta(Y)\nabla_X \xi + \nabla_X \eta(Y)\xi\}\,. \tag{3.2}$$

Since the hypersurface M has harmonic curvature, we have

$$b\{(g(\nabla_Y\xi, X) - g(\nabla_X\xi, Y))\xi + \eta(X)\nabla_Y\xi - \eta(Y)\nabla_X\xi\} = 0 . \tag{3.3}$$

Taking account of the second equation of (1.1) and $\phi\xi = 0$, we get $\nabla_\xi\xi = 0$, because the structure vector ξ is principal. If we put hence $Y = \xi$ in (3.3), then we obtain $b\nabla_X\xi = 0$, which together with (3.2) implies that the Ricci tensor is parallel. However, it is seen in [3] and [5] that there are no real hypersurfaces with parallel Ricci tensor of $M_n(c)$, $c \neq 0$, $n \geq 3$. A contradiction. Thus the proof is complete.

REMARK 3.2. It is in [4] and [7] seen that there are no real hypersurfaces with harmonic curvature of $M_n(c)$, $c \neq 0$, $n \geq 3$. Theorem 3.2 is the slight generalization, by which the last step of the proof can be omitted.

REMARK 3.3. The situation in Theorem 3.2 is quite different from that in hypersurfaces of a real space form. It is seen in [1] that there are Riemannian manifolds with harmonic Weyl tensor but not harmonic curvature. According to a theorem of Nishikawa and Maeda [10], a hypersurface of a conformally flat Riemannian manifold is also conformally flat if and only if any point of M is umbilic or it has two distinct principal curvatures one of which is simple, where a principal curvature is said to be *simple* if its multiplicity is equal to one. On the other hand, Otsuki [11] showed that there are many minimal hypersurfaces with the similar situation for principal curvatures of a real space form, which implies that they are conformally flat. Since a conformally flat hypersurface has harmonic Weyl tensor, it means that there are many minimal hypersurfaces with harmonic Weyl tensor of a real space form, which are not with harmonic curvature.

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